A Novel Joint Optimal Power Control and Channel Assignment of WLAN-based Cognitive Network

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Abstract. Channel interference caused by the Access Points (APs) of a secondary Multiple Input Multiple Output (MIMO) WLAN, is considered in this paper when they cognitively access to the TV White Space (TVWS) of primary network. This paper constructs a typical scenario where heterogeneously exists TV broadcast primary network and secondary network based on WLAN, and analyses the interference of Cognitive APs (CAPs) on primary network and the channel interference among the APs in secondary network. In order to reduce the channel interference, and meet both the communication Quality of Service (QoS) in primary network and maximum throughput of secondary network, the CAP channel assignment problem is modeled as a mixed integer linear programming problem. A joint optimal power control and channel assignment algorithm called JOPCCA is proposed. The optimal solution is obtained by Karush-Kuhn-Tucker (KKT) theory. A reasonable spectral sharing scheme is designed for CAPs access to the mixed Industrial Scientific Medical (ISM) and TVWS bands. Numerical simulation results show that the proposed algorithm can effectively reduce the channel interference of CAP on primary network and the channel interference among APs in secondary network, allocate the mixed ISM and TVWS channels reasonably and optimize the transmit power of CAPs.

Keywords: channel assignment, channel interference, cognitive radio, power control, TV white space

1 Introduction

With the increasing demand of multimedia services, cellular communication (3G/4G) is unable to meet users’ demands. The Wireless Local Area Network (WLAN) gains mobile operators’ large-scale commercial deployment because of its high rate, easy deployment and low costs. However, for the lack of unified commercial WLAN deployment and channel management standards, it is very common that the Access Points (APs) deployed by different operators in the same hot spot area are configured in the co-frequency channel, which leads to channel interference and, in worst case, may result in service interruption. Therefore, effectively solving the channel interference problem of APs in urban hot spots area is one of the key technologies on large-scale commercial deployment for WLAN.

There are mainly two ways to solve the channel interference problem of APs: the first one is to allocate suitable channels to APs in unauthorized Industrial Scientific Medical (ISM). However, the ISM has been extremely congested and the number of channels is limited. Another way is to explore the authorized but less utilized band in a cognitive way. Since the licensed spectrum for TV broadcast (e.g. the Ultra High Frequency (UHF) band) is cleared with significant capacity available for cognitive access, one of the most promising applications is that WLAN access to the TV White Space (TVWS) cognitively. TVWS are frequencies made available for unlicensed use at locations where the spectrum is not being used by licensed services, such as television broadcasting. This spectrum is distributed in the VHF (54-
216 MHz) and UHF (470-698 MHz) bands, in the US, and has characteristics that make it highly desirable for wireless communications. The regulatory document for the unlicensed operation in the TVWS took effect by the Federal Communications Commission (FCC) of the USA in 2012 [1]. In Japan, the Ministry of Internal affairs and Communications (MIC) presented a guideline to take technical discussions for effective use of the TVWS for the area limited broadcasting systems, sensor network systems, and broadband communications [2]. In accordance with these circumstances, IEEE 802.11af for WLAN has standardized wireless communication systems operating in TVWS spectrums [3]. However, research on cognitive access in TVWS is still in the initial stage and a huge number of technical issues have to be addressed before expanding WLAN into TVWS cognitively [4].

Due to the scarcity of spectrum resources, in traditional wireless communications, there are a large amount of literature study on channel assignment [5-6]. There are two algorithms of channel assignment: centric channel assignment and distribute channel assignment. The methods to achieve channel assignment are tabu search [7], genetic algorithm [8, 10] and greedy algorithm based on payloads [9].

In recent years, the research mostly focuses on how to optimize and reduce the complexity of power and channel assignment in cognitive network. An algorithm of assignment of joint channel and power is provided [11], the interference caused by secondary users for primary users can be modeled as an optimization problem of NP-Hard. A solution of assignment for cognitive network channel via weighted graphs is given [12]. An assignment of power and spectrum resource by Non-Cooperative Game Theory is put forward [13]. A comprehensive guideline of channel assignment based on cognitive networks is shown in [12]. However, there are still no practical schemes for integrated channel assignment and power assignment based on WLAN ISM and TVWS frequency band.

