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Rational Decision Making Models with Incomplete Information Based on Interval-valued Fuzzy Soft Sets

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Abstract. This paper aims to give deeper insights into decision making problem with incomplete known or complete unknown weight information based on interval-valued fuzzy soft set. Since uncertain information and incomplete knowledge coexist inevitably in the assessment process, in literature, many fuzzy multi-attribute decision making models are established to handle them simultaneously. However, very few papers have paid adequate attention to the decision maker's rationality, i.e. the optimism level and pessimism level of the decision maker. Therefore, in this paper, we provide a novel and rational multi-attribute decision making (MADM) model, which the evaluating data are expressed in interval-valued fuzzy soft set. For the incomplete information, we discuss two optimization models based on the basic ideal of traditional grey relational analysis (GRA) method, by which the attribute weights can be determined. Moreover, we compare the two optimization methods and point out that the second optimization method is more reasonable. For the special situations, where the information about criterion weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. Finally, a real life application for supplier selection is given to clarify the proposed approach which is not only valid, but also can reflect the decision makers' rationality on the influence of the final result.

Keywords: fuzzy multi-attribute decision making, grey relational analysis (GRA), incomplete weight information, interval-valued fuzzy soft set, rational decision making

1 Introduction

Multiple attribute decision making (MADM) is usually used to analyze a set of alternatives and choose the best alternative from them depending on multiple attributes. It occurs in a variety of actual situations, such as economic analysis, strategic planning, forecasting, medical diagnosis, supply chain management and many other areas. In real life, MADM problems have become even more prominent with increasing complexity of the social-economic environment. At the same time, the increasing complexity of the socio-economic environment has made it even more difficult for decision making.

The key information about multiple attribute decision making problems includes attribute values, attribute weights (reflecting the importance of each attribute to the overall decision problem) and a mechanism to synthe-size this information into an aggregated value or assessment for each alternative [1]. However, with increasing complexity in many decision situations in reality, it is often a challenge for a decision-maker (DM) to provide attribute values in a precise manner. Moreover, because of time pressure, the high of information cost or the expert's limited expertise about the problem domain, the information on criterion weights in the process of MADM is sometimes incompletely known or completely unknown. Therefore, a general trend in the literature is to investigate decision models with incomplete information [2-8].

In traditional fuzzy MADM problems with incomplete weight information, a positive ideal solution (PIS) or negative ideal solution (NIS) is chosen so as to be compared with all the alternatives. In literature, there are many fuzzy models dealing with uncertain information and incomplete knowledge. Some of recent research on the topic incorporates generalized interval-valued fuzzy numbers [9], triangular fuzzy number [10], intuitionistic fuzzy set [11-12], 2-tuple linguistic [13] and others. However, they few considered the decision makers' rational-ity. From the standpoint of a rational decision maker, not all of the attribute weights have to be configured as highly optimistic [14]. Actually, there are many decision makers with neutral or highly pessimistic viewpoints toward the assessment and this should also be considered when deriving the incomplete weight information. In real-life situations, optimistic decision-makers interpret their decision situations positively and expect favorable outcomes, whereas pessimistic decision-makers interpret these situations negatively and anticipate unfavorable outcomes [15]. Since optimistic, neutral and pessimistic experts conceive incomplete preference models with equal possibility, a rational approach method is necessary to elucidate the influences of optimism and pessimism on decision-making processes.

In addition, although these theories in literatures are very good for solving the incomplete information under uncertain environment, they are associated with an inherent limitation, which is inadequacy of the parameterization tool associated with these theories. For example, the methods of interval mathematics are not sufficiently adaptable for problems with different uncertainties. They cannot appropriately describe a smooth changing of information, unreliable, inadequate, defective information, partially contradicting aims and others. Fuzzy set is progressing rapidly but their existent posed some great difficulty. One good example is how to set the membership function in each particular case [16]. Yet, the soft set which was initiated by Molodtsov in1999 [16], a new mathematical tool can deal with uncertainties, which is free from the above limitations.

In recent years, research on soft set theory has become active and great progress has been achieved in theoretical aspect. At the same time, there has been some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making. Maji and Roy [17] introduced the definition of reduct-soft-set and described the application of soft set theory as a problem in decision-making. Mushrif et al [18] proposed a new classification algorithm of the natural textures, which was based on the notions of soft set theory. Zou and Xiao [19] presented data analysis approaches of soft set under incomplete information. Roy and Maji [20] proposed a novel method of object recognition from an imprecise multi-observer data and a decision making application of fuzzy soft set. Although the algorithm was proved incorrect by Kong et al [21], fuzzy soft sets and multi-observer concepts are valuable to successive researchers. Cagman and Enginoglu [22] defined products of soft sets and uni-int decision function. By using these new definitions, they constructed a uni-int decision making method which selected a set of optimum elements from the alternatives. Feng et al. [23] presented an adjustable approach to fuzzy soft set based on decision making and enhanced it with illustrations. Xiao et al. [17] presented a method based on interval-valued fuzzy soft set for multi-attribute group decision problems. In this paper, in order to deal with incomplete weight information, they translate interval-valued fuzzy soft set into fuzzy soft set and constructed optimal model according to the score of fuzzy soft set. Although the fuzzy soft set has been progressive in decision making, few literatures concentrated on the rational decision making with incomplete weight information.

Based on the above analysis, the aim of this paper is to extend the concept of Grey relational analysis (GRA) [24-25] to develop a methodology for solving rational MADM problems under uncertain environment, in which the attribute values take the form of interval-valued fuzzy soft set, and the information about attribute weights is incompletely known or completely unknown. In order to do that, the rest of this paper is organized as follows. In section 2, we briefly introduce some concepts of GRA method and interval-valued fuzzy soft sets. Section 3, we develop a practical method based on the traditional ideas of GRA for dealing with interval-valued fuzzy soft set decision making problem with incomplete weight information or complete unknown information. Section 4 investigates a real life application for supplier selection in order to clarify the proposed approach which is not only valid, but also can reflect the decision makers' rationality on the influence of the final result. Finally, the conclusion and discussion is given in section 5.

