Study on Temperature-Pressure Gas-Liquid Coupled Field of LPG Tank Under Fire Based on Wavelet Finite Element Method

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Abstract. In order to analyze the changing rules of temperature and pressure of the gas-liquid coupled field of LPG (Liquefied Petroleum Gas) tank under fire, the application of the wavelet finite element method on it is studied in depth. First, the research achievements of thermodynamics response of the LPG tank under fire and the wavelet finite element theory is summarized. Second the temperature-pressure gas-liquid coupled field mathematical model of LPG tank under fire is established based on the gas-liquid coupled theory and the wavelet finite element theory. Finally, the simulation analysis of the temperature-pressure gas-liquid coupled field in LPG tank under inject flame fire is carried out by the wavelet finite element method, ANSYS software and experimental test respectively, and results show that the wavelet finite element method has higher calculating efficient and precision in analyzing the temperature-pressure gas-liquid coupled field in LPG tank under fire.

Keywords: Temperature; pressure; gas-liquid coupled field; LPG tank; wavelet finite element

1 Introduction

The liquefied petroleum gas is a clean-burning fuel which is easy to use and had a large calorific capacity, and it is important in the industrial production. The liquefied petroleum gas has the flammable and explosive property, and it is generally stored in the tank. The reserves of the liquefied petroleum gas are large, and the utilization ratio of it is high, and there is a more dangerous in the stored procedure. When leakage of the LPG tank occurs, the liquefied petroleum gas escaped will do great harm to the surroundings, and the fire accident will happen in severe cases. In recent years, the explosion accidents of LPG tank occur repeatedly, which cause a large quantity of casualties and pecuniary loss, and 40% of the explosion accidents are caused by the fire. The temperature-pressure gas-liquid coupled system model of the LPG tank is shown in figure 1.

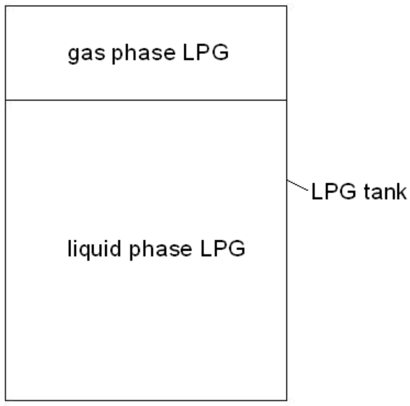

Fig. 1. The temperature-pressure gas-liquid coupled system model of the LPG tank

The temperature and pressure of the gas and liquid phase liquefied petroleum gas in LPG tank improve rapidly under the fire, and the strength of the wall of the LPG tank will reduce sharply. When a certain condition is reached, the fracture of the LPG tank will occur, and the leakage will generate. In case of naked fire the liquid phase liquefied petroleum gas blows out that will catch fire to cause fire accident. Especially when the whole crack of the LPG tank happens, and steam explosion will come into being, therefore it is every significant to prevent the fire and explosion accident of the LPG tank under fire, and the reliability and service life will improve. Under fire environment, the temperature and pressure of the gas and liquid LPG affect the danger of the

LPG tank, and there are many affecting factors such as intensity of the flame, concentration of the flue gas, and wind direction. It is necessary to establish the temperature-pressure gas-liquid coupled model of the LPG, and the changing rules of the physical quantity in the coupled field will be obtained, and the explosion of the LPG tank under fire environment will be avoided effectively. In order to improve the solution efficiency of gas-liquid coupled field analysis, and the advanced numerical simulation technology should be applied, and the coupled mechanism of the temperature-pressure gas-liquid coupled mechanism of the LPG under fire can be grasped clearly, and the explosion danger of the LPG tank will be predicted, and the theory basis of the explosion-proofed design of the LPG tank will be provided.

