

An Optimal Channel Allocation Algorithm for SU-MIMO WLAN Cognitive Network

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Abstract. Aiming at the channel allocation problem caused by the wireless Access Points (APs) in Multiple Input Multiple Output (MIMO) Wireless Local Area Network (WLAN) when accessing to the Television White Space (TVWS) authorized frequency band in cognitive way, this paper firstly models the Primary User (PU) and secondary user's network interference based on MIMO channel. Also, the number of accessed secondary users under PU outage probability is deduced. Then, by modeling the channel interference among Basic Service Set (BSS) where secondary users exist, the mutual interference degree among APs and the actual throughput of the cognitive network are analyzed. Secondly, under the premise of guaranteeing the PU Quality of Service (QoS) and ensuring the secondary users QoS ultimately, this paper models the channel allocation problem as a nonlinear integer programming problem. Besides, the optimal allocation of the secondary user channels and the transmit power are obtained according to Karush-Kuhn-Tucker (KKT) theory and Water-Filling method. Numerical analysis results verify the rationality and the validity of the proposed channel allocation algorithm in terms of the total channel capacity of cognitive network and load balancing among BSS.

Keywords: multiple input multiple output, cognitive radio, television white space, quality of service, channel interference, channel allocation

1 Introduction

Nowadays, as increasing demand for multimedia services, cellular communication (2G/ 3G network) cannot meet the needs of users, whereas the Wireless Local Area Networks (WLANs), characteristic of a high rate,

easy deployment and cost low, get mobile operators recognition. Mobile operators deploy Access Points (APs) in the hot spots and take WLANs as the most effective means to assist the 2G/3G communication.

However, due to the lack of unified commercial assignment and channel management scheme of WLANs, the same frequency channel configuration in the same hot spots with different mobile operators' APs is very common. When channel interference is serious, communication may be even interrupted. Obviously, effectively solving channel interference of APs in hot area is one of the key technologies in the large-scale commercialization of WLANs.

There are two methods to solve the channel interference of APs. The first one is in the Industrial, Scientific and Medical (ISM) / Unlicensed National Information Infrastructure (UNII) frequency band which wireless AP works in, the channel is allocated in unauthorized frequency distribution through the graph coloring [1], integer linear programming [2], the minimum spanning tree [3] method. There have been many research results and solutions to obtain the minimum interference. The summary of channel allocations can be found in [4], and researchers have demonstrated that channel allocation of WLAN APs is a NP-hard problem [5]. But due to the limitation of channels in WLAN, interference cannot be eliminated effectively in the area with high density APs. The other one is to access in the authorized frequency band with low utilization rate by cognitive radio technology (at present, the most practical spectrum is Television White Space (TVWS) band [6]), to extend the work frequency band of APs and increase the available channels. So far, the study in this field is quite few [7], that is to say, there are a large number of technical problems need to be conquered to let the cognitive network based on WLAN access to authorized frequency band.

On the other hand, in the fundamental information theory, most research of cognitive network channel capacity are based on Single Input Single Output (SISO) channel used by Primary Base Station (PBS) and secondary users [8-11], many researches of channel capacity are also about Multiple Input Multiple Output (MIMO) alone [12-15], but the MIMO channel capacity of cognitive network is not yet well studied. As Cognitive APs (CAPs) access into the Primary User (PU) frequency band in the opportunity way, the access time, space, duration and number of MIMO antennas are random, so in this case, the effect on the channel capacity of PU and channel capacity of secondary user network are dynamic. To the best knowledge of the authors, there is no special reference in this field.

In view of the above two main problems, we study on channel allocation based on opportunity access to TVWS in cognitive network with MIMO, and analyze the channel capacity of MIMO. Through modeling with the PU network and second user network, we deduce the number of access CAPs under limited Quality of Service (QoS) of PUs. Through modeling the channel interference in the Basic Service Set (BSS) with CAPs, we analyze the interference with mutual channel in BSS and the actual throughput. Finally, limited by the QoS of transmit power of secondary users, channel allocation problem will turn to a nonlinear optimization problem with inequality constraints, and then we can obtain the optimal channel and CAP power allocation through Karush-Kuhn-Tucker (KKT) theory and Water-Filling method. Numerical analysis results verify the rationality and effectively of the channel allocation algorithm in this paper.

The rest of this paper is organized as follows. Section 2 introduces system modeling and Section 3 presents throughput analysis under mutual interference of Secondary AP. An Optimal channel allocation algorithm based on KKT is described in Section 4. Section 5 gives performance evaluation of the proposed algorithm. Concluding remarks are in Section 6.

2 System Modeling

2.1 Network Topology

Fig. 1 shows the topology of coexistence with PU network and secondary user network, where the PU network is consist of PBS and the PU clients, secondary user network is consist of CAPs and secondary user clients. The PU network works in authorized frequency band, and secondary user network general works in unauthorized frequency band, referred to the ISM band in this paper. When ISM band spectrum resource is congested and PU frequency spectrum is free, CAP has a chance to access the channel of PU network, but once the PU needs spectrum resource, CAP should release the channel of PU network unconditionally.

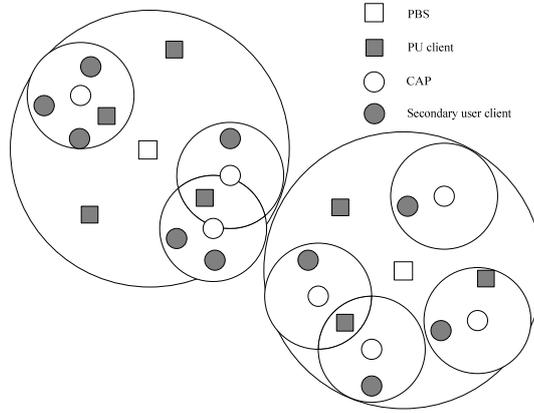


Fig. 1. The network topology of PU network and secondary user network

2.2 Channel Capacity Analysis of MIMO

There are N_t transmit antennas and N_r receive antennas in MIMO system, as shown in Fig. 2, described as $N_t \times N_r$. The channel matrix can be expressed by $\mathbf{H} = [h_{ij}]$, where h_{ij} is the channel transfer function from the j th transmit antenna to the i th receive antenna. \mathbf{H} is a random matrix, which can be characterized by an uncorrelated or correlated Rayleigh or Rician fading channel.

