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The Effects of Similarity and Randomness of Network on Opinion Formation

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Abstract. A modified opinion dynamic model is proposed which concerned two factors: similarity and randomness of network structure. The former mainly considered the common friends of individuals, and the latter considered the randomness of connections between individuals. The results of the model indicate that the node degree has a great influence on the formation of individuals' opinions. The larger node degree, the possibility of individuals' opinions come to consensus is higher. Under the same node degree, individuals' opinions are more likely to form consensus in regular network than in random network. But the less time required for individuals' opinions come to stable with the increase of randomness of network structure, which indicates that the randomness can speed up the spread of opinions. The results also represent that the small-world network that with small random links is the best place that individuals are incline to consensus when node degree is greater than 6. Our work provides some insights to the understanding of the role of similarity of network structure and the node degree in the formation of individuals' opinions.

Keywords: complex networks, opinion evolution, randomness, similarity

1 Introduction

In real world, many mankind behaviors are spread through the contacted network [1-4], and the network has become the main place where people to discuss problems and exchange opinion. Recent years, opinion dynamics has become a hot topic in the field of complex network and social computation [5-7]. It is also the basis of research of topic management and spread of information on network.

In last decade, many works on opinion dynamics have been done by a lot of researchers. The research of opinion dynamics can be divided into two classes. The first one, which supposed that the individuals' opinion is a continuous variable [8-13], is suitable to describe the diversity of the opinion on the given problem. The typical model of this class is DW (Defauant Walt) model [8], which argues that agents exchange their opinions and come to comprise only when their opinion are near. The second is discreet model [10-12], which supposed that the opinions are discreet values, is fit to describe some problems such as election, votes, and selection. The famous model, Ising Model [10], belongs to this class.

Usually, these models are very highly abstraction on the interactions between individuals, and some factors that may affect the interactive behavior are ignored. Therefore, some further works were conducted by the latter researchers [14-18]. studied the effects of network structure on opinion formation, and GUO q studied the convergence of the traditional DW model on BA and Lattice network [16]. Tao discussed the information on small world network. Centola studied the effects of network structure on the spread of individual behaviors [19].

The process of opinion diffusion is different from information spread. Before an individual forming or accepting an opinion, he/she might have interacted with many other individuals with different opinions, and he/she might also consider where the opinion is from. In real life, therefore, the interaction of individuals' opinion is affected not only by the difference of their opinions but also by their social relationship. To reflect these factors that affect opinion formation, the concept of mutual affinity was proposed by Bagnoli is introduced to describe the interpersonal relationship [20]. The method of affinity revising is given in the paper, but how to get or evaluate the affinity value is still a problem. This paper would tackle the problem by the definition of affinity as a function of the number of common neighbors and their difference in opinion. Here we call the affinity as similarity of individuals. In addition to, we also try to explore the effects of structure of network, especially the mean degree and randomness of connection, on the formation of opinions.

2 Model

In real life, when two individuals exchange their opinions on a debated issue, the difference of their opinions plays an important role on the changing of their opinions. If an individual confronts with a conflicting opinion, he/she would take one of two opposite actions to resolve such difference in opinions: (1) If the difference is great, individuals would ignore the contradictory information, and then they don't influence each other at all. (2) If the conflicting opinion is come from a close neighbor or a trustable source, according to Festinger's cognitive dissonance theory [15], the individual is naturally inclined to seek consistency among the cognitions, and consequently adjust its opinion.

From some perspectives, the interpersonal relationship can be reflected by the structure of individuals' network, and the affinity is equal to the similarity of network structure. The similarity can be represented by the number of their common friends. The higher the number, the greater similarity is, which indicates that they have more trustable relationship. The similarity of two individuals i and j can be evaluated as the following:

$$S_{ij} = \frac{\Gamma(i) \bigcap \Gamma(j)}{\Gamma(i) \bigcup \Gamma(j)} \tag{1}$$

Here, $\Gamma(i)$ and $\Gamma(j)$ denote the friends set of individual i and j. We suppose that two directly connected individuals are friends, and the S_{ij} can represents the strength of affinity of two individuals in some extent.

