Complex Stress Field of LPG Tank under Fire Based on Sobolev Orthogonal Wavelet Finite Element Method

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Abstract. In order to analyze the complex stress field of LPG tank under fire, the application of Sobolev orthogonal wavelet finite element method on it is studied in depth. First, Sobolev wavelet basis function is established, the B-spline wavelet function with the property of multi-resolution analysis is used as scale function, and the wavelet function and basis function are obtained according to the Sobolev orthogonal wavelet theory. Second, the complex stress wavelet finite element model of LPG tank wall is established by using the Sobolev wavelet basis function as the interpolating. Finally, the simulation analysis based on traditional finite element method and the traditional finite element method are carried out, and the simulation results are verified by the test value, the stress changing rules of LPG tank wall is obtained, numerical results offer good theoretical basis for fire prevention. In addition, the wavelet finite element method can obtained higher computing precision based on the comparison of results obtained from simulation analysis and test.

Key words: stress field, LPG tank, fire, wavelet finite element method

1 Introduction

Liquefied petroleum gas (LPG) is a clean fuel with high convenience and big calorific value, which has very broad applications in industry and civil. Main components of LPG consist of C3H8, C3H6, C4H10, and C4H8, therefore LPG is an inflammable and explosive dangerous good. Normally, LPG is stored in storage tank, and there are dangers in storing procession. Especially in the fire LPG tank may explode. Under fire the temperature of LPG tank wall can increase quickly, and the pressure and temperature and pressure of the liquid and gas phase LPG can increase quickly, and the stress of the LPG tank wall will increase accordingly under the effect of thermal and pressure of gas and liquid phase LPG. If the stress of LPG tank wall reaches the strength limit of the LPG tank wall substance, the tank will rupture, and the LPG will leak from the broken LPG tank, and the fire and even explosion will happen. Therefore it is necessary to study the changing rules of the complex stress of LPG tank wall under fire in depth, and stress analysis results will offer strong theoretical basis for avoiding the fire and explosion of LPG tank.

The complex stress of LPG tank under fire changes nonlinearly because the thermal stress of LPG tank wall and the pressure of LPG are non-linear. The stress test of LPG based on field test has difficulty in being carried out because the parameters of fire has the great uncertainty, therefore the numerical simulation technique is a effective tool for studying the complex stress of LPG tank wall under fire, the traditional finite element method can obtained a major error in solving nonlinear problems, an advanced numerical technique should be applied to make up the disadvantages of traditional finite element method. The wavelet finite element method is a developing technology in recent years which has a stronger advantage on solving the nonlinear problem, wavelet finite element method can be used to analyze the changing rules of complex stress of LPG tank wall under fire. In order to improve the calculating precision and save the simulation time of complex stress analysis for LPG tank under fire, the Sobolev orthogonal wavelet finite element method is applied.

2 Construction of wavelet basis function

The scale function $\phi(x)$ and the corresponding wavelet function $\psi(x)$ of multi-resolution analysis has good analysis and calculation property, the scale function or wavelet function can be used as basis function to establish the finite element, and the mathematical model can be solved based on this wavelet finite element method. Wavelet finite element method has been concerned by some scientists, and it has been applied in many engineering fields. Xuefeng Chen et al. presented an application of B-spline wavelet on interval (BSWI) finite element for one-dimension elastic wave propagation problems, the cracked rod and cracked beam BSWI element were constructed, and numerical examples verified the accuracy and efficiency of BSWI element [1]. Zhibo Yang et al. solve the free vibration and buckling problems of plates based on B-spline wavelet finite element method (BSWI), and numerical examples demonstrate that the present BSWI method achieves the high accuracy compared to the exact solution and others existing approaches in the literatures [2]. Bin Zhao studied the temperature changing rule of the crude oil in the storage tank based on the wavelet finite element method, the temperature distribution rule of the crude oil in the storage tank under different boundary conditions are obtained, the temperature changing mechanism of the crude oil was summarized finally [3]. Wen-Yu He presented the trigonometric wavelet finite beam element to analyze the bending, free vibration and buckling of beam structures, numerical examples demonstrated that the trigonometric wavelet finite element method could achieve a good accuracy with less element adopted, especially for free vibration analysis [4]. As seen from the published research results, the Daubechies wavelet, B-spline wavelet and trigonometric wavelet are used as the interpolating function to establish the finite element, in order to reduce the computing workloads, the Sobolev Orthogonal Wavelet is applied in this research, and the complex problem can be divided into several single subproblems that be solved based parallel computing.