2 Preliminaries

In the following, we briefly introduce some basic concepts related to grey relational analysis method and interval-valued fuzzy soft sets.

2.1 Grey Relational Analysis Method

GRA proposed by Deng [24] is a tool of grey system theory for analyzing the relationship between a reference series and other series. The main procedure of GRA is firstly translating the performance of all alternatives into a comparability sequence. This step is called grey relational generating. According to these sequences, a reference sequence is defined. Then the grey relational coefficient between all comparability sequences and reference sequence is calculated.

Let $X = \{x_i | i = 0,1,2,...,n\}$ be a given grey relational factor set, suppose $x_i = \{x_i(1), x_i(2),..., x_i(m)\}$ is a data series, where $x_i(k) \in X$ is the value of $x_i (i \in \{0,1,2,..., n \in N\})$ at (time) point $k (1 \le k \le m \in N)$. Suppose x_0 is the reference series and $x_1, x_2,..., x_n$ are objective series, the grey relational coefficient $\gamma(x_0(k), x_i(k))$ between the reference series x_0 and the objective series $x_i (i \in \{1,2,..., n\})$ at (time) point $k \in \{1,2,..., m\}$ was as follows:

$$\gamma(x_0(k), x_i(k)) = \frac{\min_{i \in n} \min_{k} |x_0(k) - x_i(k)| + \rho \max_{i \in n} \max_{k} |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \rho \max_{i \in n} \max_{k} |x_0(k) - x_i(k)|} \tag{1}$$

Where ρ is the distinguishing coefficient, $\rho \in [0,1]$. The purpose of the distinguishing coefficient is to expand or compress the range of the grey relational coefficient. In this paper, we take $\rho = 0.5$.

2.2 Interval-valued Fuzzy Soft Set

In this section, we shall briefly explain some basic notions being used in this paper. Throughout this paper, let U be a set of objects and E be a set of parameters with respect to objects in U. The power set of U is denoted by $\tilde{P}(U)$.

Definition 1 (see [27]) Let U be an initial universe and E be a set of parameters, a pair $(\tilde{F}, E)$ is called an interval-valued fuzzy soft set over $\tilde{P}(U)$, where $\tilde{F}$ is a mapping given by

$$\tilde{F} : E \to \tilde{P}(U) \tag{2}$$

$\forall e \in E$, $\tilde{F}(e)$ is referred as the interval fuzzy value set of parameter e, it is actually an interval-valued fuzzy set of U, where $x \in U$ and $e \in E$, it can be written as: $\tilde{F}(e) = \{< x, \mu_{\tilde{F}(e)}(x) >: x \in U\}$, here $\mu_{\tilde{F}(e)}$ is the interval-valued fuzzy degree of membership that object x holds on parameter e. If $\forall e \in E, \forall x \in U$, $\mu^{-}_{\tilde{F}(e)}(x) = \mu^{+}_{\tilde{F}(e)}(x)$, then $\tilde{F}(e)$ will degenerate to be a standard fuzzy set and then $(\tilde{F}, E)$ will be degenerated to be a traditional fuzzy soft set.

Example 1 Suppose that there are five types of cars $h_j (j = 1,2,3,4,5)$,i.e. the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters is given by $E = \{e_1, e_2, e_3, e_4, e_5\}$, where e_i stand for "dynamic", "economy", "brake", "steering stability" and "smooth-going running" respectively. Let $A = \{e_1, e_2, e_3\} \subset E$ be consisting of the parameters that Mr. X is interested in buying a car. Now all the available information on cars under consideration can be formulated as an interval-valued fuzzy soft set $(\tilde{F}, A)$ describing "attractiveness of cars" that Mr. X is going to buy. Table 1 gives the tabular representation of the interval-valued fuzzy soft set $(\tilde{F}, A)$. We can view the interval-valued fuzzy soft set $(\tilde{F}, A)$ as the collection of the following fuzzy approximations:

Table 1. Table representation of the interval-valued fuzzy soft set $(\tilde{F}, A)$

U	e_1	e_2	e_3
h_1	[0.7,0.9]	[0.6,0.7]	[0.3,0.5]
h_2	[0.6,0.8]	[0.8,1.0]	[0.8,0.9]
h_3	[0.5,0.6]	[0.2,0.4]	[0.5,0.7]
h_4	[0.6,0.8]	[0.0,0.1]	[0.7,1.0]
h_5	[0.8,0.9]	[0.1,0.3]	[0.9,1.0]

$$\tilde{F}(e_1) = \{<h_1,[0.7,0.9]>,<h_2,[0.6,0.8]>,<h_3,[0.5,0.6]>,<h_4,[0.6,0.8]>,<h_5,[0.8,0.9]>\}$$
$$\tilde{F}(e_2) = \{<h_1,[0.6,0.7]>,<h_2,[0.8,1.0]>,<h_3,[0.2,0.4]>,<h_4,[0.0,0.1]>,<h_5,[0.1,0.3]>\}$$
$$\tilde{F}(e_3) = \{<h_1,[0.3,0.5]>,<h_2,[0.8,0.9]>,<h_3,[0.5,0.7]>,<h_4,[0.7,1.0]>,<h_5,[0.9,1.0]>\}$$

The interval-valued fuzzy soft set is combined the interval-valued fuzzy set and soft set models. The interval-valued fuzzy set operations based on the arithmetic operations with membership functions do not look natural. It may occur that these operations are similar to the addition of weights and lengths [16]. The reason for the difficulties is possibly the inadequacy of the parameterization tool of the theory, while the interval-valued fuzzy soft set is free of the difficulties mentioned above.