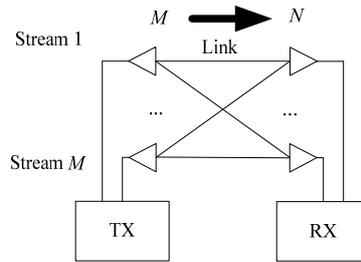


Fig. 2. MIMO System

For a MIMO system, the elements in \mathbf{H} are independent in the view of the statistics. From [12], \mathbf{H} can be expressed as two independent parts

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \quad (1)$$

where \mathbf{R}_r and \mathbf{R}_t are receive correlation and transmit correlation matrices, \mathbf{H}_w is independent Gaussian elements and unity variable matrix, and the superscript $1/2$ means the Hamilton square root of the matrix. The correlation between the lows of \mathbf{H} , and independent of transmit antennas is decided by \mathbf{R}_r . Similarly, the covariance of \mathbf{H} is decided by \mathbf{R}_t and independent of receive antennas.

The correlation matrices \mathbf{R}_r and \mathbf{R}_t can be obtained by measuring of hypothesizing the distribution around receive and transmit antennas. Assuming transmit and receive matrices are all the uniform linear arrays, \mathbf{R}_r and \mathbf{R}_t can be calculated as [13]

$$\mathbf{R}_r, \mathbf{R}_t = \begin{bmatrix} 1 & \rho & \rho^4 & \dots & \rho^{(N_x-1)^2} \\ \rho & 1 & \rho & \dots & \dots \\ \rho^4 & \rho & 1 & \dots & \rho^4 \\ \dots & \dots & \dots & \dots & \rho \\ \rho^{(N_x-1)^2} & \dots & \rho^4 & \rho & 1 \end{bmatrix} \quad (2)$$

where N_x is equal to N_r or N_t , corresponding to receive or transmit antenna array, and ρ is the fading correlation coefficient between two adjacent receive or transmit antennas, which can be calculated approximately by

$$\rho(d) \approx e^{-23\Delta^2 d^2} \quad (3)$$

where Δ is the angular spread, and d is the inter-element distance.

For a given MIMO channel \mathbf{H} , when channel noise is Gaussian with zero mean value, we can get the channel capacity according to Shannon theory [13-14]

$$C(\mathbf{H}) = B \log_2 \det \left(\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{P} \mathbf{H}^H \right) \quad (4)$$

where B is the bandwidth, \mathbf{I} is mutual information matrix, \mathbf{P} is the covariance matrix of transmit signal, the noise is Gaussian with $[0, \sigma_n^2]$, \det means the determinant of the matrix, and the total transmit power $P_{tot} = \text{tr}\{\mathbf{P}\}$.

When transmit and receive antennas can obtain all the Channel State Information (CSI), channel capacity can achieve the maximum value [15]

$$C = \sum_{i=1}^{r_H} \log_2 \left(1 + \frac{P_i}{\delta_n^2 + I} \sigma_i^2 \right) \quad (5)$$

where r_H is the rank of \mathbf{H} , \bar{I} is the average interference power, σ_i^2 is the i th singular value of \mathbf{H} , P_i is the power of the i th antenna, and δ_n^2 is noise variance.

2.3 Analysis of SAP Interference

The PBS, PU clients, CAP and secondary user clients work in Single User (SU)-MIMO. In this paper, we consider special circumstances, which PBS (the PU transmitter) is always in sending status, the PU receiver is always in receiving state, that is to say, the PU network works in ‘‘saturated’’ condition. The interference model of CAPs to the PU is shown in Fig. 3.

As shown in Fig. 3, R_{PI} is the interference radius of PU, R_{PC} is the communication radius of PU, R_{CI} is the interference radius of CAP, R_{\min} is minimum interference radius of CAP in order to guarantee the QoS of PU, and r is the actual interference radius of CAP and PU.

Hypothesis actually transmission rate of PU is C_p . The minimum rate for successful communication of PU is C_{p0} , namely, when below, communication will be terminated. The outage probability can be expressed as

$$\Pr[C_p \leq C_{p0}] \leq \beta, \quad 0 \leq \beta \leq 1 \quad (6)$$

where $\Pr(\cdot)$ is a probability function, and β is an outage probability. Eq. (6) shows the probability of C_p must be more than or equal to C_{p0} , so as to make the average throughput meet the QoS requirements of PU.

$$f(r) = \frac{2r}{R_{CI}^2 - R_{min}^2}, \quad R_{min} \leq r \leq R_{CI} \quad (13)$$

Average interference of individual CAP on the PU is [11]

$$\bar{I} = \int_{R_{min}}^{R_{CI}} f(r) I_i dr = \int_{R_{min}}^{R_{CI}} \frac{2r}{(R_{CI}^2 - R_{min}^2)} \cdot \frac{P_{st}}{r^\alpha} dr = \frac{2P_{st}}{(R_{CI}^2 - R_{min}^2)(\alpha-2)} \left(\frac{1}{R_{min}^{\alpha-2}} - \frac{1}{R_{CI}^{\alpha-2}} \right) \quad (14)$$

where P_{st} is the transmit power of CAP. In the circle area, the total number of CAPs is

$$n = \lambda_s \pi (R_{CI}^2 - R_{min}^2) \quad (15)$$

where λ_s is the density of CAPs, and obeys Poisson distribution. Then in interference area of CAPs, the mean interference of all CAPs is

$$E[I] = n \bar{I} = \frac{2\pi\lambda_s P_{st}}{(\alpha-2)} \left(\frac{1}{R_{min}^{\alpha-2}} - \frac{1}{R_{CI}^{\alpha-2}} \right) \quad (16)$$

2.4 Interference SAP Number

In order to analyze the number of interference CAP working in the frequency band of PU, we use Θ to replace $\Pr(\cdot)$.