The multi-resolution analysis is a series of subspaces $\{V_j\}_{j\in Z}$ in $L^2(\vec{R})$, which satisfied the following conditions [5]:

(1) $V_j \subset V_{j+1}$ and $f(x)\in V_j$ if and only if $f(2^{-j}x)\in V_0$

(2) $\bigcap\limits_{j\in Z} V_j = \phi$, $\overline{\bigcup\limits_{j\in Z}} V_j = L^2(\vec{R})$

Because B-spline satisfies the conditions of multi-resolution analysis, the B-spline function is used as scale function $\varphi(x)$, and the corresponding expression is listed as follows:

$$\varphi(x) = N_m(x) \tag{1}$$

where $N_m(x)$ denotes the m order B spline function, which is expressed as follows:

$$N_m(x) = \sum_{j=0}^{m+1} \frac{(-1)^j}{m_j} \binom{m+1}{j} \left[x + \frac{m+1}{2} - j \right]^m, m \geq 1 \quad (2)$$

Different Sobolev orthogonal wavelet function $\psi(x)$ can be obtained according to the following expressions in different level j [6]:

$$\sum_{m=0}^{S} \int_{-\infty}^{\infty} \lambda_m \varphi^{(m)}(x-k) \psi_{j,l}^{(m)}(x) dx = 0, j,k,l \in Z \quad (3)$$

$$\sum_{m=0}^{S} \int_{-\infty}^{\infty} \lambda_m \psi_{j,k}^{(m)}(x) \psi_{j',k'}^{(m)}(x) dx = 0, j \neq j'; j, j', k, k' \in Z \quad (4)$$

The interval periodic procession is carried out for $\varphi(x)$ and $\psi_{j,k}(x)$, and the corresponding expressions are shown in the following:

$$\varphi^*(x) = \chi_{[a,b]}(x) \sum_m \varphi(x + m(b-a)) \quad (5)$$

$$\psi_{j,k}^*(x) = \chi_{[a,b]}(x) \sum_m \psi_{j,k}(x + m(b-a)) \quad (6)$$

According to formulas (3) and (4), the following expressions can be obtained:

$$< \varphi(x-k), \psi_{j,k}^*(x) >_S = 0, \ i, j \in Z \quad (7)$$

$$< \psi_{j,k}^*(x), \psi_{j',k'}^*(x) >_S = 0, j \neq j'; j, j', k, k' \in Z \quad (8)$$

Set $V_j = span\{\varphi_{j,k}^*(x)\}$ and $W_j = span\{\psi_{j,k}^*(x), k \in Z\}$, $j,k \in Z$ and the following relations are satisfied:

$$\cdots \subset V_0 \subset V_1 \subset \cdots \subset V_n \subset \cdots \quad (9)$$

$$V_n = V_0 \oplus W_0 \oplus W_1 \oplus \cdots \oplus W_{n-1} \quad (10)$$

The Sobolev orthogonal wavelet basis is expressed as follows:

$$\phi = \{\varphi^*(x-k), \psi_{j,k}^*(x)\}, \ 0 \leq j < n, k \in Z \quad (11)$$