In order to improve the network throughput to meet users’ increasing demands, a WLAN and its cognitive way to access the TVWS is studied in this paper. We discuss the power and channel assignment of cognitive network based on WLAN which accesses to the TVWS in an opportunistic way. The main contributions of this paper are listed as below:

1. This paper constructs a typical scenario where heterogeneously exists TV broadcast primary network and secondary network based on WLAN, and analyses the interference of Cognitive APs (CAPs) on primary network and the channel interference among the APs in secondary network.

2. In order to reduce the channel interference, and meet both the Quality of Service (QoS) in primary user network and maximum throughput of secondary user network, the CAP channel assignment problem is modeled as a mixed integer linear programming problem. A joint optimal power control and channel assignment algorithm called JOPCCA is proposed. The optimal solution is obtained by Karush-Kuhn-Tucker (KKT) theory. A reasonable spectral sharing scheme is designed for APs access to mixed ISM and TVWS bands.

The rest of this paper is organized as follows. Section 2 describes the problem and model of channel interference. Section 3 presents the optimal power and channel assignment algorithm. Simulation results and discussions is presented in Section 4. Concluding remarks are in Section 5.

2 Description of problem and the model of channel interference

We introduce a communication scenario between CAPs based on WLAN and PU network in this section, and interference model of CAPs on PU receiver is given. Furthermore, we analyze the mutual interference between CAPs.

2.1 Communication model

Primary network is a TV broadcast network consisting of a Primary Base Station (PBS) and Primary User (PU) receivers. This paper takes a specific scenario as example, where PBS continuously transmits TV programs while PU receivers keep receiving. In other words, the primary network works in a “saturated” mode. The secondary network consists of a set of CAPs and WLAN users.

2.2 Analysis of PU’s saturation throughput

Assume that primary network uses Distributed Coordination Function (DCF) in MAC layer, then the saturation throughput can be expressed as [14]:

\[ \text{Saturation Throughput} = \frac{\text{Bandwidth}}{\text{Error Rate}} \]
where $E[P]$ is the average packet payload size which is determined by upper layer protocol, $\omega$ is the duration of an empty slot time which is related to the features of PHY, they are constants. $T_s$ is the average time the channel is sensed busy (i.e., the slot time lasts) because of a successful transmission. $T_c$ is the average time the channel is sensed busy by each PU during a collision. $P_{tr}$ is the probability that there is at least one transmission in the considered slot time. $P_s$ stands for the probability that exactly one PU transmits on the channel, conditioned on the fact that at least one PU transmits. $P_{tr} = 1 - (1 - r)^n$. $P_s = \frac{n \tau(1 - r)^{n-1}}{P_{tr}}$. The probability $\tau$ that a PU transmits in a randomly chosen slot time can be written as:

$$\tau = \frac{2(1 - 2p)}{(1 - 2p)(CW_{\min} + 1) + p \cdot CW_{\min} (1 - (2p)^w)}$$

where $CW_{\min}$ is minimum contention window. $p$ is the probability that at least one of the $n$-1 remaining PUs transmit when a transmitted packet encounters a collision in a time slot, it can be written as:

$$p = 1 - (1 - r)^{n-1}$$

Thus, by formulas (1), (2) and (3), we can obtain the throughput of primary network under the saturation condition.

2.3 Interference of CAPs on PU receiver

The interference model of CAPs on PU receiver is illustrated in Fig.1.

**Fig. 1. Communication interference model of CAPs on PU receiver**

As shown in Fig.1, $R_{PI}$ is the interference radius of PU, $R_{PC}$ is the communication radius of PU, $R_{CI}$ is the interference radius of CAP, $R_{\min}$ is minimum interference radius of CAP in order to guarantee the QoS of PU, and $r$ is the actual interference radius of CAP and PU. The CAPs are in a uniform distribution with a density of $\lambda$, in the ring between $R_{\min}$ and $R_{SI}$.