2 Research review of the related research

2.1 The research review of the thermodynamics response of the LPG tank under fire

In recent years, the heat transfer rules of the LPG tank under fire had been become the focus of research, and some good achievement has been acquired. The thermodynamic design charts to promote a quick computation step to decide the pressurizing period of a transport LPG tank. The pressure rise because of the boil-off phenomena is decided based on this methodology, and the pressurizing period is computed based on the charts provided [1]. A methodology for assessing the performance of the passive fire protection was proposed, and the specific experiment and numerical investigation are carried out, and the comparison was accomplished for protected and non protected tanks which evaluate the possible tank damage because of the thermal exposure [2]. A CFD model was applied in simulating the reported consequences of a recent major accident involving an LPG railcar rupture in Viareggio town, results show that both the large effect of the barriers on LPG dispersion as well as the potentials of CFD models to predict such an effect [3]. A thermal response model of LPG tank under fire is established, and the potential hazard probabilities under different fire conditions were studied according to the maximum wall temperature and media energy after the internal pressure rose to the same value. Results show that the less severe fire scenario and lower engulfing case can cause a greater probability of burst hazard because maximum wall temperature and media energy are higher [4]. A CFD-based approach was presented based on all kinds of models available for evaluating turbulence, the realizable k-ε model which is certificated by the experimental findings. The effectiveness of the method was demonstrated through simulating the dispersion and ignition of a typical vapor cloud formed for a spill from a liquid petroleum gas (LPG) tank located in a refinery. The simulation made it possible to evaluate the overpressures coming from the combustion of the flammable vapor cloud [5]. As seen from the existing research achievements, the thermal response of LPG tank under fire was studied from different angles, and the early-warning mechanism of explosion-proof of LPG tank was discussed. However the temperature-pressure gas-liquid coupled field of LPG tank under fire has not been reported so far, therefore it is significant to study the coupled mechanism of the temperature-pressure gas-liquid, and it is very difficult to study it based on the physics experiment because the fire environment around the LPG tank is difficult to be controlled. Then the numerical technology has been a critical method to study the temperature-pressure gas-liquid coupled field of the LPG tank under fire. Under fire the temperature and pressure of the gas phase and liquid phase LPG change nonlinearly, and the numerical simulation based on traditional finite element method will produce big error, therefore a advanced numerical technology should be chosen to over the disadvantages of the traditional finite element method, and the wavelet finite element method has the advantages on coping with the nonlinear problem, and the calculating efficient and correctness of coupled field analysis based on wavelet finite element method will improve.

2.2 The research review of the wavelet finite element method

Coupled mechanism of temperature-pressure gas-liquid field of the LPG tank under fire should be studied to grasp the status of the LPG tank and establish the emergency measure under fire, and the wavelet finite element method is an effective method for studying it. The wavelet finite element method is a developing numerical method, comparing with the traditional finite element method; this new method applies the scale function or wavelet function as interpolating function instead of polynomial. The multi-resolution analysis property of the wavelet is used to construct the different forms of basic function, and the corresponding basic function can be chosen according to the requirement of the different precision, and the calculating precision will be improved. The temperature changing rule of the crude oil in the storage tank was studied based on the wavelet finite element method, and the computational model and boundaries were established, and temperature distribution of the crude oil in the storage tank under different conditions was obtained, and the temperature changing mechanism of the crude oil was found out [6]. The temperature distribution of the conventional and ceramic coating diesel engine piston was analyzed respectively, and the comparison between the calculation results obtained from the wavelet finite element method and the experiment results are carried out, and the results show that the wavelet finite element was convergent and had higher analysis precision, and it can avoid the numerical oscillation though the results of the transient-state temperature fields of the diesel piston [7]. The thermal stress distribution of ceramic-coated piston is analyzed based on wavelet finite element method and the thermal stress distributions of the conventional and ceramic-coated diesel engine pistons were obtained, respectively, and results show that this method has advantage of studying the high gradient problems [8]. A new method is present to identify crack location and size based on the finite element method of second generation wavelets, and the pipe structure is dispersed into a series of nested thin-wall pipes, a new calculation method of crack equivalent stiffness is proposed to solve the stress intensity factor of the pipe structure, and the dynamic model of cracked pipe is established. And the efficiency of the presented method is verified by experiments [9]. The multivariable BSWI elements with two kinds of variables (TBS WI) for open cylindrical shell were established based on the generalized principle and B-spline wavelet on the interval (BS WI). Different from the conventional method, the generalized displacement, and stress was considered as independent variables. And differentiation and integration were avoided in computing the generalized stress and the calculating precision was improved [10]. The wavelet finite element method has a fast convergence rate and high calculating precision, and has a strong competitive advantage in solving nonlinear problems. Therefore the wavelet finite element coupled model of temperature-pressure gas-liquid of LPG tank under fire is constructed, and the coupled mechanism of temperature-pressure gas-liquid of LPG tank can be found out, and the prediction of danger and anti-explosive early-warning mechanism can be obtained.