Definition 2 (see [27]) Let (f_{ij}^-, f_{ij}^+) ($i=1,2,...n; j=1,2,...m$; i, j denote row vector and column vector of the tabular representation for interval-valued fuzzy soft set, respectively.) be the element of resultant interval-valued fuzzy soft set. Then we call $C=(c_{ij}^-, c_{ij}^+)_{n\times m}$ the score matrix of the resultant interval-valued fuzzy soft set, where $c_{ij}^- = \sum_{k=1}^{n}(f_{ij}^- - f_{kj}^-)$ and $c_{ij}^+ = \sum_{k=1}^{n}(f_{ij}^+ - f_{kj}^+)$. c_i is a choice value for each alternative h_i, such that

$$c_i = \sum_{j=1}^{m}(c_{ij}^-, c_{ij}^+) \tag{3}$$

From the formula (3) and the example 1, we can obtain the choice value c_i of each alternative,

$c_1 = \sum_{j=1}^{m}(c_{1j}^-, c_{1j}^+) = -0.2$, $c_2 = \sum_{j=1}^{m}(c_{2j}^-, c_{2j}^+) = 5.8$, $c_3 = \sum_{j=1}^{m}(c_{3j}^-, c_{3j}^+) = -4.2$,
$c_4 = \sum_{j=1}^{m}(c_{4j}^-, c_{4j}^+) = -2.7$, $c_5 = \sum_{j=1}^{m}(c_{5j}^-, c_{5j}^+) = 1.3$

Based on the choice value formula, we present the overall choice value of each alternative h_i (i=1,2,…,n):

$$r_i(w) = w_j \sum_{j=1}^{m}(c_{ij}^-, c_{ij}^+) = \sum_{j=1}^{m}(w_j c_{ij}^-, w_j c_{ij}^+) \tag{4}$$

3 An Extended GRA Method for MADM with Interval-valued Fuzzy Soft Set

This section presents a novel and rational approach to tackle MADM problems with incomplete weight information in the context of interval-valued fuzzy soft sets. Let $U=\{h_1,h_2,...,h_n\}$ be a discrete set of alternatives, consisting of n non-inferior alternatives, and $E=\{e_1,e_2,...,e_m\}$ be the set of attributes. Each alternative is assessed on the m attributes. w_j is the weighting of the attribute e_j $(j=1,2,...,m)$, where $w_j \in [0,1], \sum_{j=1}^{m} w_j = 1$. Suppose that all the evaluate values /ratings are expressed in interval-valued fuzzy soft set $(\tilde{F}, A)$, $A \subseteq E$. $\tilde{F}(\varepsilon)$ is referred as the interval fuzzy value set of parameter ε and it can be written as $\tilde{F}(\varepsilon) = \{< h_i, \tilde{F}(\varepsilon)^l(h_i), \tilde{F}(\varepsilon)^u(h_i) >: h_i \in U\}$. The decision problem is to select a most preferred alternative from set U based on the overall assessments of all alternatives on the m attributes.

For the complete optimistic decision makers, they interpret their decision situations positively and expect favorable outcomes, whereas pessimistic decision-makers interpret these situations negatively and anticipate unfavorable outcomes [14]. So a rational approach is necessary to elucidate the influences of optimism and pessimism on decision-making process. In this section, we will discuss all kinds of optimization models considering the decision makers' rationality. Below, we propose rational decision making models with incomplete information based on interval-valued fuzzy soft sets, which can be described as follows.

Step 1: Determine the positive-ideal and negative-ideal solution based on interval-valued fuzzy soft set.

Generally there are two kinds of attributes, the benefit type and the cost type. The higher the benefit type value is, the better it will be. While for the cost type, it is opposite. For the benefit type, the positive ideal alternative (PIA) $\tilde{F}(e_j)(h_{pj})$ and negative ideal alternative (NIA) $\tilde{F}(e_j)(h_{nj})$ can be defined respectively as:

$$\tilde{F}(e_j)(h_{pj}) = [\max_i \tilde{F}(e_j)(h_{ij}^l), \max_i \tilde{F}(e_j)(h_{ij}^u)]$$

$$\tilde{F}(e_j)(h_{nj}) = [\min_i \tilde{F}(e_j)(h_{ij}^l), \min_i \tilde{F}(e_j)(h_{ij}^u)],\ j = 1, 2, ..., m.$$

Step 2: Calculate the distance between the reference value and each comparison value. Then the distance between the reference value and each comparison value can be calculated using Definition 3 as follows:

Definition 3 let $(\tilde{F}, A)$ be an interval-valued fuzzy soft set, then the distance between each parameter of each object, i.e. $\tilde{d}(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj}))$ (where $i = 1, 2, ...n; j = 1, 2, ..., m$) can be defined:

$$\tilde{d}(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj})) = \sqrt{\frac{1}{2}\left(\left|\tilde{F}(e_j)(h_{ij}^l) - \tilde{F}(e_j)(h_{pj}^l)\right|^2 + \left|\tilde{F}(e_j)(h_{ij}^u) - \tilde{F}(e_j)(h_{pj}^u)\right|^2\right)} \quad \textbf{(5)}$$

where $(\tilde{F}(e_j)(h_{ij}^l), \tilde{F}(e_j)(h_{ij}^u))$ denotes the interval-valued fuzzy number of the i object under the j attribute in the interval-valued fuzzy soft set $(\tilde{F}, A)$.

Step 3: Calculate the grey relational coefficient of each alternative from PIA and NIA using the following equation, respectively. The grey relational coefficient of each alternative from PIA is given as

$$\xi_{ij}^+ = \frac{\min_{1\le i\le n}\min_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj})) + \rho \max_{1\le i\le n}\max_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj}))}{d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj})) + \rho \max_{1\le i\le n}\max_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{pj}))}, i = 1, 2, ..., n; j = 1, 2, ..., m \quad \textbf{(6)}$$

Similarly, the grey relational coefficient of each alternative from NIA is given as

$$\xi_{ij}^- = \frac{\min_{1\le i\le n}\min_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{nj})) + \rho \max_{1\le i\le n}\max_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{nj}))}{d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{nj})) + \rho \max_{1\le i\le n}\max_{1\le j\le m} d(\tilde{F}(e_j)(h_{ij}), \tilde{F}(e_j)(h_{nj}))}, i = 1, 2, ..., n; j = 1, 2, ..., m \quad \textbf{(7)}$$

where the identification coefficient $\rho = 0.5$.