PU’s QoS is represented by the outage probability. The actual transmission rate of PU is $C_P$. PU requires a minimum transmission rate of $C_{P0}$ to ensure that it can receive TV programs successfully. The
outage probability $\beta$ of PU satisfies:

$$P[C_p \leq C_p^*] \leq \beta, \quad 0 \leq \beta \leq 1$$  \tag{4}

The capacity of PU channel under the interference of CAPs is:

$$C_p = B_p \sum_{i=1}^{L_p} \log_2 \left( 1 + \frac{P_i}{\delta^2 + T} \sigma_i^2 \right)$$  \tag{5}

where $B_p$ is the bandwidth of PU channel, $r_{Pi}$ is the rank of PU’s MIMO channel matrix $H$, $\sigma_i^2$ is the $i$th singular value of $H$, $P_i$ is the power assigned to the $i$th array-antenna, $\delta^2$ is the noise variance, and $T$ is the average interference power of CAPs on PU receiver.

To calculate the average interference power $T$ of CAPs, $r$ is also referred to express the distance between any CAP and the PU receiver. With uniform location distribution, the density of $r$ is:

$$f(r) = \frac{2r}{R_{SI}^2 - R_{min}^2}, \quad R_{min} \leq r \leq R_{SI}$$  \tag{6}

The single average interference of CAPs on PU receiver is:

$$T_0 = \int_{R_{min}}^{R_{SI}^2} f(r) I_r \, dr = \int_{R_{min}}^{R_{SI}^2} \frac{2rP_{st}}{(R_{SI}^2 - R_{min}^2) r^\alpha} \, dr = \frac{2P_{st}}{(R_{SI}^2 - R_{min}^2)(\alpha - 2)} \left( \frac{1}{R_{min}^\alpha} - \frac{1}{R_{SI}^\alpha} \right)$$  \tag{7}

where $I_r$ is the CAP interference on PU receiver with a distance of $r$, $P_{st}$ is the transmit power of CAP, and $\alpha$ is the path loss factor.

The average number of CAPs accessing to TVWS band in the ring is:

$$n_s = \lambda_s \pi (R_{SI}^2 - R_{min}^2)$$  \tag{8}

where $\lambda_s$ represents density of distribution of CAPs, So the average interference of all CAPs in the interference area is:

$$T = n_s T_0 = \frac{2\pi \lambda_s P_{st}}{\alpha - 2} \left( \frac{1}{R_{min}^\alpha} - \frac{1}{R_{SI}^\alpha} \right)$$  \tag{9}

The Signal to Interference plus Noise Ratio (SINR) for a smooth communication of PU is:

$$\gamma_p = \sum_{k=1}^{L_p} P_{nk} R_p^{-\alpha} + N_p B_p$$  \tag{10}

where $\gamma_p$ is the real-time SINR of PU, $P_{mu}$ is the transmit power of PBS, $P_{nk}$ is the transmit power of AP$k$ on channel $n$, $r_k$ is the distance between AP$k$ and PU receiver, $N_p$ is the one-sided noise Power Spectral Density (PSD) of PU network, $B_p$ is the bandwidth of PU channel, and $L$ is the actual number of the accessed CAPs ($L \leq n_s$) in the ring.

The SINR of PU when interferenced by the average access of CAPs is:

$$\gamma_0 = \frac{P_i}{\delta^2 + T} \sigma_i^2 ,$$  \tag{11}

where $\gamma_0$ is the minimum SINR threshold satisfying PU’s outage probability $\beta$.

From formula (10) and (11), we can obtain the communication restriction of PU when satisfying the QoS as follows:

$$\gamma_p \geq \gamma_0$$  \tag{12}

From formula (10), (11) and (12), we can get the maximum transmit power of $P_{nk}$:
\[ P_{n,k} \leq \frac{P_m}{L\gamma_0} \left( \frac{R_{PC}}{r_k} \right)^{-\alpha} - \frac{N_p B_P}{r_k^{\alpha}} = P_{n,k,\max} \] (13)

Meanwhile, to guarantee the cognitive user’s basic communication, the SINR of AP\(_k\) on channel \(n\) is constrained by a QoS threshold:

\[ \gamma_{n,k} \geq P_{n,k} g_{n,k} \geq \gamma_{0,k} \] (14)

where \(g_{n,k}\) is the channel gain of AP\(_k\). We can get:

\[ P_{n,k} \geq \frac{\gamma_{0,k}}{g_{n,k}} = P_{n,k,\min} \] (15)

2.4 Interference model among CAPs

Two overlapped CAPs and the actual throughput subjected to the channel interference between these two CAPs are considered. All the CAPs are located in the CAP interference area, as shown in Fig.2.

