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Modeling of Takagi-Sugeno Fuzzy Control Design for Nonlinear Systems

Pu-Sheng Tsai[1], Ter-Feng Wu[2*], Nien-Tsu Hu[3], and Jen-Yang Chen[4]

[1] Department of Electronic Engineering, China University of Science and Technology,
Taipei 115, Taiwan, ROC
tps@ee2.cust.edu.tw

[2] Department of Electrical Engineering, National Ilan University,
Yilan 260, Taiwan, ROC
tfwu@niu.edu.tw

[3] Chemical Systems Research Division, National Chung-Shan Institute of Science and Technology,
Taoyuan 325, Taiwan, ROC
nientsu.hu@gmail.com

[4] Department of Electronic Engineering, Ming Chuan University,
Taoyuan 333, Taiwan, ROC
jychen@mail.mcu.edu.tw

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Abstract. In this paper, we first develop a procedure for constructing Takagi-Sugeno fuzzy systems from input-output pairs to identify nonlinear dynamic systems. The fuzzy system can approximate any nonlinear continuous function to any arbitrary accuracy that is substantiated by the Stone Weierstrass theorem. A learning-based algorithm is proposed in this paper for the identification of T-S models. Our modeling algorithm contains four blocks: fuzzy C-Mean partition block, LS coarse tuning, fine turning by gradient descent, and emulation block. The ultimate target is to design a fuzzy modeling to meet the requirements of both simplicity and accuracy for the input-output behavior. In the second part, we propose a discrete time fuzzy system that is composed of a dynamic fuzzy model and a fuzzy state feedback controller. This requires that for all the local linear models, a common positive-definite matrix P can be found to satisfy the Lyapunov stability criterion, although this is an extremely difficult problem for all systems. Thus in this paper, Fuzzy controller design is divided into two procedures. In the first step, we express the fuzzy model by a family of local state space models, and the controller is designed by state feedback control law for each local linear state space model. In the second step, we establish a global stability condition to guarantee the stability of the global closed loop system in order to circumvent the problem of determining the common P.

Keywords: fuzzy modeling, linear matrix inequality, Lyapunov stability criterion, maximum gradient descent, parameter identification

1 Introduction

The essential function of fuzzy systems is to formulate expert knowledge and experience in order to make a strategic decisions. Expert knowledge may be classified into two categories: conscious knowledge and subconscious knowledge. Conscious knowledge can be explicitly expressed in words, but subconscious knowledge is difficult to express precisely in words. When the expert is providing knowledge, we can view him as a black box and measure the input-output data pairs.

* Corresponding Author

A fuzzy controller with expert knowledge or experience is sufficient to provide solutions to highly nonlinear complicated [1-3], and unknown systems. This paper presents a systematic design method to identify a system model using a set of input-output data, thereby allowing the fuzzy model to satisfy the requirement of accuracy and minimum rules under the cluster analysis. In addition, this paper proposes a design for the fuzzy controller based on fuzzy models, thus creating the guidelines for the global stability of a fuzzy system under the Lyapunov stability criterion [4-5]. According to the Stone-Weierstrass theorem [6], a fuzzy system is capable of approximating any continuous function [7] and can be used as the basis for fuzzy modeling theory. For the fuzzy identification, the modeling architecture presented by Takagi and Sugeno in 1985 [8] is becoming increasingly important and has been successfully applied to nonlinear modeling [9]. The output of a T-S model is a linear combination of input variables, and this model can be represented as state equations, which are more suitable for analysis of stability and robustness.

2 Fuzzy Modeling

2.1 Takagi-Sugeno Fuzzy Model

The T-S model can be expressed as follows:

$$\begin{aligned} &If\ x(k)\ is\ A_1^i\ and .....and\ x(k-n+1)\ is\ A_n^i\ ,Then \\ &x^i(k+1)=a_0^i+a_1^i x(k)+...+a_n^i x(k-n+1), \end{aligned} \tag{1}$$

where $i=1,...,m$ represents the number of fuzzy rules, $j=1,...,n$ represents the number of input variables, and A_j^i represents the membership function of the ith rule and jth input variable. One characteristic of this model is that the consequence can be expressed as the linear combination of the input variables of the premise. In addition, in this the consequence is expressed as the linear combination of input variables of the premise.