From eq. (6) and eq. (7)

$$\begin{aligned} \Theta &= \Pr \left[r_H B_P \log_2 \left(1 + \frac{P_T}{\delta^2 + I} \sigma_P^2 \right) \leq C_{P0} \right] \\ &= \Pr \left[I \geq \frac{\frac{C_{P0}}{2^{r_H B_P}} - 1}{\sigma_P^2} P_T \right] \end{aligned} \quad (17)$$

From above eq. (17), if there is no CAP interference and then we set $I = 0$

$$\frac{P_T \sigma_P^2}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2 = 0 \rightarrow P_R = r_H P_T \sigma_P^2 = r_H \delta^2 \left(\frac{C_{P0}}{2^{r_H B_P}} - 1 \right) \quad (18)$$

So the actual communication distance is the longest interference distance of PU R_{PI} . Eq. (18) is substituted into eq. (9)

$$R_{PI} = \left(\frac{r_H P_T G_t G_r h_t^2 h_r^2}{\left(\frac{C_{P0}}{2^{r_H B_P}} - 1 \right) \delta^2} \right)^{1/4} \quad (19)$$

$$R_{PC} \leq \left(\frac{r_H P_T G_t G_r h_t^2 h_r^2}{\left(\frac{C_{P0}}{2^{r_H B_P}} - 1 \right) \delta^2} \right) = R_{PI} \quad (20)$$

In order to calculate the number of CAPs accessed to frequency band of PU when there is a certain throughput requirement of the PU, we first present a lemma:

Lemma 1 (Markov inequality): assuming X is the non-negative random variable in the sample space and has limited expectation, then $\forall \varepsilon > 0$, $P(X \geq \varepsilon) \leq E(X)/\varepsilon$, the inequality can be only an equation when $P(X \in \{0, \varepsilon\}) = 1$.

Obviously I can meet the requirement of random variable,

$$I = X \geq \varepsilon = \frac{P_R}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2 \quad (21)$$

Eq. (17) is substituted into the Markov inequality, we can obtain

$$\Theta = \Pr[I \geq \varepsilon] \leq \frac{E[I]}{\varepsilon} = \frac{E[I]}{\frac{P_R}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2} \quad (22)$$

From eq. (16), when $R_{CI} \rightarrow \infty$, the upper limit of $E[I]$ is

$$E[I]_{UB} = \frac{2\pi\lambda_s P_{st}}{(\alpha-2) R_{min}^{\alpha-2}} \quad (23)$$

When eq. (23) is substituted into the eq. (22), and combining with eq. (6) $\Theta \leq \beta$,

$$\Theta \leq \beta \leq \frac{E[I]_{UB}}{\varepsilon} = \frac{\frac{2\pi\lambda_s P_{st}}{(\alpha-2) R_{min}^{\alpha-2}}}{\frac{P_R}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2} \quad (24)$$

Simplify eq. (24)

$$\left(\frac{P_R}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2 \right) \beta \leq \frac{2\pi\lambda_s P_{st}}{R_{min}^{\alpha-2}} \quad (25)$$

Then simplify eq. (25)

$$\frac{P_R \beta}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \frac{2\pi\lambda_s P_{st}}{R_{min}^{\alpha-2}} - \delta^2 \beta \leq 0 \quad (26)$$

If eq. (26) = 0, the number of accessed CAPs is

$$\frac{P_R \beta}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \frac{2\pi\lambda_s P_{st}}{R_{min}^{\alpha-2}} - \delta^2 \beta = 0 \quad (27)$$

The CAP density can be calculated as [17]

$$\lambda_s = \frac{R_{min}^{\alpha-2} \beta \left(\frac{P_R}{\frac{C_{P0}}{2^{r_H B_P}} - 1} - \delta^2 \right)}{2\pi P_{st}} \quad (28)$$

Then the number of CAPs in this corresponding circle area is

$$n = \text{int}(\lambda_s \pi (R_{CI}^2 - R_{min}^2)) \quad (29)$$

where $\text{int}(\cdot)$ is rounded down function.

3 Throughput Analysis under Mutual Interference of Cognitive SAP

3.1 The Hypothesis

In an ideal condition, the average throughput of terminals in BSS is so intimately associated with Medium Access Control (MAC) protocol. In SU-MIMO channel, the ideal throughput with the IEEE 802.11e MAC is approximately as [18]:

$$G_i \approx r_H \frac{p_{si} E[P_{load}]}{E[I_d] + \sum_{i=0}^3 p_{si'} (T_s + AIFS[ACi']) + (1 - \sum_{i=0}^3 p_{si'}) T_c} \quad (30)$$

where P_{load} is the length of the load, $E[I_d]$ is the average number of spare slots before transmission, p_{si} and $p_{si'}$ are the probability of condition success transmission of ACi and ACi' , T_s is the average time required for the successful transmission, T_c is the collision time, $AIFS [ACi']$ is the number of slots taken up with the interval between arbitration frames of ACi ($AIFS$).

In addition, more hypothesis are as follows.

- (1) Don't consider fast fading channel, channel is stable and BSS interference is a main factor of packet loss.
- (2) Only consider the overlap zone of two CAPs, not of three or more CAPs.
- (3) Assuming MAC transmission obeys the RTS (Request to Send) / CTS (Clear to Send) / DATA /ACK mechanism, so that it can avoid "hidden terminal" and "exposed terminal".

3.2 Inter-BSS Channel Interference Modeling

Two overlapped CAPs and the actual throughput subjected to the channel interference between these two CAPs are considered. All the CAPs we analyze are located in the CAP interference area in Fig. 3.