![Communication model in overlap area](image)

Fig. 2. Communication model in overlap area

In Fig.2, \(M\) is the number of terminals in each AP communication area, AP\(_i\)’s communication area is \(A_i\), and the adjacent APs (AP\(_i\), AP\(_j\)) have an interference area \(A_{ij}\) (the shadow area in the middle of Fig.2). \(M_{ij}\) is the number of terminals associated with AP\(_i\) but interfered by AP\(_j\), and vice versa. The channel set of ISM is \(F_{\text{ISM}} = \{f_1, ..., f_{11}\}\) while that of TVWS is \(F_{\text{TVWS}} = \{f_{tvws,1}, ..., f_{tvws,v}\}\). \(F = F_{\text{ISM}} \cup F_{\text{TVWS}}\) is used to express the channel set of the mixed ISM and TVWS. The interference of AP\(_j\) over AP\(_i\) is represented by an Interference Penalty (IP) index \(IP_{ij}(f_i, f_j)\):

\[ IP_{ij}(f_i, f_j) = \frac{A_{ij}}{A_i} \times \rho(f_i, f_j) \] (16)

\[ \rho(f_i, f_j) = \begin{cases} \max \left(1 - \frac{1}{O \times |f_i - f_j|}, 0\right) & \text{if } f_i, f_j \in F_{\text{ISM}} \\ 0 & \text{else} \end{cases} \] (17)

where \(f_i\) and \(f_j\) are the frequencies assigned to AP\(_i\) and AP\(_j\) respectively, \(\rho(f_i, f_j)\) is channel overlap factor, \(|f_i - f_j|\) is the absolute value of the channel designator corresponded to \(f_i\) and \(f_j\), and \(O\) is the orthogonal channel interval number. In ISM band, channel 1, 6 and 11 are orthogonal channels, so \(O=5\) [15].

3 Optimal power and channel assignment algorithm

In this section, we present the objective function, that is, the optimal power and channel assignment of CAPs. Then the solving process of the proposed objective function is given in details, and the spectrum
sharing scheme of CAPs in ISM and TVWS bands is also presented.

3.1 Solving the objective function

The aim of this paper is to find a joint power and channel assignment to meet the outage probability of the PU as well as the QoS of the secondary users.

Let $N$ denote the total channel number of ISM and TVWS channels, we can define a target function as below:

$$ f = \max \overline{C} $$

where $\max \overline{C}$ is the maximum value of the channel capacity in cognitive network. $\overline{C} = C / B_S$ is spectral efficiency of the cognitive network, which $C$ is channel capacity of the cognitive network, $B_S$ is bandwidth of each CAP’s channel. $\overline{C}$ is given as:

$$ \overline{C} = \sum_{k=1}^{L} \sum_{n=1}^{N} \sum_{l=1}^{N_0} \log_2 \left( 1 + \frac{P_{l,n,k} \sigma^2_{l,n,k}}{N_0 B_S + IP_{n,k}} \right) $$

where $P_{l,n,k}$ is the $i$th array-antenna power of $P_{n,k}$, $\sigma^2_{l,n,k}$ is the $i$th singular value of $H_0$ of $AP_k$ on channel $n$, $c_{n,k} = \{0,1\}$ denotes the assignment of channel $n$ to $AP_k$, $m \geq 1$ is a fuzzy weighted factor, $N_0$ is the one-sided noise PSD of secondary network, and $r_{nk}$ is the rank of CAP’s MIMO channel matrix.

Assuming each CAP has the same transmit power $P_{n,k} = P_{l,n,k}$ and the same channel gain $g_{n,k} = \frac{\sigma^2_{l,n,k}}{N_0 B_S + IP_{n,k}}$. Therefore, equation (19) can be simplified as:

$$ C_{\text{norm}} = r_{nk} \sum_{l=1}^{L} \sum_{n=1}^{N} \log_2 \left( 1 + P_{n,k} g_{n,k} \right) $$

For all $n, k$, subject to:

$$ \sum_{k=1}^{L} c_{n,k} = 1, c_{n,k} (1 - c_{n,k}) = 1, \sum_{n=1}^{N} P_{n,k} \leq P_{k,\text{max}} $$

$$ P_{n,k} \geq c_{n,k} P_{n,k,\text{min}}, P_{n,k} \leq c_{n,k} P_{n,k,\text{max}} $$

where $P_{k,\text{max}}$ is $AP_k$’s maximum transmit power, $P_{n,k,\text{min}}$ and $P_{n,k,\text{max}}$ are respectively determined by the of formula (15) and the formula (13).