The subspaces $\{V_j^1\}$, $\{V_j^2\}$, and $\{V_j^3\}$ are generated by wavelet basis function $\phi^1(\varepsilon)$, $\phi^2(\eta)$ and $\phi^3(\gamma)$, and the higher level subspace is generated based on the tensor product of the above subspaces, which is expressed as follows:

$$V_j = V_j^1 \otimes V_j^2 \otimes V_j^3 \tag{12}$$

3 Complex stress wavelet finite element model of LPG tank wall

Under fire the linear strain of LPG tank wall can produce for the thermal expansion, and the complex stress analysis of the LPG tank wall belongs to the three-dimensional problem, the original strain ε_0 is calculated by the following expression [7]:

$$\varepsilon_0 = \alpha(T - T_0)\begin{bmatrix}1 & 1 & 1 & 0 & 0 & 0\end{bmatrix}^T \tag{13}$$

where α denotes the thermal expansion coefficient, $1/K$; T_0 denotes the original temperature field of LPG tank wall, K; T denotes the transient temperature field, K.

The interpolating function is expressed as follows:

$$W_{m,n,l} = \phi_m^1(\varepsilon) \otimes \phi_n^2(\eta) \otimes \phi_l^3(\gamma) = \Phi_{m,n,l} \tag{14}$$

The transient temperature field T can be obtained through the interpolation of nodal temperature of element, and the corresponding expression is expressed as follows:

$$T = \Phi_{m,n,l} T^e \tag{15}$$

where T^e denotes the nodal temperature of element, K.

The stress-strain relation is expressed as follows:

$$\vec{\sigma} = D(\vec{\varepsilon} - \vec{\varepsilon}_0) \tag{16}$$

where D denotes the elastic coefficient matrix.

The functional expression is expressed as follows [8]:

$$\Pi^e(\vec{u}) = \iiint_{V^e} (\frac{1}{2}\vec{\varepsilon}^T D\vec{\varepsilon} - \vec{\varepsilon}^T D\vec{\varepsilon}_0 - \vec{u}^T \vec{f})dV - \iint_{\Gamma^e} \vec{u}\vec{F}d\Gamma \tag{17}$$

where $\vec{u}$ denotes the motion vector; $\vec{f}$ denotes body force vector; $\vec{F}$ denotes the surface force vector.

And the corresponding finite element formulation can be obtained through setting $\Pi^e(\vec{u}) = 0$, which is expressed as follows:

$$\vec{K}^e \vec{q}^e = \vec{P}^e \tag{18}$$

where $\vec{K}^e$ denotes stiffness matrix, $\vec{K}^e = \sum_e \iiint_{V^e} \vec{B}\vec{D}\vec{B}dV$, $\vec{B}$ denotes the geometry matrix, $\vec{q}^e$ denotes the undetermined coefficient, $\vec{q}^e = \{u^e, v^e, w^e\}^T$, $\vec{P}^e$ denotes the load vector, which is expressed as follows:

$$\vec{P}^e = \vec{P}_v + \vec{P}_T + \vec{P}_p \quad (19)$$

where $\vec{P}_v$ denotes the body load N; $\vec{P}_T$ denotes temperature load, $\vec{P}_T = \sum_e \iiint_{V^e} \vec{B}^T \vec{D}\vec{\varepsilon} dV$ N; $\vec{P}_p$ denotes the pressure of gas and liquid phase LPG.

The global finite element formulation can be obtained through superimposing the element matrixes, which is expressed as follows [9]:

$$\vec{K}\vec{q} = \vec{P} \quad (20)$$

The Sobolev orthogonal wavelet finite element can be applied in the problem with high gradient, nonlinearity and singularity, and this simulation method can be applied in the complex stress analysis of LPG tank under fire.

4 Complex stress simulation of LPG tank under fire

4.1 Simulation parameters

The structure sizes of the LPG tank that is used to carry out complex stress analysis are listed as follows: the height of the LPG tank is 812mm, the inner diameter of the LPG tank is 400mm, and the thickness of the LPG tank is 6mm. The filling rate of LPG tank is 50%. The performance parameters of LPG tank wall substance are listed as follows: the thermal conductivity is 50 W, the heat capacity is 460 $J/kg \cdot K$, the density is 7850 kg/m^3, the Poisson's ratio is 0.3, the elastic modulus is 201 GPa. The pool fire is used in this research, and the corresponding parameters are listed as follows: the radiation intensity is 110000 W/m^2, the initial temperature condition is 285 K.