In Fig. 4, M is the number of terminals in each AP communication area, AP_i 's communication area is A_i , and the adjacent CAPs (AP_i, AP_j) have an interference area $A_{i,j}$ (the shadow area in the middle of Fig. 4). $M_{i,j}$ is the number of terminals associated with AP_i but interfered by AP_j , and vice versa. The channel set of ISM is $F_{ISM} = \{f_1, \dots, f_{11}\}$ while that of TVWS is $F_{TVWS} = \{f_{TVWS,1}, \dots, f_{TVWS,v}\}$. $F = F_{ISM} \cup F_{TVWS}$ is used to express the channel set of the mixed ISM and TVWS. The interference of AP_j over AP_i is represented by an Interference Penalty (IP) index $IP_{i,j}(f_i, f_j)$

$$IP_{i,j}(f_i, f_j) = A_{i,j} / A_i \times \rho(f_i, f_j) \quad (31)$$

$$\rho(f_i, f_j) = \begin{cases} \max(1 - 1/O \times |f_i - f_j|, 0), & \text{if } f_i, f_j \in F_{ISM} \\ 0, & \text{else} \end{cases} \quad (32)$$

where f_i and f_j are the frequencies assigned to AP_i and AP_j respectively, $\rho(f_i, f_j)$ is channel overlap factor, $|f_i - f_j|$ is the absolute value of the channel designator corresponding to f_i and f_j , and O is the orthogonal channel interval number. In ISM band, channel 1, 6 and 11 are orthogonal channels, so $O=5$ [3].

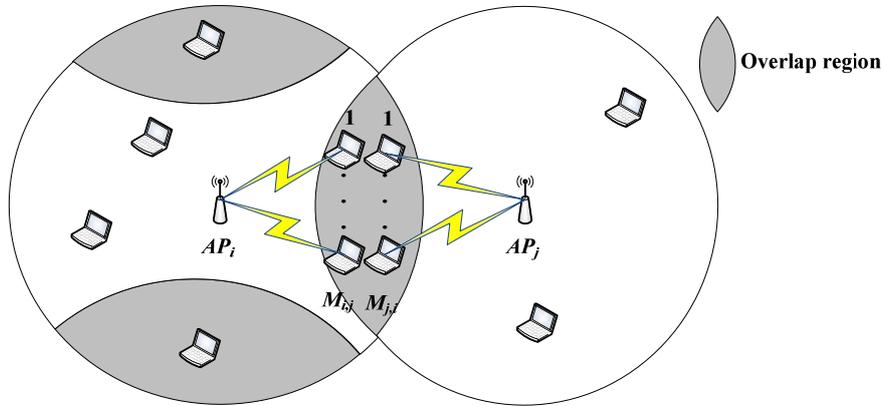


Fig. 4. Channel interference model among BSSs

Overlap regions $A_{i,j}$ of the two adjacent AP is calculated as shown in Fig. 5.

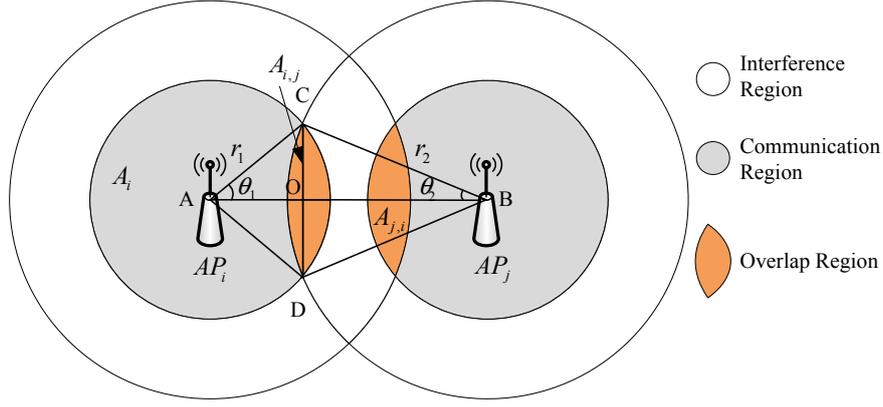


Fig. 5. Model of BSS overlap region

In Fig. 5, we set the distance between the adjacent AP_i and AP_j is d_{AB} , the communication radius of AP_i is r_1 , interference radius of AP_j is r_2 , r_1 and r_2 can be calculated as [13]

$$r_x = 10^{\frac{P_i - P_r - L_0}{10\alpha}} \quad (33)$$

where L_0 is the channel attenuation in 1 m from receiver.

The area of $A_{i,j}$ is expressed as

$$A_{i,j} = \frac{1}{2}r_1^2[2\theta_1 - \sin(2\theta_1)] + \frac{1}{2}r_2^2[2\theta_2 - \sin(2\theta_2)] \quad (34)$$

According to the obtained $A_{i,j}$ and A_i , we can gain the interference penalties $IP_{i,j}$ of AP_j to AP_i , similarly gain the $IP_{j,i}$ of AP_i to AP_j .

Since AP_i is interfered by other APs, its throughput is decreased by

$$W_i = \sum_{j=1, i \neq j}^{j=L-1} IP_{i,j} S_{i,j} = \sum_{j=1, i \neq j}^{j=L-1} IP_{i,j} \sum_{k=1}^{M_{i,j}} G_{k,l} \quad (35)$$

where $S_{i,j}$ is the throughput of $M_{i,j}$ terminals associated with AP_i in the overlap area, $G_{k,l}$ is the normalized throughput of the k th terminal in the l th AC.

The actual throughput of AP_i equals to the interference-free throughput minus the throughput decrease W_i caused by the interferences

$$t_i = \sum_{k=1}^M G_{k,l} - W_i \quad (36)$$

4 Optimal Channel Allocation Algorithm Based on the Theory of KKT

4.1 Problem Formulation

We can get the maximum number of accessed CAPs under the constraints of PU outage probability from Section 2.4. From Section 3.2 of channel interference modeling, we know the actual throughput of terminal with QoS in the BSS under interference. The above problem is described as an optimization problem.

Under the constraints of the PU outage probability and the QoS of cognitive users, channel allocation is optimized to maximum the overall throughput in each BSS and balance the load among BSS.