The problem can be modeled as a mixed-integer nonlinear programming problem, which can be solved by the classical KKT theory [16].

We first construct a Lagrangian function:

$$ L_o(c_{n,k}, P_{n,k}, \lambda_k, \beta_n, u_{n,k}) $$

$$ = C_{\text{norm}} + \sum_{n=1}^{N} \beta_n \left( \sum_{k=1}^{L} c_{n,k} - 1 \right) + \sum_{n=1}^{N} \sum_{k=1}^{L} \beta_n c_{n,k} (1 - c_{n,k}) + \sum_{k=1}^{L} \lambda_k (P_{k,\text{max}} - \sum_{n=1}^{N} P_{n,k}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\text{min}}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu'_{n,k} (c_{n,k} P_{n,k,\text{max}} - P_{n,k}) $$

$$ = C_{\text{norm}} + \sum_{n=1}^{N} \beta_n \left( \sum_{k=1}^{L} c_{n,k} - 1 \right) + \sum_{n=1}^{N} \sum_{k=1}^{L} \beta_n c_{n,k} (1 - c_{n,k}) + \sum_{k=1}^{L} \lambda_k (P_{k,\text{max}} - \sum_{n=1}^{N} P_{n,k}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\text{min}}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu'_{n,k} (c_{n,k} P_{n,k,\text{max}} - P_{n,k}) $$

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$$ = C_{\text{norm}} + \sum_{n=1}^{N} \beta_n \left( \sum_{k=1}^{L} c_{n,k} - 1 \right) + \sum_{n=1}^{N} \sum_{k=1}^{L} \beta_n c_{n,k} (1 - c_{n,k}) + \sum_{k=1}^{L} \lambda_k (P_{k,\text{max}} - \sum_{n=1}^{N} P_{n,k}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\text{min}}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu'_{n,k} (c_{n,k} P_{n,k,\text{max}} - P_{n,k}) $$

$$ = C_{\text{norm}} + \sum_{n=1}^{N} \beta_n \left( \sum_{k=1}^{L} c_{n,k} - 1 \right) + \sum_{n=1}^{N} \sum_{k=1}^{L} \beta_n c_{n,k} (1 - c_{n,k}) + \sum_{k=1}^{L} \lambda_k (P_{k,\text{max}} - \sum_{n=1}^{N} P_{n,k}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\text{min}}) + \sum_{n=1}^{N} \sum_{k=1}^{L} \mu'_{n,k} (c_{n,k} P_{n,k,\text{max}} - P_{n,k}) $$

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Solving the first order partial derivative of $P_{n,k}$ and $C_{n,k}$ in formula (23) respectively, we can obtain the KKT conditions as follows:

$$\lambda_k \geq 0, \mu_{n,k} \geq 0, \mu_{n,k}' \geq 0$$  \hspace{1cm} (24)

$$\frac{\partial L}{\partial P_{n,k}} = m(c_{n,k})^m - \frac{\gamma H_0}{\lambda_k} \ln 2 \left(1 + P_{n,k} g_{n,k}\right) + \beta_n$$

$$+ \beta_n' \left(1 - 2c_{n,k}\right) - \mu_{n,k} P_{n,k,\min} + \mu_{n,k}' P_{n,k,\max} = 0$$  \hspace{1cm} (25)

$$\frac{\partial L}{\partial C_{n,k}} = 0 \rightarrow \gamma H_0 \left(\frac{c_{n,k}}{\lambda_k}\right)^m - \frac{g_{n,k}}{1 + P_{n,k} g_{n,k}} - \lambda_k + \mu_{n,k} - \mu_{n,k}' = 0$$  \hspace{1cm} (26)

$$\sum_{k=1}^L \lambda_k (P_{n,k,\max} - \sum_{n=1}^N P_{n,k}) = 0$$  \hspace{1cm} (27)

$$\sum_{n=1}^N \sum_{k=1}^L \mu_{n,k} (P_{n,k} - c_{n,k} P_{n,k,\min}) = 0$$  \hspace{1cm} (28)

$$\sum_{n=1}^N \sum_{k=1}^L \mu_{n,k}' (c_{n,k} P_{n,k,\max} - P_{n,k}) = 0$$  \hspace{1cm} (29)