Step 4: Estimate the soft relative degree of grey relational coefficient, using the following equation, respectively:

$$\xi_i^+ = \sum_{j=1}^{m} w_j \xi_{ij}^+, i = 1, 2, ..., n \quad \textbf{(8)}$$

$$\xi_i^- = \sum_{j=1}^{m} w_j \xi_{ij}^-, i = 1, 2, ..., n \quad \textbf{(9)}$$

The basic principle of the GRA method is that the chosen alternative should have the "largest degree of grey relation" from the positive-ideal solution and the "smallest degree of grey relation" from the negative-ideal solu-tion. Obviously, for the weight vector given, the larger the values ξ_i^+ and the smaller the values ξ_i^-, the better the alterative h_i is. For this purpose, we can establish the following multiple objective optimization models to obtain the weight information:

Case 1: If the weight information is incomplete known, we can establish the following multiple objective optimization models to obtain the weight information:

In the actual decision making process, the decision makers could not be completely optimistic or pessimistic. And the decision makers are always partly optimistic and pessimistic at the same time. Therefore, in the next decision optimization model, the degree of optimistic is represented by parameter α, and the degree of pessimism is represented by β, ($0 \le \alpha \le 1$, $0 \le \beta \le 1$, $\alpha + \beta \le 1$). So we can establish the optimization model:

$$\text{(M-1)} \begin{cases} \max \alpha\xi_i^+ = \sum_{j=1}^{m} \alpha w_j \xi_{ij}^+, i = 1,2,...,n \\ \min \beta\xi_i^- = \sum_{j=1}^{m} \beta w_j \xi_{ij}^-, i = 1,2,...,n \\ s.t: \sum_{j=1}^{m} w_j, w_j \ge 0, j = 1,2,...,m \end{cases},$$

i. If we put all the individual as a whole, we can construct the following optimization model from the point view of global:

We can translate the above multiple objective optimization model into the following single-objective optimization model:

$$\textbf{(M-2)} \begin{cases} \max \xi = \sum_{i=1}^{n} \sum_{j=1}^{m} w_j (\alpha\xi_{ij}^+ - \beta\xi_{ij}^-) \\ s.t.: w \in H \end{cases}$$

By solving the model (M-2), we get the optimal solution $w = (w_1, w_2, ..., w_m)$, which can be used as the weight vector of attributes.

We can see that it does not consider the individual situation, although the optimization model is simple. There is an extreme case that when only one individual achieve optimum, the whole achieve optimum. At this moment, others don't achieve optimum, so this result might come to the worst. In order to overcome this shortcoming, we can establish another optimization model.

ii. From both aspects of the individual and the whole to consider, we can construct the model:

We can translate the above multiple objective optimization model into the following single-objective optimization model:

$$\textbf{(M-3)} \begin{cases} \max(\alpha\xi_i^+ - \beta\xi_i^-) = \sum_{j=1}^{m} w_j (\alpha\xi_{ij}^+ - \beta\xi_{ij}^-), i = 1,2,...,n \\ s.t: w \in H, \sum_{j=1}^{m} w_j = 1 \end{cases}$$

By solving the **(M-3)** model, we obtain the optimal solution $w^{(i)} = (w_1^i, w_2^i, ..., w_m^i)^T$ corresponding to the alternative h_i. However, in the process of determining the weight vector $w = (w_1, w_2, ..., w_m)^T$, we also need to consider all the alternatives $h_i (i = 1,2,...,n)$ as a whole. Thus, we construct weight $W = (w_j^{(i)})_{n \times m}$ of the optimal solutions $w^{(i)} = (w_1^{(i)}, w_2^{(i)}, ..., w_m^{(i)})^T (i = 1,2,...,n)$ as:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix},$$

and we calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ of the matrix $((\alpha\xi^+ - \beta\xi^-)W)^T$ $((\alpha\xi^+ - \beta\xi^-)W)$. Then we can construct a combined weight vector as follows:

$$w = W\omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}$$

$$= \omega_1 w^{(1)} + \omega_2 w^{(2)} + ... + \omega_n w^{(n)}$$

Case 2: If the weight information is complete unknown, we can establish the following multiple objective optimization models to obtain the weight information:

$$\textbf{(M-4)} \begin{cases} \max \alpha\xi_i^+ = \sum_{j=1}^{m} \alpha w_j \xi_{ij}^+, i = 1, 2, ..., n \\ \min \beta\xi_i^- = \sum_{j=1}^{m} \beta w_j \xi_{ij}^-, i = 1, 2, ..., n \\ s.t: \sum_{j=1}^{m} w_j = 1, w_j \geq 0, j = 1, 2, ..., m \end{cases}.$$

Since each alternative is non-inferior, so there exists no preference relation on all the alternatives. Therefore, we can aggregate the above multiple objective optimization model into the following optimization model:

$$\textbf{(M-5)} \begin{cases} \max \xi_i(w) = \sum_{j=1}^{m} [w_j(\alpha\xi_{ij}^+ - \beta\xi_{ij}^-)]^3 \\ s.t.: \sum_{j=1}^{m} w_j = 1, w_j \geq 0, j = 1, 2, ..., m \end{cases}$$

Similarly, we may aggregate the above optimization models with equal weights into the following single-objective optimization model:

$$\textbf{(M-6)} \begin{cases} \max \xi(w) = \sum_{i=1}^{n} \xi_i(w) = \sum_{i=1}^{n} \sum_{j=1}^{m} [w_j(\alpha\xi_{ij}^+ - \beta\xi_{ij}^-)]^3 \\ s.t. \sum_{j=1}^{m} w_j = 1 \end{cases}$$

To solve the above model, referring to [13], we construct the Lagrange function of the constrained optimization problem (M-6):

$$L(w, \lambda) = \sum_{i=1}^{n} \sum_{j=1}^{m} [w_j(\alpha\xi_{ij}^+ - \beta\xi_{ij}^-)]^3 + 3\lambda(\sum_{j=1}^{m} w_j - 1) \quad \textbf{(10)}$$