Each AP can only work in one channel, so the allowed minimum Signal to Interference plus Noise Ratio (SINR) of k th AP communicated in the n th channel is

$$\gamma_{n,k} = \frac{P_{n,k} \kappa_{n,k}}{IP_{n,k} + N_S B_S} \geq \gamma_{th,k} \quad (37)$$

where $P_{n,k}$ is the power of cognitive AP_k in the channel n , $\kappa_{n,k}$ is channel gain, N_S is the noise power spectral density of cognitive user, B_S is channel bandwidth of cognitive network, $IP_{n,k}$ is the interference of cognitive AP_k in the n th channel by the adjacent channel.

$$P_{n,k} \geq \frac{(IP_{n,k} + N_S B_S) \gamma_{th,k}}{\kappa_{n,k}} = P_{n,k,\min} \quad (38)$$

Eq. (6) is equivalent to the SINR of PU

$$\gamma_P = \frac{P_R}{\sum_{k=1}^K P_{n,k} d_k^{-\alpha} + N_P B_P} \geq \gamma_{P0} \quad (39)$$

where d_k is the distance between the cognitive AP_k and the PU receiver, N_P is noise power spectral density of PU, B_P is the bandwidth of PU, γ_{P0} is the allowed minimal SINR of the PU network.

Assuming the distance of all the CAPs to PU receiver is the same and the transmit power also is the same,

$$\gamma_P = \frac{P_R}{K P_{n,k} d^{-\alpha} + N_P B_P} \geq \gamma_{P0} \quad (40)$$

Also, eq. (40) can be converted

$$P_{n,k} \leq \frac{P_R}{K \gamma_{P0} d^{-\alpha}} - \frac{N_P B_P}{K d^{-\alpha}} = P_{n,k,\max} \quad (41)$$

4.2 Establishment and Solution of the Objective Function

Considering the limitation of cognitive network resources, cognitive users and transmit power of PU, we can obtain a combined objective function with channel capacity of cognitive user and load balancing.

N is channel resources for cognitive users, K is the number of CAPs (from eq. (29)), objective function is

$$f = \arg \max C + \arg \max \eta = \sum_{k=1}^K \sum_{n=1}^N \sum_{i=1}^{t_{in}} (c_{n,k})^m B_S \log_2 \left(1 + \frac{P_{i,n,k} \sigma_{i,n,k}^2}{N_S B_S + IP_{n,k}} \right) + \left[\frac{\left(\sum_{i=1}^K \sum_{j=1}^N c_{n,k} t_i \right)^2}{K \sum_{i=1}^K \sum_{j=1}^N (c_{n,k} t_i)^2} \right] \quad (42)$$

Former part expresses the overall channel capacity of cognitive network. Latter half means the fairness of throughput between each BSS in the cognitive network. It can be measured by the Jain index η [19], which is in $[0, 1]$. When η is close to 1, it means the throughput between each BSS is roughly same, otherwise, load distribution is extremely equilibrium. When the allocation of channel and power are optimized, C and η achieve maximum.

Assuming each AP with MIMO has the same antenna state. Each AP has L antennas, whose power and gain are all the same, i.e. $P_{n,k} = P_{i,n,k}$, so it can be simplified as

$$f = L \sum_{k=1}^K \sum_{n=1}^N (C_{n,k})^m \log_2 \left(1 + \frac{P_{n,k} \kappa_{n,k}}{N_S B_S + IP_{n,k}} \right) + \left[\frac{\left(\sum_{i=1}^K \sum_{j=1}^N c_{n,k} t_i \right)^2}{K \sum_{i=1}^K \sum_{j=1}^N (c_{n,k} t_i)^2} \right] \quad (43)$$

Subject to:

$$\sum_{k=1}^K c_{n,k} = 1, \quad \text{for all } n \quad (44)$$

$$IP_{n,k} \leq c_{n,k} IP_{k,\max}, \quad \text{for all } n \quad (45)$$

$$\sum_{n=1}^N P_{n,k} \leq P_{k,\max}, \quad \text{for all } n \quad (46)$$

$$P_{n,k} \geq c_{n,k} P_{n,k,\min}, \quad \text{for all } n, k \quad (47)$$

$$P_{n,k} \leq c_{n,k} P_{n,k,\max}, \quad \text{for all } n, k \quad (48)$$

where $c_{n,k}$ is a binary instruction function, which means that n th channel is assigned to n th AP. M is the fuzzy factor not less than one. $IP_{k,\max}$ is the max interference threshold of AP, $P_{k,\max}$ is the max transmit power of AP. $P_{n,k,\max}$ and $P_{n,k,\min}$ can be obtained from eq. (38) and eq. (41).

Above problem is a nonlinear integer programming problem with inequality constraints, which can be solved by the KKT theory, namely KKT-CA algorithm. To obtain the optimal solution, Lagrange function and the KKT coefficients must be obtained.

We define Lagrange function as

$$\begin{aligned} L(P_{n,k}, c_{n,k}, \lambda_k, \beta_n, \mu_{n,k}) = & L \sum_{k=1}^K \sum_{n=1}^N (C_{n,k})^m \log_2 \left(1 + \frac{P_{n,k} \kappa_{n,k}}{N_s B_s + IP_{n,k}} \right) + \left[\frac{\left(\sum_{i=1}^K \sum_{j=1}^N c_{n,k} t_i \right)^2}{K \sum_{i=1}^K \sum_{j=1}^N (c_{n,k} t_i)^2} \right] \\ & + \sum_{n=1}^N \beta_n \left(\sum_{k=1}^K c_{n,k} - 1 \right) + \sum_{n=1}^N \sum_{k=1}^K \beta'_{n,k} (c_{n,k} IP_{k,\max} - IP_{n,k}) + \sum_{k=1}^K \lambda_k \left(P_{k,\max} - \sum_{n=1}^N P_{n,k} \right) \\ & + \sum_{n=1}^N \sum_{k=1}^K \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\min}) + \sum_{n=1}^N \sum_{k=1}^K \mu'_{n,k} (c_{n,k} P_{n,k,\max} - P_{n,k}) \end{aligned} \quad (49)$$