3 Temperature-pressure gas-liquid coupled field mathematical model of LPG tank under fire

VOF (Volume of Fluid) model follows the free fluid level; it can simulate the movement of gas-liquid flow through solving the continuity and momentum equations. The VOF model is expressed as follows [11]:

$$\frac{\partial \alpha_g}{\partial t} + u_i \frac{\partial \alpha_g}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \alpha_l}{\partial t} + u_i \frac{\partial \alpha_l}{\partial x_i} = 0 \tag{2}$$

where α_g and α_l denote the volume fraction of the gas and liquid phase LPG, u_i denotes the velocity component, x_i denotes coordinate, $i = x, y$.

$$\rho = \alpha_l \rho_l + \alpha_g \rho_g \tag{3}$$

$$\mu = \alpha_l \mu_l + \alpha_g \mu_g \tag{4}$$

$$\mu_T = \alpha_l \mu_{Tl} + \alpha_g \mu_{Tg} \tag{5}$$

Momentum equation can be expressed as follows [12]:

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla(\rho \vec{v}\vec{v}) = -\nabla p + \nabla[\mu(\nabla \vec{v} + \nabla \vec{v}^T)] + \rho G + \vec{F} \tag{6}$$

Energy equation can be expressed as follows [13]:

$$\frac{\partial (\rho E)}{\partial t} + \nabla[\vec{v}(\rho E + p)) = \nabla[k_{eff} \nabla T) + S_h \tag{7}$$

where S_h denotes the volumetric heat source.

When the condition $\alpha_1 = 100\%$ is satisfied, the equations above correspond to liquid-phase. While the condition $\alpha_2 = 100\%$ is satisfied, the equations above correspond to gas-phase.

$RNG\ k-\varepsilon$ model is established based on renormalization group theory of Navier-Stokes equation, and the effect of average strain rate is considered, it can be applied to analyze the turbulent flow. The transport model of the turbulent kinetic energy is expressed as follows [14]:

$$\frac{\partial k}{\partial t}+\frac{\partial(ku_i)}{\partial x_i}=\frac{1}{\rho}\left(\mu+\frac{\mu_T}{\sigma_k}\right)\frac{\partial}{\partial x_j}\left(\frac{\partial k}{\partial x_j}\right)+\frac{G_k+S_k}{\rho}-\varepsilon \tag{8}$$

The transport model of turbulent dissipation rate is expressed as follows:

$$\frac{\partial k}{\partial t}+\frac{\partial(\varepsilon u_i)}{\partial x_i}=\frac{1}{\rho}\left(\mu+\frac{\mu_T}{\sigma_\varepsilon}\right)\frac{\partial}{\partial x_j}\left(\frac{\partial \varepsilon}{\partial x_j}\right)+C_1\frac{\varepsilon}{k}G_k-C_2\frac{\varepsilon^2}{k} \tag{9}$$

where G_k denotes the turbulent kinetic energy term，which is expressed as follows:

$$G_k=2\mu_T D_{ij}D_{ij} \tag{10}$$

where D_{ij} denotes average strain rate tensor, which is expressed as follows:

$$D_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{11}$$

4 Wavelet finite element model of temperature-pressure gas-liquid coupled field in LPG tank under fire

Multi-resolution subspaces $\{S_1\}$ and $\{S_2\}$ can be generated by Daubechies wavelet scale functions $\phi^1(\alpha)$ and $\phi^2(\beta)$ respectively. And the higher order spaces can be generated through the tensor product of the sub-spaces $\{S_1\}$ and $\{S_2\}$, which can be expressed as follows [15]:

$$S=S_1\otimes S_2 \tag{12}$$

The Hilbert space $L^2(R^2)$ can be formed by space $\{S\}$, and the corresponding expressions of $\{S_1\}$ and $\{S_2\}$ can be expressed as follows:

$$\vec{\phi}^1=\{\phi^1(\alpha),\phi^1(\alpha+1),\cdots,\phi^1(\alpha+(N-2))\} \tag{13}$$

$$\vec{\phi}^2=\{\phi^2(\beta),\phi^2(\beta+1),\cdots,\phi^2(\beta+(N-2))\} \tag{14}$$