Differentiating Eq. (10) with respect to λ and $w_j (j = 1, 2, ..., m)$, and setting these partial derivatives equal to zero, we can obtain the following set of equations:

$$\begin{cases} \dfrac{\partial L}{\partial w_j} = \sum_{i=1}^{n} w_j^2 (\alpha \xi_{ij}^+ - \beta \xi_{ij}^-)^3 + \lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = \sum_{j=1}^{m} w_j - 1 = 0 \end{cases} \tag{11}$$

By solving Eq. (11), we get a simple and exact formula for determining the criteria weights as follows:

$$w_j^* = \frac{\left[\sum_{j=1}^{m} \left(\left| \sum_{i=1}^{n} (\alpha \xi_{ij}^+ - \beta \xi_{ij}^-)^3 \right|^{1/2} \right)^{-1} \right]^{-1}}{\left| \sum_{i=1}^{n} (\alpha \xi_{ij}^+ - \beta \xi_{ij}^-)^3 \right|^{1/2}} \tag{12}$$

Step 5: Construct the resultant weighted interval-valued fuzzy soft set $(\tilde{F}, (wA))$ according to the interval-valued fuzzy soft set $(\tilde{F}, A)$.

Step 6: According to the Eq. (12), we can get the relative score $r_i(w)$ of h_i. Then decision is h_k, if $r_k(m) = \max r_i(w)$.

4 Illustrative Example

4.1 The Decision Steps

This section presents a case study to illustrate the application of the proposed solution. Let us suppose that there is a manufacturing business, which wants to select a best global supplier for one of its most critical parts used in assembling process (adapted from Chan and Kumar [28]). Detailing discussion on every criterion, based on primary discussion, five critical criteria have been identified. They are cost risk (e_1); quality risk (e_2); service risk (e_3); risk of supplier's profile (e_4); external risk (e_5). Through the screening, 5 suppliers are determined the final candidates. The five possible alternatives $h_i (i = 1, 2, 3, 4, 5)$ are to be evaluated using the interval-valued fuzzy soft set by the decision maker for the above five attributes, as listed in the following Table 2.

Table 2. Tabular form of the interval-valued fuzzy soft set $(\tilde{F}, A)$

U	e_1	e_2	e_3	e_4	e_5
h_1	[0.1,0.3]	[0.0,0.2]	[0.2,0.4]	[0.3,0.5]	[0.4,0.5]
h_2	[0.2,0.3]	[0.1.0.2]	[0.0,0.2]	[0.2,0.3]	[0.2,0.4]
h_3	[0.1,0.2]	[0.0,0.1]	[0.1,0.3]	[0.3,0.4]	[0.2,0.3]
h_4	[0.2,0.4]	[0.1,0.3]	[0.0,0.1]	[0.3,0.5]	[0.2,0.5]
h_5	[0.0,0.1]	[0.0,0.2]	[0.2,0.3]	[0.2,0.4]	[0.3,0.4]

Case 1: If the information about the attribute weights is incompletely known as follows:

$H = \{w_1 \le 0.3, 0.2 \le w_3 \le 0.5, 0.1 \le w_2 \le 0.2, w_5 \le 0.4, w_3 - w_2 \ge w_5 - w_4, w_4 \ge w_1,$
$w_3 - w_1 \le 0.1, 0.1 \le w_4 \le 0.3\}$

Step 1: Determine the positive-ideal and negative-ideal solution:

$\tilde{F}(\varepsilon)(h_{pj}) = ([0.0, 0.1], [0.0, 0.1], [0.0, 0.1], [0.2, 0.3], [0.2, 0.3])$

$\tilde{F}(\varepsilon)(h_{nj}) = ([0.2, 0.4], [0.1, 0.3], [0.2, 0.4], [0.3, 0.5], [0.4, 0.5])$

Step 2: Calculate the distance between the reference value and each comparison value. Then the distance between the reference value and each comparison value can be calculated using Definition 3 as follows (Table 3):

Table 3. The grey relational coefficient of each alternative from PIS and NIS

$\tilde{d}(\tilde{F}(\varepsilon)(h_{ij}), \tilde{F}(\varepsilon)(h_{pj}))$	e_1	e_2	e_3	e_4	e_5
h_1	0.1581	0.0707	0.2550	0.1518	0.2000
h_2	0.2000	0.1000	0.0707	0.0000	0.0707
h_3	0.1000	0.0000	0.1518	0.1000	0.0000
h_4	0.2550	0.1518	0.0000	0.1518	0.1414
h_5	0.0000	0.0707	0.2000	0.0707	0.1000
$\tilde{d}(\tilde{F}(\varepsilon)(h_{ij}), \tilde{F}(\varepsilon)(h_{nj}))$	e_1	e_2	e_3	e_4	e_5
h_1	0.1000	0.1000	0.0000	0.0000	0.0000
h_2	0.0707	0.0707	0.2000	0.1518	0.1518
h_3	0.1518	0.1518	0.1000	0.0707	0.2000
h_4	0.0000	0.0000	0.2550	0.0000	0.1414
h_5	0.2550	0.1000	0.0707	0.1000	0.1000

Step 3：Calculate the grey relational coefficient of each alternative from PIA and NIA using the Eq.(6) and Eq.(7), which is represented in Table 4.