Seeking the first-order partial derivatives with $P_{n,k}$ and $C_{n,k}$ respectively, we get the KKT conditions as follows:

$$\lambda_k \geq 0, \quad \mu_{n,k} \geq 0, \quad \mu'_{n,k} \geq 0 \quad (50)$$

$$\frac{\partial L}{\partial P_{n,k}} = 0 \rightarrow L \frac{(c_{n,k})^m}{\ln 2} \frac{\kappa_{n,k}}{N_s B_s + IP_{n,k} + P_{n,k} \kappa_{n,k}} - \lambda_k + \mu_{n,k} - \mu'_{n,k} = 0 \quad (51)$$

$$\frac{\partial L}{\partial c_{n,k}} = 0 \rightarrow m(c_{n,k})^{m-1} L \log_2 \left(1 + \frac{P_{n,k} \kappa_{n,k}}{N_s B_s + IP_{n,k}} \right) + \beta_n + \beta'_{n,k} (1 - 2c_{n,k}) - \mu_{n,k} P_{n,k,\min} + \mu'_{n,k} P_{n,k,\max} = 0 \quad (52)$$

$$\sum_{n=1}^N \sum_{k=1}^K \beta'_{n,k} (c_{n,k} IP_{k,\max} - IP_{n,k}) = 0 \quad (53)$$

$$\sum_{k=1}^K \lambda_k \left(P_{k,\max} - \sum_{n=1}^N P_{n,k} \right) = 0 \quad (54)$$

$$\sum_{n=1}^N \sum_{k=1}^K \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\min}) = 0 \quad (55)$$

$$\sum_{n=1}^N \sum_{k=1}^K \mu'_{n,k} (c_{n,k} P_{n,k,\max} - P_{n,k}) = 0 \quad (56)$$

Because there are many parameters in the KKT conditions, it is difficult to get the optimal solution. For simplicity, we can get the approximate solution by Water-Filling method

$$\lambda'_k = \lambda_k - \mu_{n,k} + \mu'_{n,k} \quad (57)$$

The Lagrange function with partial derivative to $P_{n,k}$ can be simplified as

$$P_{n,k} = (c_{n,k})^m \frac{L}{\lambda'_k \ln 2} - \frac{N_S B_S + IP_{n,k}}{\kappa_{n,k}} \quad (58)$$

$$= \begin{cases} P_{n,k}^0 = \frac{L}{\lambda'_k \ln 2} - \frac{N_S B_S + IP_{n,k}}{\kappa_{n,k}} & \lambda'_k < \frac{L \kappa_{n,k}}{\ln 2 (N_S B_S + IP_{n,k})}, c_{n,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$

where $P_{n,k}^0$ is the initial transmit power value of the Water-Filling iterations. The above eq. is substituted into the eq. (51)

$$\lambda'_k = \frac{|\Omega_k|}{\ln 2 \left(P_{k,\max} + \sum_{n \in \Omega_k} \frac{N_S B_S + IP_{n,k}}{L \kappa_{n,k}} \right)} \quad (59)$$

where Ω_k is the channel set allocated to the k th AP, $c_{n,k}=1$ for all.

From eq. (54), eq. (55)

$$P_{n,k} = \max \left(\min \left(P_{n,k}^0, P_{n,k,\max} \right), P_{n,k,\min} \right), \quad \text{if } c_{n,k} = 1 \quad (60)$$

Above formulas constitute all the equations with Water-Filling method.

5 Performance Evaluation

We assume that all of the CAPs have the same maximum transmit power $P_{k,\max}$. SINR limit of CAP is $\gamma_{th,k}=5$ dB. The noise power of CAP is $N_S B_S = 0.1$ dBm. Received useful power of the PU receiver is $P_r = 2.5$ w. Noise power of the PU receiver is $N_P B_P = 0.2$ dBm. The distance between the PBS and PU receiver is 1 Km, cognitive user density λ_s is 0.005. Maximum number of accessed CAPs is 80. The maximum interference radius of CAP is 300 m. The minimum access radius of PU is 20 m. Path loss factor $\alpha=2$. SINR limit of PU receiver is $\gamma_0=5$ dB. The numbers of PBS and CAPs antennas are both 4. MAC protocol is IEEE 802.11e. The main parameters configuration is shown in **Table 1**.

Table 1. Simulation parameters settings

Parameter	Settings
Load length	1000 bytes
PHY head	24 bytes
MAC head	28 bytes
ACK	14 bytes
Data rate	11 Mbps
Basic rate	1 Mbps
Slot	9 us
SIFS	16 us
DIFS	38 us
<i>AIFS</i> [AC3]	3 Slots
<i>AIFS</i> [AC2]	5 Slots

Assuming that the total number of unauthorized ISM frequency band (11 channels, 1-11 channel) and authorized TVWS band [6] (2 channels, 14-15 channel) is $N = 13$. Assuming that there are 6 CAPs. Channel gain vector $g_n = \tau/N$, τ is random variables between (0, 1), so we can get