2.2 Fuzzy Modeling

To obtain the T-S model corresponding to various types of input-output data, the process of fuzzy modeling can be divided into structural recognition and parameter identification. The purpose of structural recognition is to determine the number of input variables, thereby partitioning input space; determining the number of rules; building up the initial distribution of membership functions of input variables and the consequential parameters; and determining the architecture for the approximate models. Parameter identification is used to eliminate the difference between models and physical systems, thereby obtaining a complete and accurate model via a fine tuning algorithm of parameter. This paper partitions the modeling process into 4 blocks, as shown in Fig. 1.

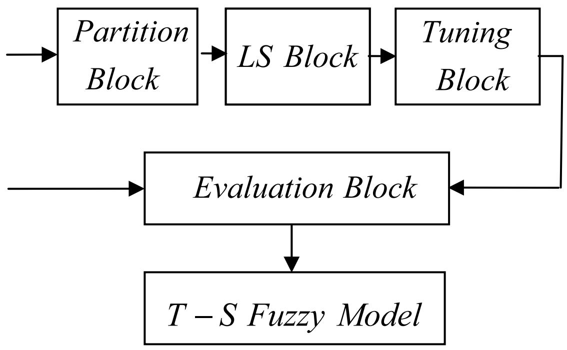


Fig. 1. Block diagram of fuzzy modeling

Partition block. The blocks of input data and output data are partitioned first. This study adopts the fuzzy C-Mean cluster method incorporating the concept of optimization to partition blocks. Let $X=\{x_1,x_2,.....,x_n\}$ be the data to be identified, $c\in\{2,3,.....,n-1\}$,$1\le k\le n, 1\le i\le c$ be the number of clusters, Matrix $U=[u_{ik}]$ be the level of membership of k[th] data to i[th] cluster and $V=(v_1,v_2,.....,v_c)$ be the central vector of each cluster.

According to the following operation:

(1) objective: to find $U=[u_{ik}]\in M_f$ and $V=(v_1,v_2,.....,v_c)$ such that

(2) minimize: $J(U,V)=\sum_{k=1}^{n}\sum_{i=1}^{c}(u_{ik})^m\left\|x_k-v_i\right\|^2, m\in(1,\infty)$ is a weighted value,

(3) subject to: $\sum_{i=1}^{c}u_{ik}=1,\quad \forall k=\{1,2\cdots,n\},\ u_{ij}\geq 0,\quad 1\leq i\leq n,\quad 1\leq j\leq c,$

we can locate

$$u_{ik}=\frac{1}{\sum_{j=1}^{c}\left(\frac{\left\|x_k-v_i\right\|}{\left\|x_k-v_j\right\|}\right)^{\frac{2}{m-1}}},1\leq i\leq c,\ 1\leq k\leq n$$
$$v_i=\frac{\sum_{k=1}^{n}(u_{ik})^m x_k}{\sum_{k-1}^{n}(u_{ik})^m},1\leq i\leq c \tag{2}$$

For the requirement of minimum number of rules in structural recognition, an optimal balance between the minimum number of rules and approximate errors must be found. In the beginning, all data is divided into 2 categories (c=0) in order to construct the simplest model, and whether the system requirement can be satisfied is determined by the final model evaluation. If it fails, the number of partitioned blocks must be increased, which means increasing the number of rules for the fuzzy model.

Tuning parameters by the least squares method. The model parameters are tuned by the least squares method in order to determine the initial parameters of the premise, as well the consequence of the fuzzy rules.

(1) Parameters of membership function for premise: As shown by Gauss distribution, parameters m_j^i and σ_j^i need to be determined.

$$A_j^i(m_j^i,\sigma_j^i)=\exp\left\{-\left(\frac{x_j-m_j^i}{\sigma_j^i}\right)^2\right\}$$
$$m_j^i=\frac{\sum_{k=1}^{n}u_k^i x_{kj}}{\sum_{k=1}^{n}u_k^i}\quad,\quad \sigma_j^i=\sqrt{\frac{\sum_{k=1}^{n}u_k^i(x_{kj}-m_j^i)^2}{\sum_{k=1}^{n}u_k^i}} \tag{3}$$

where $i=1,\cdots,c$ denotes the number of cluster (rules), and $u_k^i=u_{ik}$.