The scale function in space $\{S\}$ is expressed as follows:

$$\vec{\phi}=\vec{\phi}^1\otimes\vec{\phi}^2 \tag{15}$$

The temperature-pressure gas-liquid coupled field of LPG in tank can be simplified as two two-dimensional models, and the solution domain of coupled field can be divided into several finite elements, and physical quantity of any nodes can be got using Daubechies wavelet scale function as interpolating function, and the temperature and pressure can be formulated by the following expressions respectively [16]:

$$T(\alpha,\beta)=\vec{\phi}\cdot\vec{a}_e=\sum_{i,j=0}^{N-2}\phi_i(\alpha)\phi_j(\beta)\cdot\vec{a}_e \tag{16}$$

$$p(\alpha,\beta)=\vec{\phi}\cdot\vec{b}_e=\sum_{i,j=0}^{N-2}\phi_i(\alpha)\phi_j(\beta)\cdot\vec{b}_e \tag{17}$$

where $\vec{a}_e$ and $\vec{b}_e$ are the corresponding vector of the wavelet filter coefficients, which can be expressed as follows:

$$\vec{a}_e = [a_0, a_1, \cdots, a_{-(N-2)}]^T \tag{18}$$

$$\vec{b}_e = [b_0, b_1, \cdots, b_{-(N-2)}]^T \tag{19}$$

According to the energy equation, the functional equation can be expressed as follows [17]:

$$\Pi(T,p) = \iint_S (\frac{\partial(\rho E)}{\partial t} + \nabla[\vec{v}(\rho E + p)) - \nabla[k_{eff}\nabla T) + S_h)dxdy \tag{20}$$

where S denotes the computational domain of the temperature-pressure coupled field of LPG in tank.

To make the wavelet finite element agree with $\vec{a}^e$ the node configuration is established for the hexahedron element. The relationship between the local coordinate (α, β) and the global coordinate (x, y) can be expressed as follows:

$$\alpha = \frac{x - x_1}{x_2 - x_1} \tag{21}$$

$$\beta = \frac{y - y_1}{y_2 - y_1} \tag{22}$$

where x_1 and x_2 are the minimum and maximum coordinate value of the hexahedron element in x direction respectively, y_1 and y_2 are the minimum and maximum coordinate value of the hexahedron element in y direction respectively.

The functional equation of the wavelet finite element can be expressed as follows:

$$\Pi^e(T,p) = \iint_{Se} (\frac{\partial(\rho E)}{\partial t} + \nabla'[\vec{v}(\rho E + p)) - \nabla'[k_{eff}\nabla' T) + S_h)d\alpha d\beta \tag{23}$$

where $\nabla' = (\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2})$, e denotes the wavelet finite element.

The differential equation of the two dimension thermal response can be obtained through the condition $\delta\Pi^e(T,p) = 0$, and the corresponding expression can be expressed as follows:

$$\iint_{Se} (\frac{\partial W}{\partial \alpha}\frac{\partial p}{\partial \alpha} + \frac{\partial W}{\partial \beta}\frac{\partial p}{\partial \beta}) - k_{eff}(\frac{\partial W}{\partial \alpha}\frac{\partial T}{\partial \alpha} + \frac{\partial W}{\partial \beta}\frac{\partial T}{\partial \beta})d\alpha d\beta = 0 \tag{24}$$

where W denotes the weight function. Applying the interpolating function as weight function can get the following equation:

$$\vec{D}^e\vec{a}^e + \vec{F}^e\vec{a}^e = \vec{G}^e \tag{25}$$

where $\vec{a}^e$ denotes the column vector of unknown wavelet coefficient, $\vec{D}^e$, $\vec{F}^e$ and $\vec{G}^e$ denote the characteristic matrix corresponding to the temperature or pressure, and the corresponding expression can be expressed as follows [18]:

$$\vec{D}^e_{p,q,h,l} = \int_0^1\int_0^1 \Phi_{p,q}\Phi_{h,l}d\alpha d\beta \tag{26}$$

$$\vec{F}^e_{p,q,h,l} = \int_0^1\int_0^1 (\frac{\partial \Phi_{p,q}}{\partial \alpha}\cdot\frac{\partial \Phi_{h,l}}{\partial \alpha} + k_{eff}\frac{\partial \Phi_{p,q}}{\partial \alpha}\cdot\frac{\partial \Phi_{h,l}}{\partial \alpha})d\alpha d\beta \tag{27}$$

$$\vec{G}^{e}_{p,q,h,l} = \int_0^1 \int_0^1 (\Phi_{p,q} + k_{eff}\Phi_{h,l}) d\alpha d\beta \tag{28}$$

where $\Phi_{p,q} = \phi_p \phi_q, \Phi_{h,l} = \phi_h \phi_l$.

The global differential equation of thermal response can be constructed through superimposing the wavelet finite element matrix and handling the boundary conditions, which can be expressed as follows:

$$\vec{D}\vec{T} + \vec{F}\vec{T} = \vec{G} \tag{29}$$

$$\vec{D}\vec{p} + \vec{F}\vec{p} = \vec{G} \tag{30}$$

where $\vec{D}$, $\vec{F}$ and $\vec{G}$ denotes the global characteristic matrix of thermal response of the gas and liquid LPG in tank.

The algorithm of calculating the temperature and pressure of the gas or liquid LPG can be expressed as follows:

$$\frac{1}{2}\left(\left.\frac{\partial T}{\partial t}\right|_t + \left.\frac{\partial T}{\partial t}\right|_{t-\Delta t} \right) = \frac{T_t - T_{t-\Delta t}}{\Delta t} + O(\Delta t^2) \tag{31}$$

$$\frac{1}{2}\left(\left.\frac{\partial p}{\partial t}\right|_t + \left.\frac{\partial p}{\partial t}\right|_{t-\Delta t} \right) = \frac{p_t - p_{t-\Delta t}}{\Delta t} + O(\Delta t^2) \tag{32}$$

where Δt denotes the time step, $O(\Delta t^2)$ denotes the truncation error. The simultaneous equation of the formula (29) and formula (31) in t and $t-\Delta t$ can be expressed as follows:

$$\left(\vec{D} + \frac{\vec{F}}{\Delta t} \right) \vec{T}_t = \left(\vec{G}_t + \vec{G}_{t-\Delta t} \right) + \left(\frac{2\vec{F}}{\Delta t} - \vec{D} \right) \vec{T}_{t-\Delta t} \tag{33}$$

The simultaneous equation of the formula (30) and formula (32) in t and $t-\Delta t$ can be expressed as follows:

$$\left(\vec{D} + \frac{\vec{F}}{\Delta t} \right) \vec{P}_t = \left(\vec{G}_t + \vec{G}_{t-\Delta t} \right) + \left(\frac{2\vec{F}}{\Delta t} - \vec{D} \right) \vec{P}_{t-\Delta t} \tag{34}$$

The temperature and pressure of the gas or liquid LPG can be obtained according to formula (33) and formula (34).

5 Simulation analysis of the temperature-pressure gas-liquid coupled field in LPG tank under fire

In order to verify the correctness of the wavelet finite element analysis and the test of temperature and pressure of the gas or liquid state LPG in tank under fire is designed, and the test platform is shown in figure 2.