$$g_{n,k} = \begin{matrix} & \text{AP1} & \text{AP2} & \text{AP3} & \text{AP4} & \text{AP5} & \text{AP6} \\ \begin{matrix} \left[\begin{array}{cccccc} 0.0468 & 0.0039 & 0.0143 & 0.0767 & 0.0325 & 0.0390 \\ 0.0364 & 0.0403 & 0.0468 & 0.0065 & 0.0702 & 0.0260 \\ 0.0273 & 0.0598 & 0.0208 & 0.0338 & 0.0143 & 0.0052 \\ 0.0637 & 0.0715 & 0.0507 & 0.0078 & 0.0208 & 0.0091 \\ 0.0455 & 0.0104 & 0.0533 & 0.0741 & 0.0117 & 0.0143 \\ 0.0416 & 0.0442 & 0.0572 & 0.0013 & 0.0104 & 0.0143 \\ 0.0702 & 0.0364 & 0.0351 & 0.0598 & 0.0663 & 0.0182 \\ 0.0221 & 0.0013 & 0.0065 & 0.0624 & 0.0442 & 0.0325 \\ 0.0585 & 0.0260 & 0.0182 & 0.0663 & 0.0416 & 0.0039 \\ 0.0572 & 0.0611 & 0.0702 & 0.0065 & 0.0117 & 0.0689 \\ 0.0286 & 0.0234 & 0.0117 & 0.0312 & 0.0650 & 0.0728 \\ 0.0442 & 0.0403 & 0.0637 & 0.0611 & 0.0481 & 0.0377 \\ 0.0052 & 0.0741 & 0.0032 & 0.0728 & 0.0273 & 0.0377 \end{array} \right] \end{matrix} & \begin{matrix} \text{CH1} \\ \text{CH2} \\ \text{CH3} \\ \text{CH4} \\ \text{CH5} \\ \text{CH6} \\ \text{CH7} \\ \text{CH8} \\ \text{CH9} \\ \text{CH10} \\ \text{CH11} \\ \text{CH12} \\ \text{CH13} \end{matrix} \end{matrix}$$

Initial channel distribution $C = [C_{n,k}]$ and the corresponding transmit power P of CAPs are

$$[C_{n,k}] = \begin{matrix} & \text{AP1} & \text{AP2} & \text{AP3} & \text{AP4} & \text{AP5} & \text{AP6} \\ \begin{matrix} \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right] \end{matrix} & \begin{matrix} \text{CH1} \\ \text{CH2} \\ \text{CH3} \\ \text{CH4} \\ \text{CH5} \\ \text{CH6} \\ \text{CH7} \\ \text{CH8} \\ \text{CH9} \\ \text{CH10} \\ \text{CH11} \\ \text{CH12} \\ \text{CH13} \end{matrix} \end{matrix}, \quad P_{n,k} = \begin{matrix} & \text{AP1} & \text{AP2} & \text{AP3} & \text{AP4} & \text{AP5} & \text{AP6} \\ \begin{matrix} \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0.1946 \\ 0 & 0.1770 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1611 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0085 & 0 \\ 0 & 0.0197 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0480 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0193 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0266 \\ 0 & 0 & 0 & 0 & 0.1635 & 0 \\ 0 & 0.1442 & 0 & 0 & 0 & 0 \\ 0.0791 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1308 & 0 & 0 & 0 & 0 \\ 0 & 0.0988 & 0 & 0 & 0 & 0 \end{array} \right] \end{matrix} & \begin{matrix} \text{CH1} \\ \text{CH2} \\ \text{CH3} \\ \text{CH4} \\ \text{CH5} \\ \text{CH6} \\ \text{CH7} \\ \text{CH8} \\ \text{CH9} \\ \text{CH10} \\ \text{CH11} \\ \text{CH12} \\ \text{CH13} \end{matrix} \end{matrix}$$

And $P_{n,k,max}=P_{k,max}=0.2$ mw. Total channel capacity without a channel allocation algorithm is 23.7004 bits/s/Hz.

With the KKT-CA algorithm, the corresponding $C=[C_{n,k}]$ and power allocation are respectively:

$$[C_{n,k}] = \begin{matrix} & \text{AP1} & \text{AP2} & \text{AP3} & \text{AP4} & \text{AP5} & \text{AP6} \\ \begin{matrix} \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \end{matrix} & \begin{matrix} \text{CH1} \\ \text{CH2} \\ \text{CH3} \\ \text{CH4} \\ \text{CH5} \\ \text{CH6} \\ \text{CH7} \\ \text{CH8} \\ \text{CH9} \\ \text{CH10} \\ \text{CH11} \\ \text{CH12} \\ \text{CH13} \end{matrix} \end{matrix}, \quad P_{n,k} = \begin{matrix} & \text{AP1} & \text{AP2} & \text{AP3} & \text{AP4} & \text{AP5} & \text{AP6} \\ \begin{matrix} \left[\begin{array}{cccccc} 0.1000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0585 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0585 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0585 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1000 & 0 \end{array} \right] \end{matrix} & \begin{matrix} \text{CH1} \\ \text{CH2} \\ \text{CH3} \\ \text{CH4} \\ \text{CH5} \\ \text{CH6} \\ \text{CH7} \\ \text{CH8} \\ \text{CH9} \\ \text{CH10} \\ \text{CH11} \\ \text{CH12} \\ \text{CH13} \end{matrix} \end{matrix}$$

After the optimization, $P_{n,k,max}=0.1000 \text{ mw} < P_{k,max}$, and spectral efficiency is 28.2616 bits/s/Hz. Obviously, the spectral efficiency in KKT-CA is better than that without channel allocation algorithm by about $(28.2616-23.7004)/23.7004 \approx 19.2\%$. Corresponding the actual AP channel allocation situation is shown in Fig. 6.

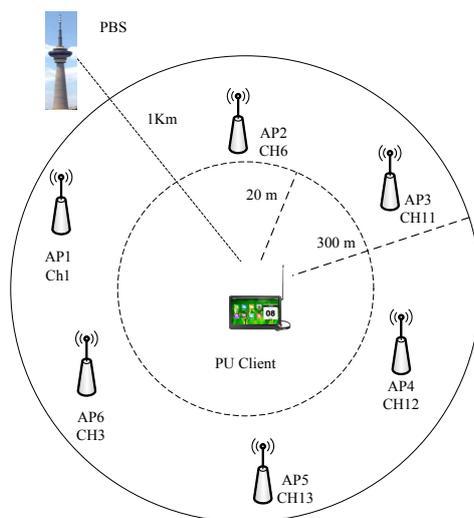


Fig. 6. KKT-CA channel allocation results

From Fig. 6, assigned channels with the KKT-CA are mutually orthogonal, which is the best channel allocation. In addition, through large number of simulations, comparing the KKT-CA, Hsum [1] algorithm based on the weighted coloring, and SU-MIMO JCERA (Joint Channel Estimation and Resource Allocation for MIMO) [20]. The change of spectral efficiency with the number of CAPs K and the number of channels N is shown in Fig. 7. Spectral efficiency is changed with the transmit power shown in Fig. 8. Fig. 9 shows the change of load balance with the CAPs.