The traditional limited confidence model (DW) set a same constant threshold for every individual. If the difference in opinion is greater than the given threshold, the individuals would keep their original opinions, and only when the difference is less than the threshold, they would exchange and adjust their opinion towards a middle value. Obviously, according to the theory of Festinger's cognitive theory, if the conflicting opinion comes from a trustable or a closer friendship neighbor, the individual might raise the threshold to allow the interaction taking place, which aims to eliminate or minimize the difference in opinion for each other.

Here, a dynamic threshold is closely related with the similarity is proposed. Based on similarity, the greater value of S_{ij}, individual i and j would give a bigger compromise for each other. Therefore, the update rule of opinion can be defined as following:

$$\begin{cases} O_i(t+1) = O_i(t) + u \cdot f(S_{ij}, \Delta O_{ij}^t) \cdot \Delta O_{ij}^t \\ O_j(t+1) = O_j(t) - u \cdot f(S_{ij}, \Delta O_{ij}^t) \cdot \Delta O_{ij}^t \end{cases} \tag{2}$$

Here, μ is a convergence coefficient, and f is a switch function, it can be defined as:

$$f(S_{ij}, \Delta O_{ij}^t) = \begin{cases} 0 & (\Delta O_{ij}^t - \tanh(S_{ij})) \geq 0 \\ 1 & (\Delta O_{ij}^t - \tanh(S_{ij})) < 0 \end{cases} \tag{3}$$

$\Delta O_{ij}^t = O_j(t) - O_i(t)$, that is the difference in opinions between individual i and j. The dynamic threshold $\varepsilon_{ij} = \tanh(S_{ij})$ is plotted in Fig.1.

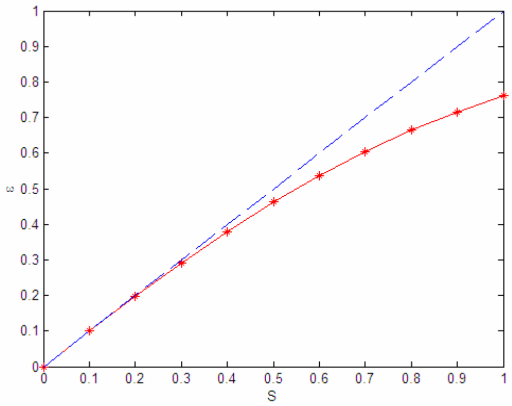

Fig. 1. Dynamic threshold based on similarity

The slope of the curve decreases with the increase of S_{ij}, which can reflect the decreasing of sensitivity of dynamic threshold to the similarity. When the S_{ij} less than 0.3, ε_{ij} is approximate to S_{ij}, and it begin to less than the S_{ij} while it greater than 0.4. It can implicate that if similarity is small, the interaction of exchanging opinion is determined by the value of S_{ij}, it has greater role on determining the opinion exchanging of two individuals. While the extent of the contribution is weaken with increase of S_{ij}.

3 Experiments and results

To explore the effects of similarity and randomness of network structure on the opinion formation, many experiments are conducted on three different networks: regular network, small-world network and random network. The following gives the network topology of such three networks with different randomness.

The left plot in Fig.2 is a typical regular network with n nodes, where each connected to k nearest neighbors by undirected edges, namely to k/2 neighbors clockwise and counterclockwise. For clarity, n = 20 and k = 4 in the schematic figure. The middle plot is the small-world networks, which is generated by randomly rewiring each edge with links probability p on a regular network, and it keep the degree of each node unchanged [20]. Obviously, as p increases, the network topology becomes increasingly disordered, and it turn to a random network when p = 1, as it showed in the right plot. One main feature of regular network is that it is highly clustered, while small world network has similar structure, but with small characteristic path length like it in a random network [21].