Table 4. The distance from the positive ideal solution and negative ideal solution

ξ_{ij}^{+}	e_1	e_2	e_3	e_4	e_5
h_1	0.4464	0.6433	0.3333	0.4565	0.3893
h_2	0.3893	0.5604	0.6433	1.0000	0.6433
h_3	0.5604	1.0000	0.4565	0.5604	1.0000
h_4	0.3333	0.4565	1.0000	0.4565	0.4742
h_5	1.0000	0.6433	0.3893	0.6433	0.5604
ξ_{ij}^{-}	e_1	e_2	e_3	e_4	e_5
h_1	0.5604	0.5604	1.0000	1.0000	1.0000
h_2	0.6433	0.6433	0.3893	0.4464	0.4464
h_3	0.4464	0.4464	0.5604	0.6433	0.3893
h_4	1.0000	1.0000	0.3333	1.0000	0.4742
h_5	0.3333	0.5604	0.6433	0.5604	0.5604

If the decision is complete optimist, that is $\alpha = 1, \beta = 0$, utilize the Model (M-3) to obtain the optimal weight vectors $w^{(i)} = (w_1^{(i)}, w_2^{(i)}, w_3^{(i)}, w_4^{(i)}, w_5^{(i)})^T$ corresponding to the alternatives $h_i (i = 1,2,3,4,5)$:

$w^{(1)} = (0.3, 0.2, 0.2, 0.3, 0.0)^T$;

$w^{(2)} = (0.1, 0.1, 0.2, 0.3, 0.3)^T$;

$w^{(3)} = (0.1, 0.1, 0.2, 0.25, 0.35)^T$;

$w^{(4)} = (0.2667, 0.1, 0.3667, 0.2667, 0.0)^T$;

$w^{(5)} = (0.3, 0.2, 0.2, 0.3, 0.0)^T$ 。

So

$$W = \begin{pmatrix} 0.3 & 0.1 & 0.1 & 0.2667 & 0.3 \\ 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.3667 & 0.2 \\ 0.3 & 0.3 & 0.25 & 0.2667 & 0.3 \\ 0.0 & 0.3 & 0.35 & 0.0 & 0.0 \end{pmatrix}$$

$$((\alpha\xi^+ + \beta\xi^-)W)^T((\alpha\xi^+ + \beta\xi^-)W) = \begin{pmatrix} 1.8119 & 1.8373 & 1.8353 & 1.7708 & 1.8119 \\ 1.8373 & 1.8854 & 1.8849 & 1.8005 & 1.8373 \\ 1.8353 & 1.8849 & 1.8852 & 1.7976 & 1.8353 \\ 1.7708 & 1.8005 & 1.7976 & 1.7469 & 1.7708 \\ 1.8119 & 1.8373 & 1.8353 & 1.7708 & 1.8119 \end{pmatrix}$$

We can calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ of the matrix $((\alpha\xi^+ - \beta\xi^-)W)^T((\alpha\xi^+ - \beta\xi^-)W)$.
$\omega = (0.1993, 0.2032, 0.2030, 0.1953, 0.1993)^T$,

Then we construct a combined weight vector as follows:

$$w = W\omega = \begin{pmatrix} 0.3 & 0.1 & 0.1 & 0.2667 & 0.3 \\ 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.3667 & 0.2 \\ 0.3 & 0.3 & 0.25 & 0.2667 & 0.3 \\ 0 & 0.3 & 0.35 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0.1993 \\ 0.2032 \\ 0.2030 \\ 0.1953 \\ 0.1993 \end{pmatrix}$$

$$= (0.2123, 0.1399, 0.2326, 0.2833, 0.1320)^T$$

Step 4: Construct the resultant weighted interval-valued fuzzy soft set $(\tilde{F}, (wA))$ according to the interval-valued fuzzy soft set $(\tilde{F}, A)$, which is following in the Table 5.

Table 5. Tabular representation of the resultant weighted interval-valued fuzzy soft set $(\tilde{F}, (wA))$

U	w_1e_1	w_2e_2	w_3e_3	w_4e_4	w_5e_5
h_1	[0.0740,0.0925]	[0.0771,0.0881]	[0.1369,0.1825]	[0.1111,0.1333]	[0.1274,0.1529]
h_2	[0.0925,0.1110]	[0.0661,0.0991]	[0.1141,0.1597]	[0.1555,0.1777]	[0.1019,0.1784]
h_3	[0.0555,0.0925]	[0.0881,0.0991]	[0.1141,0.1597]	[0.1333,0.1555]	[0.0510,0.0764]
h_4	[0.0370,0.0925]	[0.0771,0.0881]	[0.1369,0.1597]	[0.1111,0.1555]	[0.1274,0.2038]
h_5	[0.0555,0.0740]	[0.0661,0.0771]	[0.1597,0.1825]	[0.1111,0.1333]	[0.1529,0.1784]

According to Eq.(4), we can obtain: $r_1(w) = 0.5876$; $r_2(w) = -0.3245$; $r_3(w) = -0.2268$; $r_4(w) = 0.2263$; $r_5(w) = -0.2625$.

Since $r_1 > r_4 > r_3 > r_5 > r_2$, then $h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$. Therefore, the most suitable supplier is h_1.

Due to the coexistence of optimism and pessimism for the same decision maker, the overall rational decision making model with (α, β) level of optimism and pessimism could be applied to this problem. There are many combinations (as long as $\alpha + \beta \le 1$). In order to simple explain that the decision result is changing as the decision makers' rationality, we only enumerate $\alpha + \beta = 1$ combinations in this paper. For simplicity, the results of other rational models with incompletely known information are listed in Table 6.

For situations where parameters (α, β) dissatisfy the equation $\alpha + \beta = 1$, we only enumerate several combinations in this paper, which is shown in Table 7.

From the results in Table 6 and Table 7, we can conclude that not only the weight vector of attributes always changes with the variation of the parameters (α, β), but also the supplier final sort may also change with the variation of the parameters (α, β). Moreover, in Table 7 the weight vector of the parameters $\alpha = \beta = 0.5$ is exactly the same as $\alpha = \beta = 0.3$. This is not coincidence, because the objective function of the model is

$$\max \xi = \sum_{j=1}^{5} w_j(\alpha\xi_{ij}^+ + \beta\xi_{ij}^-), i = 1, 2, ..., 5$$

Table 6. Tabular representation of the resultant weighted interval-valued fuzzy soft set $(\tilde{F},(wA))$