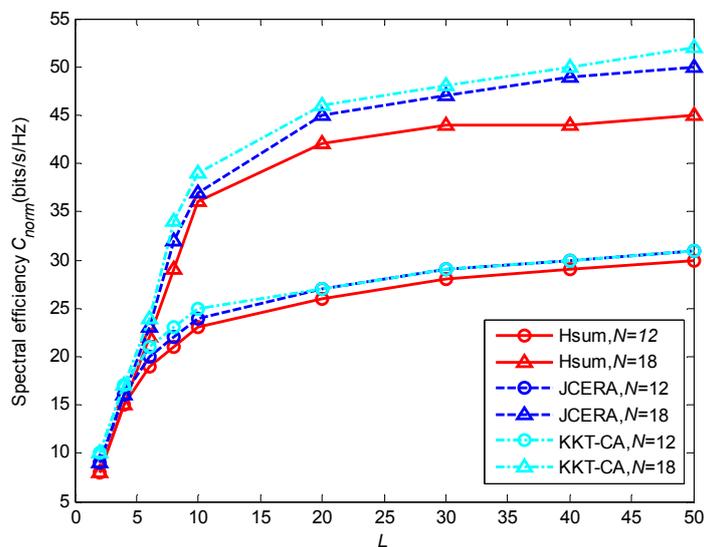


Fig. 7. Channel capacity under different N and K

As shown in Fig. 7, KKT-CA has a larger throughput than Hsum and SU-MIMO JCERA when they have a same channel number. Because Hsum works in the SISO channel, SU-MIMO JCERA only has transmit power optimum, while KKT-CA works in MIMO channel with a QoS distinguish. For a same channel number in a certain algorithm, the system total throughput increases gradually with the increasing number of the APs. Be-

cause as the number of APs increases, the number of terminals associated with AP will increase and also the communication business. However, when the number of APs is large enough, the system tends to be saturated, which results a slower increasing of the system throughput.

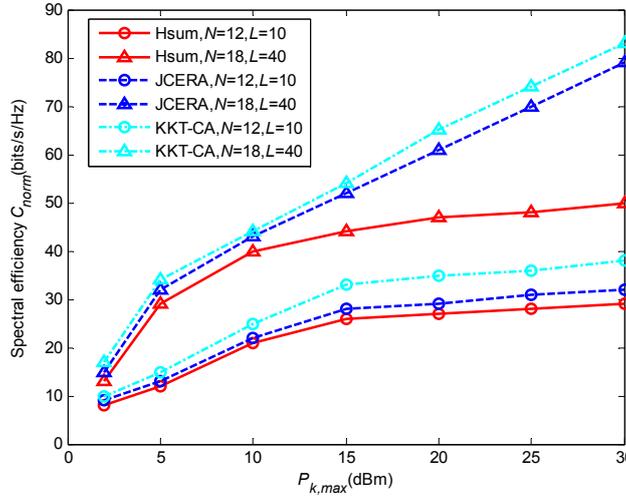


Fig. 8. The relations between the transmit power and the spectral efficiency

In Fig. 8, with a same transmit power, AP and channel number, KKT-CA has a larger throughput than Hsum and SU-MIMO JCERA. Because KKT-CA optimizes the transmit power and reduce the AP mutual channel interference. For a same channel allocation algorithm, the throughput will increase with the increasing of the transmit power when we increase the number of APs and channels in a certain degree. Especially, when AP number is far less than channel number ($N = 18, K = 40$), there will be a much increase of the system total throughput, because each AP works in orthogonal channel, which can maximize the throughput.

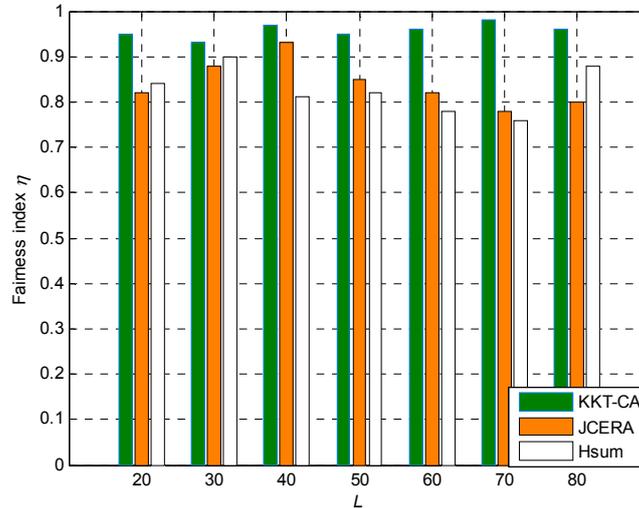


Fig. 9. The load distribution among different CAPs when $N=14$

As shown in Fig. 9, with a certain AP transmit power and channel number (such as $P_{k,max}=200$ mw, $N=14$), the fairness index of Hsum and SU-MIMO JCERA are uncertain and instable with the effect of terminal user associated with AP, because no load balance problem is considered in these two algorithms. So KKT-CA is

better than Hsum and JCERA in load fairness index. From Fig.9 we can see that the fairness index of KKT-CA keeps a value larger than 0.9.

6 Conclusion

The channel interference of PU receiver from CAPs in MIMO channel is studied in this paper. The expressions of communication quality of PU under the channel interference of CAPs as well as the mutual channel interference among BSS are obtained through the mathematical modeling. Also, a channel allocation algorithm which balances the PU channel capacity, the secondary user channel capacity and the load balance is proposed. Numerical analysis shows that the proposed algorithm can reduce the channel interference among CAPs effectively when guaranteeing the QoS of PU. Besides, the system throughput of cognitive network can be increased when considering the throughput fairness among BSS.

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