(2) Setting up regression parameters for consequence: The output of the fuzzy model is the linear combination of input variables. The following recursive formulation of least square method determines the parameters for each rule *i*:

$$P^i=[a_0^i\ \ a_1^i\ \ \cdots\ \ a_n^i]^T$$

$$y^i=a_0^i+a_1^i x_1+a_2^i x_2+.....+a_n^i x_n$$
$$y^i=X^T P^i\qquad i=1,2,\cdots,c$$
$$X=[1\ \ x_1\ \ \cdots\ \ x_n]^T$$
$$P^i=[a_0^i\ \ a_1^i\ \ \cdots\ \ a_n^i]^T$$

$$P_{k+1}^i=P_k^i+K_k[y_{k+1}-X_{k+1}^T P_k^i] \tag{4}$$

$$\square K_k = S_{k+1}X_{k+1} = \frac{S_k X_{k+1}}{(\mu_k^i)^{-1} + X_{k+1}^T S_k X_{k+1}} \quad \textbf{(5)}$$

$$S_{k+1} = [1 - K_k X_{k+1}^T]S_k \quad k = 1,2,\cdots,n \quad \textbf{(6)}$$

(3) Tuning parameters by the maximum gradient descent: In the maximum gradient descent, the difference between the fuzzy model and the object of identification is considered a performance index, pushing parameters upward for least errors, and thereby systematically adjusting the model parameters by recursive equation. Fundamentally, the learning effect of parameters or convergence speed is dependent on 3 factors – accuracy of structural recognition, quality of learning laws, and the parameter η_α of learning speed. The parameters to be tuned in this paper include the parameters (m_j^i, σ_j^i) of membership function in the rules of the premise, as well as the combinative parameters (α_j^i) of consequence. Initially, the error function is defined as $E = \frac{1}{2}[y_p(k) - y_m(k)]^2$, with $y_p(k)$ as the output sequence of the real system, and $y_m(k)$ as the output sequence of fuzzy model.

$$\alpha_0^i(k+1) = \alpha_0^i(k) - \eta_\alpha \frac{\partial E(k)}{\partial_0^i(k)} \quad \textbf{(7)}$$

$$\alpha_j^i(k+1) = \alpha_j^i(k) - \eta_\alpha \frac{\partial E(k)}{\partial \alpha_j^i(k)} \quad \textbf{(8)}$$

$$m_j^i(k+1) = m_j^i(k) - \eta_m \frac{\partial E(k)}{\partial m_j^i(k)} \quad \textbf{(9)}$$

$$\sigma_j^i(k+1) = \sigma_j^i(k) - \eta_\sigma \frac{\partial E(k)}{\partial \sigma_j^i(k)} \quad \textbf{(10)}$$

Further, the chain rule is used to derive

$$\frac{\partial E(k)}{\partial \alpha_0^i(k)}, \ \frac{\partial E(k)}{\partial \alpha_j^i(k)}, \ \frac{\partial E(k)}{\partial m_j^i(k)}, \ \frac{\partial E(k)}{\partial \sigma_j^i(k)} \quad \textbf{(11)}$$

and the maximum gradient descent is used to find the learning laws as

$$\alpha_0^i(k+1) = \alpha_0^i(k) + \eta_\alpha (y_p(k) - y_m(k))\phi^i(k) \quad \textbf{(12)}$$

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \eta_\alpha (y_p(k) - y_m(k))\phi^i(k)x_j(k) \quad \textbf{(13)}$$

$$\begin{aligned} m_j^i(k+1) &= m_j^i(k) + \\ \eta_m &\frac{(y_p(k) - y_m(k))(y_m^i(k) - y_m(k))\phi^i(k)(x_j(k) - m_j^i(k))}{(\sigma_j^i(k))^2} \end{aligned} \quad \textbf{(14)}$$

$$\begin{aligned} \sigma_j^i(k+1) &= \sigma_j^i(k) + \\ \eta_\sigma &\frac{(y_p(k) - y_m(k))(y_m^i(k) - y_m(k))\phi^i(k)(x_j(k) - m_j^i(k))^2}{(\sigma_j^i(k))^3} \end{aligned} \quad \textbf{(15)}$$

3 Fuzzy State Feedback Controller

For convenience, the model in (1) is rewritten as follows:

$$\begin{aligned} &If\ x(k)\ is\ A_1^i\ and .....and\ x(k-n+1)\ is\ A_n^i\quad Then \\ &x^i(k+1)=a_1^i x(k)+\cdots+a_n^i x(k-n+1)+b^i u(k) \\ &\qquad\qquad = A_i x(k)+B_i u(k) \end{aligned} \tag{16}$$

where $A_i=\begin{bmatrix} a_1^i & a_2^i & \cdots & a_{n-1}^i & a_n^i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ and $x(k)=[x(k)\quad x(k-1)\cdots x(k-n+1)], i=1,\cdots,m$.