4.2 Test design

In order to verify the effectiveness of this method, the test is carried out, and three testing points are chosen, and the stress sensor is fixed on the testing point, and the stress data are collected through data collector, and schematic of test equipment is shown in Fig. 1.

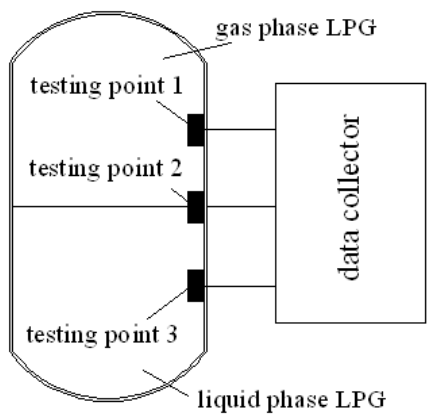

Fig. 1. Schematic of test equipment

4.3 Mesh generation

In addition, the traditional finite element method is applied in computing the complex stress of LPG tank under fire, and the simulation analysis is carried out based on ANSYS software, the finite element model is shown in Fig. 2, which consists of 19824 elements and 26440 nodes. The wavelet finite element program is compiled by MATLAB software, and the wavelet finite element model concludes 3430 elements and 5352 nodes.

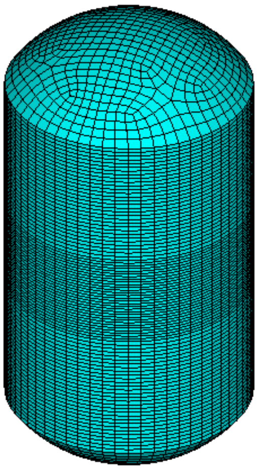

Fig. 2. Finite element model of LPG tank

4.4 Simulation results

Complex stress analysis is carried out based on the thermal response analysis and the changing rules of temperature and pressure of LPG is used as boundary conditions of complex stress analysis.

The stress distribution diagram of LPG tank wall after 400s is shown in Fig. 3, as seen from Fig. 3, the stress of LPG tank wall near the gas-liquid interface is highest, and the stress of LPG tank wall near gas phase LPG is higher than that of LPG tank wall near liquid phase LPG, because the temperature gradient of gas-liquid interface is highest, and the temperature and pressure of gas phase LPG increase faster than that of liquid phase LPG.

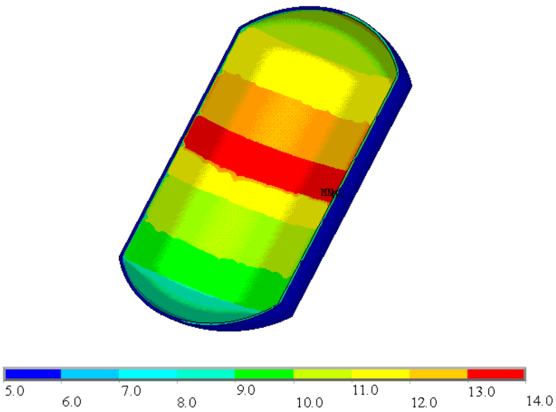

Fig. 3. Stress distribution diagram of LPG tank wall after 400s Stress changing rules of testing point 1, testing point 2 and testing point 3 on LPG tank wall are shown in Fig. 4, Fig. 5 and Fig. 6 respectively.

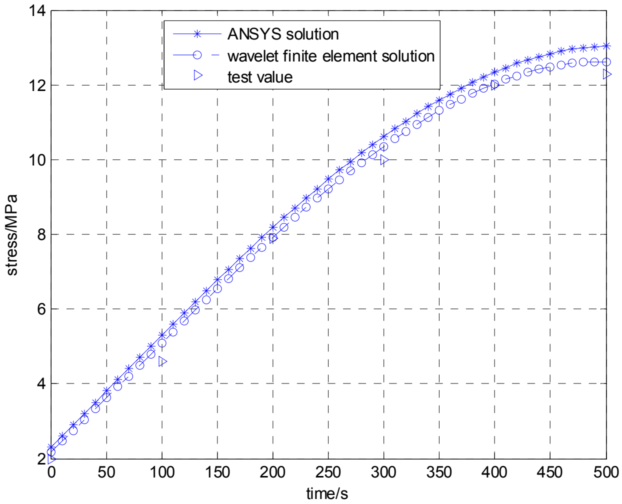

Fig. 4. Stress changing curves of testing point 1 from ANSY, wavelet finite element and experiment