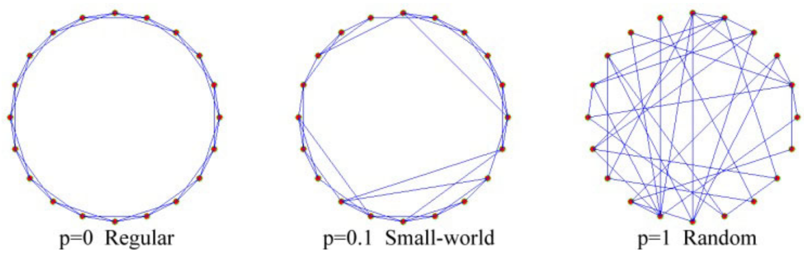

Fig. 2. Network structure and its Randomness

To investigate the effects of network structure on the similarity of individuals, we analyzed the mean similarity of individuals $\overline{S}$ on three types of network with different node degree. Here, the mean similarity is the average of all individuals' similarity. In figure 3, the three plot lines represent the mean similarity values corresponding with k from 2 to 10 when p = 0, 0.1 and 1.0 respectively. From the results, we found that the node degree have great influence on the similarity. With increased of k, $\overline{S}$ increased obviously. While the value of k is small, such as 2 or 3, the similarity values are very near for the same node degree k, but when k is more than 7, the similarity of individuals on a regular network is greater than it on random network. From the results, we can also find that individuals are easier to share with more common friends to form a small clique in regular network than it in random network.

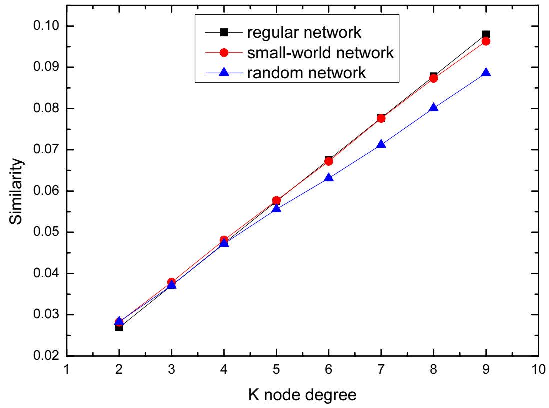

Fig. 3. The average similarity and node degree

All simulations are performed 20 times under the same parameters, and the results are the average values of the 20 times. Fig.4 show the process of individuals' opinions formation on such three networks with different k node degree, it is just one result of 20 simulations, and the horizon axial is the time step of evolution of opinions, and the vertical axial is the opinions distribution of individuals. Form the results we can find that the individuals are easier to come to consensus with the increase of k. When k is greater than 6, the consensus would be formed among individuals on regular network. While the small-world and random network with more randomness, there exist more clustered opinions among the individuals. When k is greater than 8, for small world network, the consensus are begin to emerge among individuals. Although the number of opinion clusters in the random network is less than it in the regular network, as increasing of k, the amount of opinions decreased obviously, it is still difficult to come to consensus.

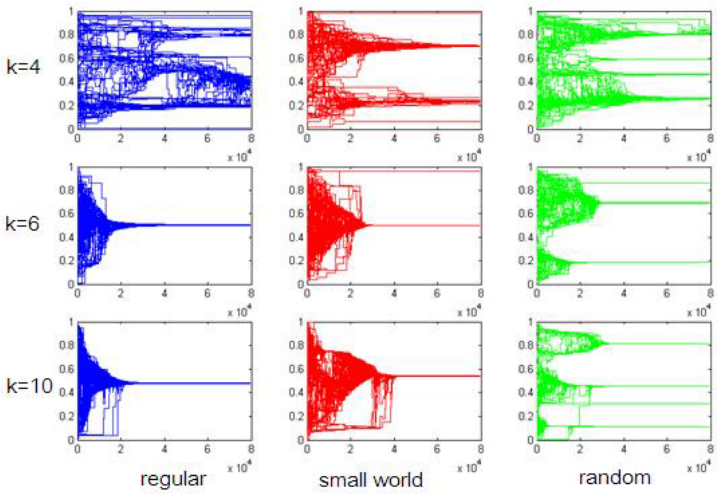

Fig.4. The opinion formation of 100 individuals in three different networks with k = 4, 6, 10