The state feedback fuzzy controller u is designed as follows:

$$\begin{aligned} &If\ x(k)\ is\ A_1^i\ and .....and\ x(k-n+1)\ is\ A_n^i \\ &Then\quad u(k)=-K_i x(k) \end{aligned} \tag{17}$$

After the process of fuzzy inference, the closed-loop system is integrated as

$$x(k+1)=\sum_{i=1}^{m}\sum_{j=1}^{m}\lambda_i(x)\lambda_j(x)(A_i-B_iK_j)x(k) \tag{18}$$

[Theorem 1] If the fuzzy system is asymptotically stable at the equilibrium point, there must exist a common positive symmetric matrix P to satisfy:

$$(A_i+B_iK_j)^T P(A_i+B_iK_j)-P=-Q_{ij}\quad i,j=1,2,\cdots,r \tag{19}$$

where Q_{ij} is a positive definite matrix. Each rule of the fuzzy system is considered a sub-system. The controller must satisfy not only the local stability of all sub-systems, but also the global stability of the overall system. Therefore, how to find a common P matrix to satisfy all rules and the fuzzy state matrix between all rules is the bottleneck for the design of fuzzy controller.

4 Stability Analysis for the Fuzzy System

In this paper, we attempt to establish the criterion of global stability for fuzzy system from a different viewpoint. In equation (17), assuming that rth rule has the highest weighting and $r=\arg\ \max[\mu_j(x)]$, $j=1,2\cdots m$, then the output of controller can be denoted as:

$$\begin{aligned} u(k)&=-K_r x(k) \\ x(k+1)&=\sum_{i=1}^{m}\mu_i[A_i x(k)+B_i u(k)] \\ &=\sum_{i=1}^{m}\mu_i[A_i x(k)-B_iK_r]x(k) \\ &=(\mu_r A_{rr}+\sum_{\substack{i=1\\ i\neq r}}^{m}\mu_i A_{ir})x(k) \\ &=[A_{rr}+\sum_{\substack{i=1\\ i\neq r}}^{m}\mu_i(A_{ir}-A_{rr})]x(k)=[A_{rr}+\Delta A_{ir}]x(k) \end{aligned} \tag{20}$$

where $A_{ir} = A_i - B_i K_r$, $A_{rr} = A_r - B_r K_r$, and $\Delta A_{ir} = \sum_{\substack{i=1 \\ i \neq r}}^{m} \mu_i (A_{ir} - A_{rr})$.

According to Theorem 1, there must be a positive symmetric matrix P_r for K_r to satisfy

$$A_{rr}^T P_r A_{rr} - P_r + 2I = 0 \quad \textbf{(21)}$$

Further, define a Lyapunov equation $V_r[x(k)] = x^T(k) P_r x(k)$, so that $\Delta V_r = V_r[x(k+1)] - V_r[x(k)] < 0$ is the criterion for asymptotical stability. We obtain

$$\begin{aligned}
\Delta V_r &= V_r[x(k+1)] - V_r[x(k)] \\
&= x^T(k+1) P_r x(k+1) - x^T(k) P_r x(k) \\
&= [(A_{rr} + \Delta A_{ir}) x(k)]^T P_r [(A_{rr} + \Delta A_{ir}) x(k)] \\
&\quad - x^T(k) P_r x(k) \\
&= x^T(k)[(A_{rr} + \Delta A_{ir})^T P_r (A_{rr} + \Delta A_{ir}) - P_r] x(k) \\
&= x^T(k)[-2I + \Delta A_{ir}^T P_r A_{rr} + \\
&\qquad A_{rr}^T P_r \Delta A_{ir} + \Delta A_{ir}^T P_r \Delta A_{ir}] x(k) \\
&= 2x^T(k)[-I + \sum_{i=1, i \neq r}^{m} Q_{ri} + \sum_{i=1, i \neq r}^{m} R_{rij}] x(k)
\end{aligned} \quad \textbf{(22)}$$

where $\sum_{i=1, i \neq r}^{m} \mu_i Q_{ri} = \dfrac{\Delta A_{ir}^T P_r A_{rr} + A_{rr}^T P_r \Delta A_{ir}}{2}$ and $\sum_{i=1, i \neq r}^{m} \mu_i \mu_j R_{rij} = \dfrac{\Delta A_{ir}^T P_r \Delta A_{ir}}{2}$

[Theorem 2] If a closed-loop discrete fuzzy system satisfies $\sum_{i=1, i \neq r}^{m} \mu_i \lambda_{\max}(Q_{ri}) + \sum_{i=1, i \neq r}^{m} \sum_{j=1, j \neq r}^{m} \mu_i \mu_j \lambda_{\max}(R_{rij}) < 1$, then this fuzzy system has global asymptotical stability where $\lambda_{\max}(Q_{ri})$ and $\lambda_{\max}(R_{rij})$ are the maximum eigenvalues of matrices Q_{ri} and R_{rij}, respectively.