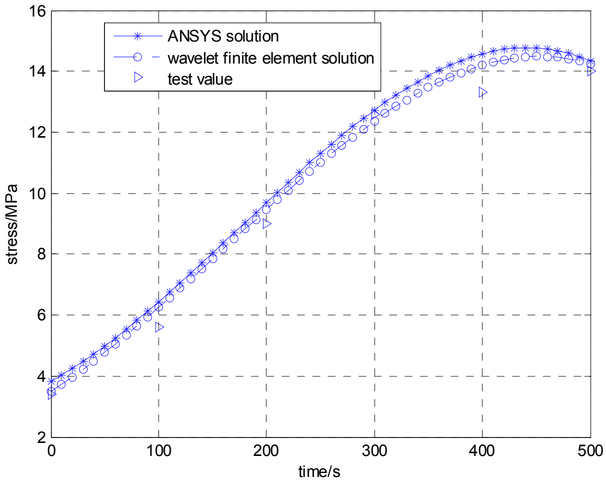

Fig. 5. Stress changing curves of testing point 2 from ANSY, wavelet finite element and experiment

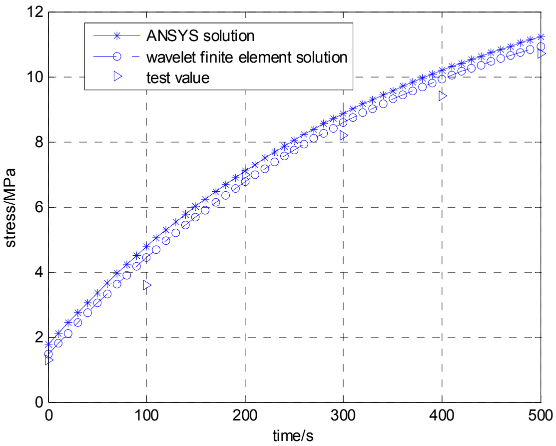

Fig. 6. Stress changing curves of testing point 3 from ANSY, wavelet finite element and experiment

As seen from Fig. 4, Fig. 5 and f Fig. 6, the stress of LPG tank wall first increases quickly, and then increases slowly with time, and the explosion fatalness of LPG tank under fire increases accordingly. Some fire precautions should be used to avoid the exploration of LPG tank. Wavelet finite element solution is closer to the test value than ANSYS solution; these results show that the wavelet finite element method has higher computing precision than traditional finite element method. The wavelet finite element method use less elements and nodes to get higher computing precision than traditional finite element method in ANSYS software. Results show that the Sobolev orthogonal wavelet finite element method has better numerical stability, which can be suit for solving singularity problem, and it can obtain the correct simulation results of complex stress field of LPG tank under fire.

4 Conclusions

The Sobolev orthogonal wavelet function is used as interpolating function to establish the wavelet finite element, and the complex stress wavelet finite element formulation is constructed. The stress changing rules of LPG tank under fire is obtained based on ANSYS software, the wavelet finite element method and test respectively, the wavelet finite element method has higher computing precision than traditional finite element method, it can use less elements and nodes to get more accurate solution than traditional finite element method. The complex stress distribution rules of LPG tank wall is obtained, the stress of LPG tank wall near gas-liquid interface is highest, and the stress of LPG tank wall near the gas phase LPG is higher than the stress of LPG tank wall near the liquid phase LPG, and the stress changing rules of LPG tank wall with time is obtained, these simulation results can offer good theoretical guidance for fire prevention.

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