Since the KKT conditions formula (25) and formula (26) have many parameters and it is difficult to obtain the optimal solution. This can be approximately solved by the Water-Filling [17] as follows.

We can derive $P_{n,k}$ as a Water-Filling:

$$P_{n,k} = (c_{n,k})^m \frac{\gamma H_0}{\lambda_k} - \frac{1}{g_{n,k}}$$

$$= \begin{cases} P_{n,k}^0 = \frac{\gamma H_0}{\lambda_k^0} - \frac{1}{g_{n,k}} & \text{if } \lambda_k^0 < \frac{\gamma H_0 g_{n,k}}{\ln 2}, \ c_{n,k} = 1 \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (30)

where $\lambda_k^0 = \lambda_k - \mu_{n,k} + \mu_{n,k}'$.

Assuming $\lambda_k$ is constant relative to $n$, we can get:

$$\lambda_k' = \frac{|\Omega_k|}{P_{k,\max}^0 + \sum_{n \in \Omega_k} \frac{1}{g_{n,k}}}$$  \hspace{1cm} (31)

where $\Omega_k$ is the set of channels that are assigned to AP $k$.

Power constraints are considered in formula (19). After estimating the power by formula (27), we then force the estimated power to satisfy the power constraints:

$$P_{n,k} = \max(\min(P_{n,k}^0, P_{n,k,\min}), P_{n,k,\min})$$  \hspace{1cm} (32)

An iterative Water-Filling algorithm for JOPCCA is given as follows, and the algorithm converge to the optimal values within a few iterations.
Algorithm: JOPCCA

Input:
$c_{n,k} \leftarrow \emptyset, i \leftarrow 1$

Iteration:

Count channel gain $g_{n,k}$
Calculate $P_{n,k,min}$, $P_{n,k,\max}$ by formula (15) and (13)
while (true)
  for each channel $n=1$ to $N$
    NoUsing$= \emptyset$, Using$= \emptyset$
    for each CAP, $k=1$ to $L$
      if $P_{n,k,min} \leq P_{n,k,\max}$ and $P_{n,k,min} \leq P_{k,\max}$
        $c_{n,k} \leftarrow 1$, Using$=\text{Using}\cup\{k\}$
        $\Omega = \Omega \cup \{n\}$
        Calculate $\lambda' k$ by formula (31)
        Calculate $P_{n,k}$ by formula (30) and (32)
        $c_{n,k} \leftarrow \emptyset$, $\Omega = \Omega \setminus \{n\}$
      else // this channel is not assigned
        $c_{n,k} \leftarrow \emptyset$, NoUsing$=\text{NoUsing}\cup\{k\}$
      end if
    end for
    if Using$\neq \emptyset$
      $k' \leftarrow \max_{\{k'\in \text{Using}\}} P_{n,k}$
      $c_{n,k'} \leftarrow 1$, $\Omega = \Omega \cup \{n\}$
      Calculate $\lambda' k'$ by formula (31)
      Calculate $P_{n,k'}$ by formula (30) and (32)
    end if
  end for
  $C(i) \leftarrow \{c_{n,k}\}$
  Calculate $C(i)$ by formula (20)
  if ($d(C(i), C(i-1))$ is below a threshold)
    break
  end if
  $i \leftarrow i+1$
end while.

Output:
$\lambda' k$, $P_{n,k}$, $P_{n,k'}$.

In the pseudo code, $d(C(i), C(i-1))$ is the Hamming distance between the two matrices.

3.2 Spectrum sharing scheme

The spectrum sharing scheme in mixed ISM and TVWS bands when adopting JOPCCA is described as follows:

(1) CAPs are allocated in ISM band for the orthogonal channels (1, 6 and 11), idle channels, the adjacent channels and the co-frequency channels in order.