Optimism		Weight					Ranking order
α	β	w_1	w_2	w_3	w_4	w_5	
1.0	0.0	0.2123	0.1399	0.2326	0.2833	0.1320	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$
0.9	0.1	0.2124	0.1602	0.2326	0.2833	0.1115	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$
0.8	0.2	0.1724	0.1604	0.2326	0.2834	0.1513	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.7	0.3	0.1725	0.1804	0.2326	0.2834	0.1311	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.6	0.4	0.1598	0.1800	0.2200	0.2900	0.1502	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.5	0.5	0.1783	0.1801	0.2016	0.2900	0.1500	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.4	0.6	0.2203	0.1801	0.2000	0.2900	0.1096	$h_1 \succ h_4 \succ h_3 \succ h_2 \succ h_5$
0.3	0.7	0.1801	0.1801	0.2000	0.3000	0.1397	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$
0.2	0.8	0.1802	0.1802	0.2000	0.3000	0.1396	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$
0.1	0.9	0.2103	0.1602	0.2299	0.3000	0.0996	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$
0.0	1.0	0.2106	0.1601	0.2299	0.2901	0.1092	$h_1 \succ h_4 \succ h_3 \succ h_5 \succ h_2$

Table 7. Rational supplier's risk assessment results $(\alpha+\beta\neq 1)$

Optimistic		Weight					Ranking
α	β	w_1	w_2	w_3	w_4	w_5	
0.7	0.2	0.1724	0.1604	0.2326	0.2834	0.1512	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.4	0.4	0.1783	0.1801	0.2016	0.2900	0.1500	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.5	0.3	0.1728	0.1603	0.2328	0.2834	0.1508	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$
0.3	0.3	0.1783	0.1801	0.2016	0.2900	0.1500	$h_1 \succ h_4 \succ h_5 \succ h_3 \succ h_2$

When $\alpha=\beta$, the above objective function can transform to the following function:

$$\max \xi = \sum_{j=1}^{5} \alpha w_j (\xi_{ij}^{+} + \xi_{ij}^{-}), i = 1,2,...,5$$

We can see that the objective function is a same function except a constant coefficient. So, when $\alpha=\beta$, the weight vector doesn't change. In order to understand the trends of the weight vectors with the variations of optimism parameter α, we draw the Fig. 1.

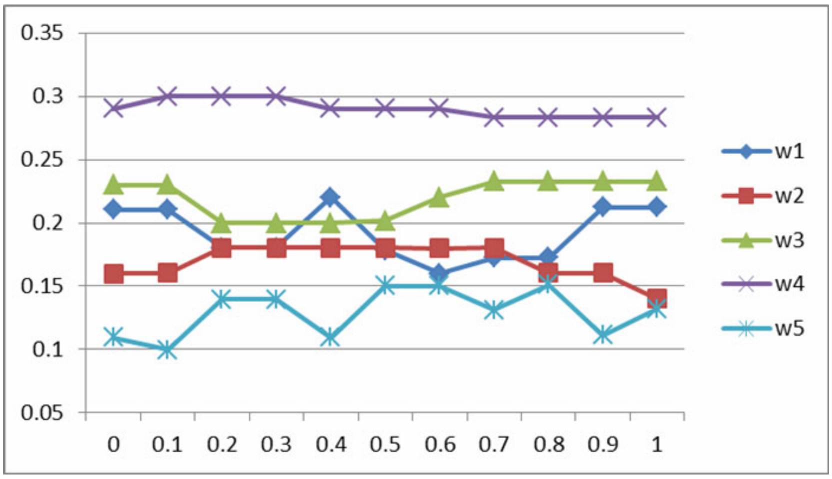

Fig. 1. Weight vectors with respect to α

Observations of Fig. 1 inform us that the index of cost and external environment risk fluctuate severely with the increase of optimistic coefficient. Compared to cost and external environment risk, supplier quality risk mildly changes. This shows whether evaluators are optimistic or pessimistic, they are more focused on cost and the external environment risk.

Case 2: If the weight information is complete unknown

If the decision is complete optimist, that is $\alpha = 1, \beta = 0$, according to the Eq.(12), we can get

$w = (0.2288, 0.1729, 0.2156, 0.1895, 0.1931)^T$.

So the resultant interval-valued fuzzy soft set is as following in Table 8.

Table 8. Tabular representation of the resultant weighted interval-valued fuzzy soft set $(\tilde{F}, (wA))$.

U	w_1e_1	w_2e_2	w_3e_3	w_4e_4	w_5e_5
h_1	[0.0212,0.0637]	[0.0000,0.0386]	[0.0435,0.0870]	[0.0444,0.0740]	[0.0916,0.1146]
h_2	[0.0425,0.0637]	[0.0193,0.0386]	[0.0000,0.0435]	[0.0296,0.0444]	[0.0458,0.0916]
h_3	[0.0212,0.0425]	[0.0000,0.0193]	[0.0218,0.0653]	[0.0444,0.0592]	[0.0458,0.0687]
h_4	[0.0425,0.0850]	[0.0193,0.0580]	[0.0000,0.0218]	[0.0444,0.0740]	[0.0458,0.1146]
h_5	[0.0000,0.0212]	[0.0000,0.0386]	[0.0435,0.0653]	[0.0296,0.0592]	[0.0687,0.0916]

According to the Eq.(4), we can get the score： $r_1(w) = 0.5843$； $r_2(w) = -0.2134$；
$r_3(w) = -0.3681$； $r_4(w) = 0.2171$； $r_5(w) = -0.2200$.

Since $r_1 > r_4 > r_2 > r_5 > r_3$, then $h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$. Therefore, the most suitable supplier is h_1.

There are many combinations (as long as $\alpha + \beta \le 1$). In order to simple explain that the impact of decision makers' rationality on the decision result, we only enumerate $\alpha + \beta = 1$ combinations in this paper. For simplicity, the results of other rational models with complete unknown information are listed in Table 9.

Table 9. Rational supplier's risk assessment results $(\alpha + \beta = 1)$

Optimism		Weight					Ranking order
α	β	w_1	w_2	w_3	w_4	w_5	
0.0	1.0	0.2124	0.1932	0.2175	0.1479	0.2291	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.1	0.9	0.2157	0.1890	0.2199	0.1500	0.2255	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.2	0.8	0.2189	0.1851	0.2219	0.1525	0.2216	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.3	0.7	0.2219	0.1816	0.2233	0.1556	0.2175	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.4	0.6	0.2246	0.1786	0.2242	0.1592	0.2134	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.5	0.5	0.2268	0.1762	0.2243	0.1633	0.2093	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.6	0.4	0.2285	0.1743	0.2238	0.1679	0.2055	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.7	0.3	0.2295	0.1731	0.2226	0.1730	0.2018	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.8	0.2	0.2299	0.1725	0.2207	0.1783	0.1986	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.9	0.1	0.2297	0.1724	0.2184	0.1839	0.1956	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
1.0	0.0	0.2288	0.1729	0.2156	0.1895	0.1931	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$

For situations where parameters (α, β) dissatisfy the equation $\alpha + \beta = 1$, we only enumerate several combinations in this paper, which is shown in Table 10.