To reflect the change of the number of opinion clusters with the node degree k, the Fig.5 plot the numbers of opinion clusters on the three networks with different node degree, k = 4, 6, …., 10. From the results, we can find that the clusters are decreased rapidly with the increase of node degree, especially when the node degree is less than 5. The trend is more obvious on regular network than it on two other networks. In regular network, individuals are inclined to come to consensus when k > 8. When the node degree is less than 5, the number of opinion clusters on random and small networks is smaller obviously than it on regular networks. But when the degree is greater than 6, the cluster number is decreased to 1 for individuals in regular network, while it keep 3~5 for individuals in small-world or random network. The results indicate that the consensus only appeared in some cliques in regular networks, while it can emerge among inter-cliques on small-world or random networks. Another interesting phenomena is that with the increase of degree and it exceed 6, the individuals' opinions come to consensus in regular network, but there exist more opinion clusters in small-world or random networks. The results indicate that the randomness can reduce the pressure caused by the difference of opinions of individuals in the same clique. Therefore, with the randomness increased, the individuals' opinions are easier to form polarized opinion clusters.

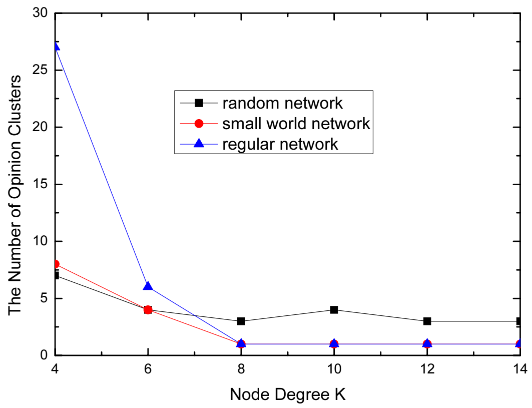

Fig.5. The number of opinion clusters under different k

At each time step, two individuals are randomly selected to exchanging their opinions, and the process is called interaction. At the beginning, the individuals' opinions are distributed randomly. After many interactions, their opinions come to order and stable status, such as consensus, polarized opinions. From the intermediate results during every experiment, we found that there exist many interactions that two individuals did not change their opinions at all. To evaluate the speed and extend of the opinion evolution, we define the interaction that the two individuals changed their opinions as a valid interaction. Fig.6 gives the number of valid interactions when all individuals' opinions come to stable where individuals in the regular networks, random networks and small-world networks respectively. From the results, we found that there exist an obvious inflexion in the valid interactions times when individuals in regular network. With the increase of node degree, the valid interactions also increased, and when k = 5, it come to the maximum, and then it began to decrease with k increasing.