(2) When reaching the maximum $IP$ threshold $IP_{S\_MAX}$ in ISM band, CAPs access to TVWS band in opportunistic way. In such case the idle channels, in which all channels are orthogonal, are firstly adopted.

(3) When all the PU idle channels are allocated, CAPs access to the co-frequency channels in TVWS band. However, the maximum $IP$ threshold of PU $IP_{P\_MAX}$ should not be exceeded all the time and the successful communication QoS of PU in existing channels must be guaranteed always.

4 Simulation results and discussions

In this section, we verify the effectiveness of JOPCCA algorithm by simulation, it shows that JOPCCA can improve spectrum efficiency of CAPs in ISM and TVWS bands, and also, it has an ability to optimize CAPs’ transmit power and reduce mutual interference effectively.

In the simulation, the distance between PBS and PU receiver $R_{PC}$ is 1 km, the distance from the CAP to PU $R_{st}$ is 20 m, PBS transmit power $P_{pt}=5$ W, all the CAPs have the same maximum transmit power.
$P_{k,\text{max}}=200\text{ mW}$, the path loss factor $\alpha$ is 2, and PU’s outage probability $\beta$ is 0.05. TVWS band is set to have 40 orthogonal channels with a bandwidth of 5 MHz. ISM band is to set 11 overlapping channels with 22 MHz each. The other simulation parameters are shown in Table 1.

Table 1. Simulation parameters settings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of the network topology</td>
<td>$1.5\text{ km} \times 1.5\text{ km}$</td>
</tr>
<tr>
<td>PU’s outage probability $\beta$</td>
<td>0.05</td>
</tr>
<tr>
<td>mean square error of noise $\delta$</td>
<td>0.01 mW</td>
</tr>
<tr>
<td>minimum transmission rate $C_{P0}$</td>
<td>4 Mbps</td>
</tr>
<tr>
<td>the bandwidth of PU channel $B_P$</td>
<td>5 MHz</td>
</tr>
<tr>
<td>the rank of MIMO channel matrix $r_H$</td>
<td>1</td>
</tr>
<tr>
<td>Number of orthogonal channels of TVWS</td>
<td>40</td>
</tr>
<tr>
<td>Number of overlapping channels of ISM</td>
<td>11</td>
</tr>
<tr>
<td>minimum interference radius of CAP $R_{\text{min}}$</td>
<td>10 m</td>
</tr>
<tr>
<td>the interference radius of CAP $R_{\text{CI}}$</td>
<td>100 m</td>
</tr>
<tr>
<td>density of distribution of CAPs $\lambda$</td>
<td>0.0005</td>
</tr>
<tr>
<td>The maximum number of CAP which could be allowed to access to TVWS by primary users</td>
<td>15</td>
</tr>
</tbody>
</table>

4.1 The relationship of iterations and spectrum efficiency

The relationship of iterations and spectrum efficiency when $N=12$ is shown in Fig.3. We can conclude that (1) the optimal channel and power control are effective by JOPCCA, that is, it can achieve a stable convergence by finite number of iterations; (2) the number of iterations increases with $L$ increases when $N$ remains unchanged, spectrum efficiency, instead, decreases.

4.2 The relationship among spectrum efficiency, the number of channels and users

For a better verification of the performance of the proposal in this paper, we analyze JOPCCA algorithm and MST algorithm which is provided [15] in numerical way.

![Fig. 3. The relationship of iterations and spectrum efficiency](image)

In Fig.4, given the same $N$ and $L$, when the number of the orthogonal channels is larger than $L$, the JOPCCA and MST achieve the same channel capacity $C_{\text{norm}}$. When $L$ is greater than the number of the orthogonal channels, JOPCCA has a higher $C_{\text{norm}}$ than MST. This is because MST considers $IP$ only, but
JOPCCA takes into account both $IP$ and transmit power. When $N$ is fixed, $C_{\text{norm}}$ increases while $L$ increases.

![Graph](image_url)

**Fig. 4.** Spectral efficiency of cognitive network under different $N$ and $K$

This can be understood as, when $L$ increases, the associated terminals increases thus $C_{\text{norm}}$ increases accordingly. When $L$ reaches the average tolerance number $n_s$ (as shown in formula (8)), the system is saturated and $C_{\text{norm}}$ increases slowly.