Table 10. Supplier's risk rational assessment results $(\alpha + \beta \ne 1)$

Optimistic		Weight					Ranking
α	β	w_1	w_2	w_3	w_4	w_5	
0.7	0.2	0.2299	0.1725	0.2212	0.1771	0.1993	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.4	0.4	0.2268	0.1762	0.2243	0.1633	0.2093	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$
0.5	0.3	0.2288	0.1740	0.2236	0.1692	0.2045	$h_1 \succ h_4 \succ h_2 \succ h_5 \succ h_3$

From Table 9 and Table 10, we can obtain that the various parameters (α,β) always change the weight vector, although it doesn't change the final ranking order. In a similar way, when $\alpha=\beta$, the weight vector is the same.

From Fig. 2, we can infer that the index of low overall cost and good of supplier's profile risk increase sharply, while the index of high quality and low risk decrease sharply. At the same time, we can conclude that the more optimistic the decision maker is, the more emphasis should be given the low overall cost and good of supplier's profile risk. The more pessimistic the decision maker is, the more emphasis should be given to the high quality and low risk.

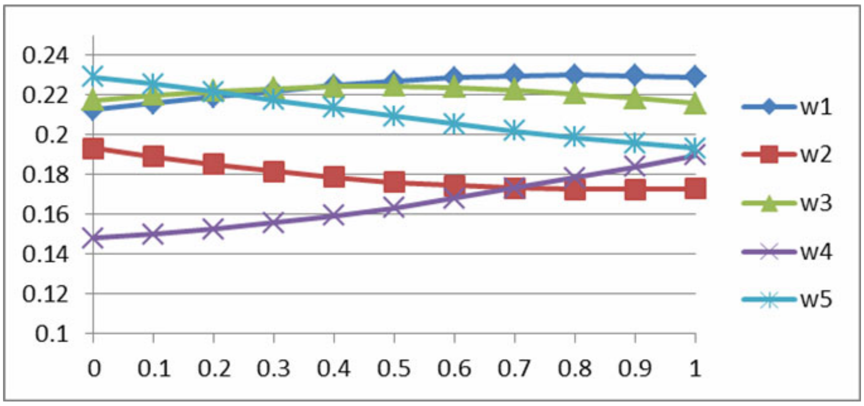

Fig. 2. Weight vectors with respect to α

4.2 Comparative Analysis and Discussion

Wei [34] investigated the problem of intuitionistic fuzzy multiple attribute decision making with incompletely known attribute weight information. In his paper, he aggregated the multiple objective optimization models with equal weights into a single-objective optimization model.
According to his transformation method, we can translate the Model (M-3) into the Model (M-3)':

(M-3)' Minimize: $\alpha\tilde{d}_i^+(w)+\beta\tilde{d}_i^-(w)=\sum_{i=1}^{n}\sum_{j=1}^{m}w_j(\alpha\tilde{d}_i^+(\tilde{F}(\varepsilon_j)(h_{ij}),\tilde{F}(\varepsilon_j)(h_{pj}))+\beta\tilde{d}_i^-(\tilde{F}(\varepsilon_j)(h_{ij}),\tilde{F}(\varepsilon_j)(h_{nj})))$

Subject to: $w=(w_1,w_2,...,w_m)^T\in H$, $w_j\geq 0, i=1,2,...,m, \sum_{j=1}^{m}w_j=1$

From above optimization model, we can see that it does not consider the individual situation, although the optimization model is simple. There is an extreme case that only one individual achieve optimum, when the whole achieve optimum. And others not only don't achieve optimum but also might come to the worst. The following is compare results according to two optimized methods, which is shown in Table 11.

Table 11. The initial weight about comparative study above two optimization approaches

		w_1	w_2	w_3	w_4
$\alpha=0.0,\beta=1.0$	(M-3)	0.32	0.23	0.30	0.15
	(M-3)'	0.28	0.28	0.30	0.15
$\alpha=0.2,\beta=0.8$	(M-3)	0.31	0.22	0.32	0.15
	(M-3)'	0.28	0.28	0.30	0.15
$\alpha=0.5,\beta=0.5$	(M-3)	0.25	0.20	0.37	0.19
	(M-3)'	0.25	0.20	0.37	0.19
$\alpha=0.7,\beta=0.3$	(M-3)	0.34	0.13	0.35	0.18
	(M-3)'	0.35	0.10	0.37	0.19
$\alpha=1.0,\beta=0.0$	(M-3)	0.34	0.13	0.35	0.18
	(M-3)'	0.35	0.10	0.37	0.19

From above Table 6, we can see that there exists great difference between the two optimization models. The reason is that the optimization Model (M-3)' doesn't consider the individual and just unilaterally consider the whole. While the optimization Model (M-3) not only consider the individual but also consider the whole. So we think that the Model (M-3) is more reasonable.

5 Conclusion

In this paper, we have investigated the MADM problems with unknown criterion weights, in which the information taking the form of interval-valued fuzzy soft set considering the rational behavior of decision makers. For the incomplete information, we establish two optimization models from the point view of global and from both aspects of the individual and the whole, respectively. Moreover, we compare the two optimization methods and point out that the second optimization method is more reasonable. For the special situations, where the information about attribute weights is completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. Finally, a practical supplier selection problem is given to verify the developed approach. The results show that different rational behavior of decision makers concern different indicators. In future research, our work will focus on the application of the proposed method in the similar decision problems, such as site selection, material selection, risk assessment and so on.

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