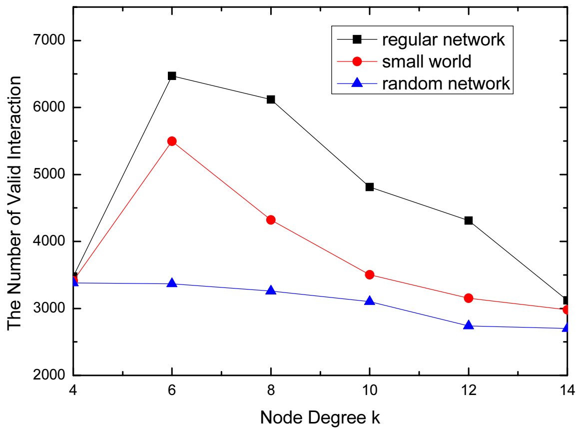

Fig.6. The valid interactions when opinions come to stable

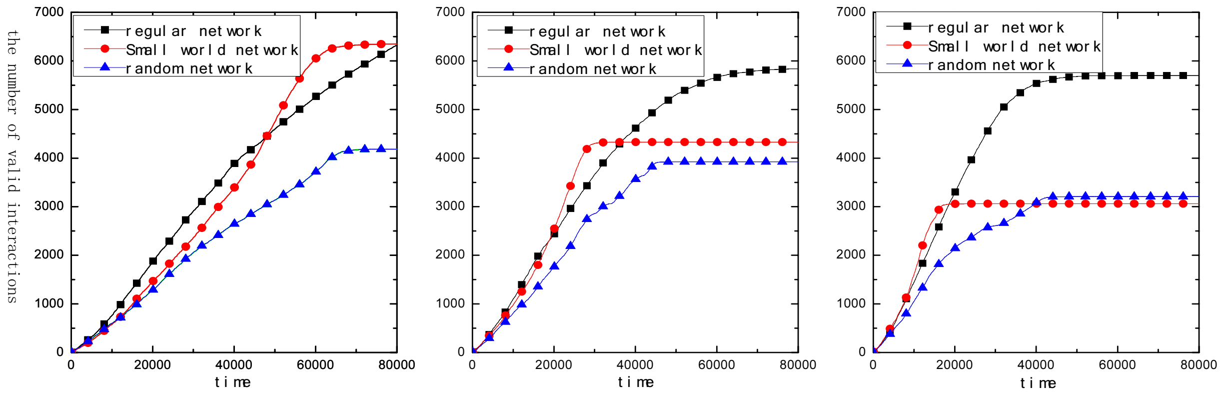

Fig.7. The time step of opinion evolution when k = 6, 8, 10

At same time, we also analyzed the time step that is required for individuals' opinions come to stable and the results are plotted in Fig.7. From the results, we found that the individuals' opinions in random network are the fastest come to stable, while it is the slowest when individuals in regular network at the same node degree. With the increase of node degree, the time that the individuals' opinions come to stable is shorten obviously. It indicates that the node degree have greater effect on the formation of opinions, it also implicate that the randomness of network is helpful and can speed up the diffusion of opinion. But we also found that there is no association between the speed of convergence and the clusters of opinions.

Finally, to investigate the influence of the scale of individuals on the formation of opinions, we also conducted these above experiments for 200 individuals, ie. N = 200. With the increase of k, on regular network, the number of opinion cluster is decreased, but it would not be reduced to 1, namely it cannot form consensus among individuals. The suitcase also exists in random network, and small network. Therefore, with increase of the scale of individuals, it is more difficult to come to consensus, and it is coincide with the real case that the opinion formation on internet.

4 Conclusions

In past decade, many research works on opinion formation have been done, and many classical models have been proposed, which explained the mechanism and many phenomena of public opinion formation, and revealed that the essence of dynamics of opinion evolution. In recent years, some researchers studied the opinion formation on network, and found some interesting features of opinion spread in small world network. But the structure of network is how to influence the formation of opinion is deserve to be studied. This paper studied the influence of similarity and randomness of network on opinion formation in the view of network topology.

Under the same population scale, node degree and the initial opinions distribution, individuals are easiest to come to consensus on regular network. With the increase of degree, the probability of consensus is also increase. The randomness can speed up the spread of opinion, which can shorten the time required for opinion formation, but it would also make the individuals' opinions more diversity. It implicates that there are more types of opinions among individuals who have more random connections. The results also implicate that the small world is the best place where consensus is easier to form, and the time required for opinions come to stable is shorter compared with random or regular networks. Therefore, the small world network which is reconstructed from the regular network by introduced some little randomness between individuals is very fit for opinion spread.

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