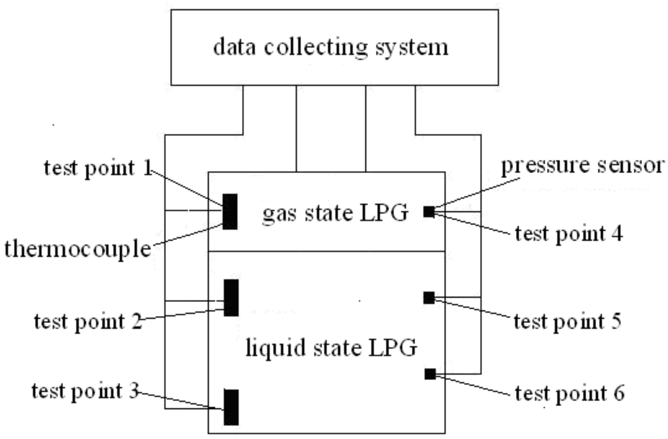

Fig. 2. The test system diagram of the temperature pressure gas-liquid state coupled field of LPG tank under fire

The temperature of test points in the LPG tank under fire chosen can be obtained through thermocouple, and the pressure of the test points in the LPG tank under fire chosen can be obtained through pressure sensor, and the experimental data can input into the data collecting system, and then the processing procedure of the collecting data is carried out.

And the numerical simulation is carried out by wavelet finite element method and ANSYS software respectively. The wavelet finite element analysis program is made by MATLAB software, and the computational domain is meshed by Daubechies wavelet finite elements and FLUID141 in ANSYS software respectively. And the kind of fire environment is jet flame fire.

And then the numerical analysis of the temperature-pressure gas-liquid coupled field of LPG tank under fire is carried out. The calculating conditions are listed as follows: the height of the stand tank is 1m, and the diameter of the base surface of the LPG tank is 0.2m, and the fill level is 70%, and the fire kind is jet flame, the fuel is LPG, and its mass flow is 85kg/s. The injecting location locates at the bottom of the LPG tank, and the diameter of the spray nozzle is 0.01m. The original temperature is 295K, and the original pressure is 0.66MPa. The interior media is Propane. The solution domain is meshed by 56 wavelet finite elements and 245 FLUID141 elements in ANSYS software.

The changing rules of the temperature of the media in LPG tank is shown in figure 3 when the increasing of the height of the LPG tank.

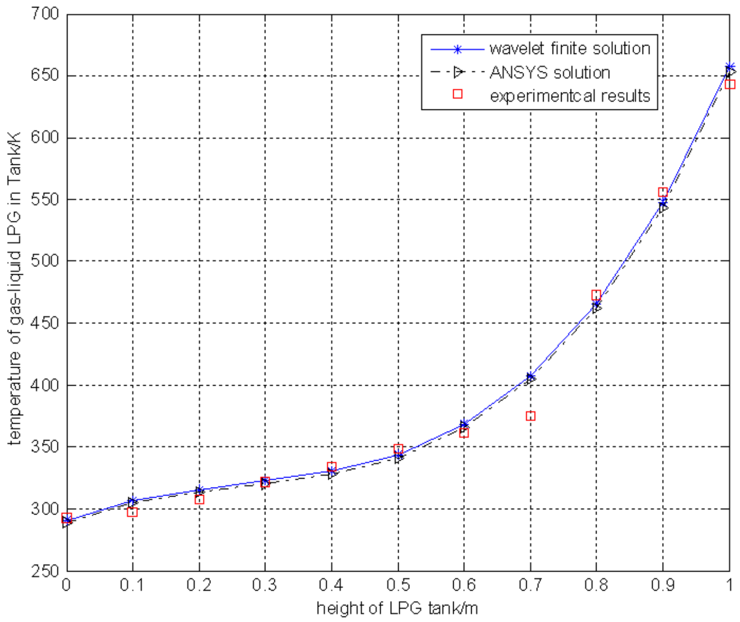

Fig. 3. The changing rules of the temperature of the media in LPG tank

As can be seen from figure 3 the temperature of the gas or liquid state LPG increases with increasing of the height of LPG tank. This results show that the temperature stratification is obvious. And the temperature stratification level of the liquid state LPG is bigger.

The temperature and pressure of the gas-liquid LPG in tank under inject fire is calculated, and the three points (test point 1, test point 2 and test point 3) which are shown in figure 2 are chosen. And the simulation results are shown in figure 4 and figure 5.