![Graph](image_url)

**Fig. 5.** Correlation of CAP spectral efficiency and maximum transmit power

In Fig.5, with the same $N$, $L$ and $P_{k,\text{max}}$, the $C_{\text{norm}}$ of JOPCCA is higher than that of MST. Since there is no power control in MST, the increasement of mutual interference among CAPs affects the increasement of $C_{\text{norm}}$ seriously. When $P_{k,\text{max}}>15\text{dBm}$, the system adopting MST is saturated. Whereas with power control in JOPCCA, transmit power of CAPs can be optimized according to the channel interference, even if when $L$ is far more than $N$ (i.e. $L=40$, $N=18$), $C_{\text{norm}}$ can also achieve a higher throughput.
4.3 The mutual interference between primary users and cognitive users

To measure the impact that JOPCCA method has on the performance of system interference, we define that the average rate of interference which receiver receives from channel interference as $\eta$:

$$\eta = \frac{P[I]}{P[I] + P}$$

(33)

where $P[I]$ means average interference of users who receive from a network (PU network or cognitive network), $P$ stands for available signal power of users who receive from corresponding transmitter.

Also, we define the average rate of interference of the whole system as $\sigma$:

$$\sigma = \frac{\sum_{i=1}^{M} \eta_{p_i} + \sum_{j=0}^{N-M} \eta_{c_j}}{N}$$

(34)

where $\eta_{p_i}$ represents the average rate of interference of primary users which work in TVWS band, $M$ means the number of receivers who work in primary user network, $\eta_{c_j}$ stands for the average rate of interference of primary users which work in ISM band of the network whose number of receivers is $N-M$, the number of interference transmitters (number of CAP) in the system is $N$.

The results of simulation are shown in Fig 6.

![Graph showing average interference of the system under different numbers of cognitive APs](image-url)

Fig. 6. The average interference of the system under different numbers of cognitive APs

The $x$-axis is the number of CAP ($N$), $y$-axis means $\sigma$. The values $\sigma$ of two algorithms are the same before mutual interference of users is more than $IP_{c_{\text{max}}}$, the reason is that the ISM band resource is enough when in low number of CAP. Particularly, the interference is 0 when the number of AP is no more than 3, because of 3 orthorhombic channels (1/6/11) of ISM.

The interference of MST is higher than JOPCCA’s with the number of CAP increases after mutual interference of users is more than $IP_{c_{\text{max}}}$. The reason is that MST method only can allocate ISM band, the channel interference increases with the number of CAP under the condition that mutual interference is over than $IP_{c_{\text{max}}}$, as a consequence, the average interference of the system increases. However, JOPCCA method considers both ISM band and TVWS band. The maximum number of CAP who are allowed to access to TVWS band is 15 without having an impact on QoS of primary users before mutual interference is more than $IP_{p_{\text{max}}}$. JOPCCA keeps on allocating channels following the rule of minimum interfer-
ence after orthorhombic channels of TVWS band run out. Nevertheless, MST algorithm is worse than JOPCCA before \( IP_{p_{\text{max}}} \).

After MST algorithm is more than \( IP_{p_{\text{max}}} \), JOPCCA won’t allocate channels to new CAP, instead, it will do it in ISM band in order to protect primary users, but MST keeps on allocating in TVWS band, so the average interference of MST is higher than JOPCCA’s after \( IP_{p_{\text{max}}} \).

5 Conclusion

This paper proposes a joint optimal power and channel assignment algorithm to meet both the outage probability of the primary TV broadcast network and the QoS of the secondary WLAN. The transmit power of the CAPs is optimized simultaneously. This algorithm allows the existing WLAN in the congested ISM band access the less utilized channels in TVWS cognitively. This simulation results show that, this algorithm is able to increase the throughput of the WLAN and keep load balance of inter-BSS while guarantee the QoS of the primary TV broadcast network. The further work of this paper is to optimize antenna transmit power, channel allocation and downlink beamforming for secondary WLAN with QoS guarantee of the primary network, in the condition of considering primary network and secondary WLAN that both work in MU-MIMO system.

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References


A Novel Joint Optimal Power Control and Channel Assignment of WLAN-based Cognitive Network