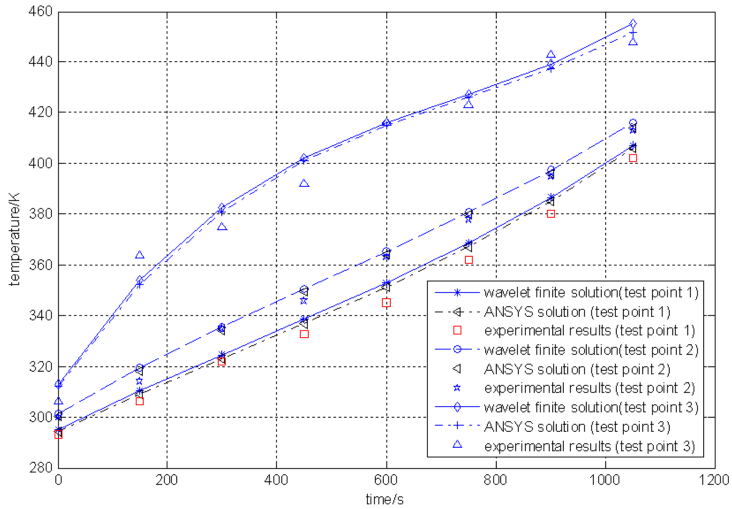

Fig. 4. The temperature changing curves of the gas-liquid LPG with time.

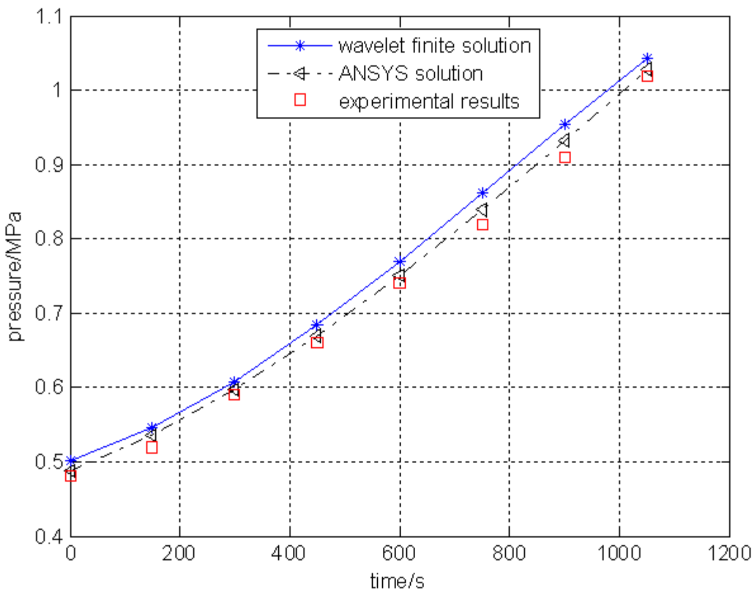

Fig. 5. The pressure response of the gas-liquid coupled field of LPG tank with time

Figure 4 show the temperature changing curves of the gas-liquid LPG with time. As can be seen from figure 4, the maximum temperature of the liquid LPG exists on the vapor-liquid interface. And the temperature of the liquid LPG almost increased linearly. The temperature of the gas state LPG increases quickly before 400s, and then the temperature of the gas state LPG increases slowly.

Figure 5 show the pressure response of the gas-liquid coupled field of LPG tank with time, as can be seen from figure 5, the pressure of the coupled field of LPG tank increase with time.

According to the above results, the solution form the wavelet finite element method is closer to the experimental results; these results show that the wavelet finite element method has higher calculating precision. At the same time the wavelet finite element method uses less finite elements than the FLUID141 elements in ANSYS software, so the wavelet finite element method has also a higher calculating precision. It is effective to apply the wavelet finite element method in the analysis of the temperature-pressure gas-liquid coupled field of LPG tank under fire.

6 Conclusions

The temperature-pressure gas-liquid coupled wavelet finite element of LPG tank under fire is established based on the wavelet finite element method and gas-liquid coupled theory. The changing rules of the temperature and pressure of the gas or liquid LPG in tank are obtained by the wavelet finite element method, ANSYS software, and experimental test. And results show that the results from wavelet finite element method agree with the experimental results, and the wavelet finite element method has a better calculating efficient and gets a higher calculating precision in analyzing the temperature-pressure gas-liquid coupled field of LPG tank under fire.

Acknowledgments

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