5 Simulation Results

[Example 1] The approximate model presented by Wang and Mendel [10] is applied to a real case of auto-parking:

$$\begin{aligned}
x(t+1) &= x(t) + \cos[\phi(t) + \theta(t)] + \sin[\theta(t)] \sin[\phi(t)] \\
y(t+1) &= y(t) + \sin[\phi(t) + \theta(t)] - \sin[\theta(t)] \cos[\phi(t)] \\
\phi(t+1) &= \phi(t) - \sin^{-1}[\frac{2\sin(\theta(t))}{b}]
\end{aligned}$$

where $x \in [0, 20]$, $\phi \in [-90^{\circ}, 270^{\circ}]$, $\theta \in [-40^{\circ}, 40^{\circ}]$, and b denotes car length.

In Fig. 2, ϕ denotes the angle between the car's movement and the x axis, and θ denotes the angle between the wheels and the y axis. The angle created by the rotation of steering wheel is limited as $\theta \in [-40^{o}, 40^{o}]$. We desired to identify the trajectory x(t) through a fuzzy model. The procedures are as follows:

Step1: Select $x(t)$, $\phi(t)$, and $\theta(t)$ as possible input variables.

Step2: Normalize all variables.

$$x(t) = \frac{x(t)}{20}, \phi(t) = \frac{\phi(t)}{2\pi}, \theta(t) = \frac{\theta(t)}{4/9\,\pi}$$

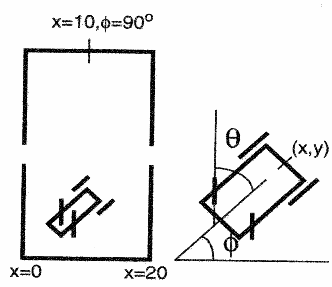


Fig. 2. Mobile robot configuration

Step3: Partition data by Fuzzy C-Mean and set 800 points as the size of data.

inputs: $\theta(t) = -(2\pi/9) + (k \cdot \pi/180)$, $x(t)$, and $\phi(t)$

Output: x(t+1)

Step4: Let the number of clusters be 5.

Step5: Simulation results are as Fig. 3.

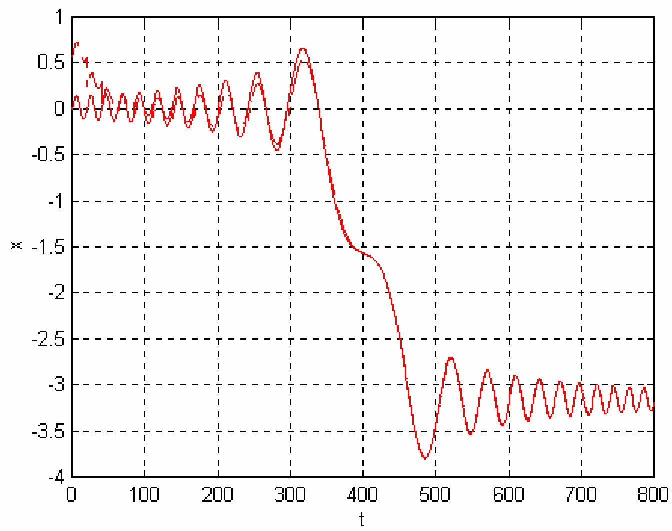


Fig. 3. Fuzzy modeling of car's trajectory on x Axis with '-' Denoting System Output and '- -' Denoting Model Output

In this figure, assume that angle θ starts from -40° and increases 0.1° each increment time until 40°, so the steering wheel starts from -40° and moves around a circle. As the steering wheel is turned to face front gradually, the radius increases gradually until $\theta = 0^\circ$. As soon as the steering wheel faces forward, the car moves straightforward. Then the car starts circling as the steering wheel turns to the other side. The fuzzy model derived from cluster analysis makes a perfect approximating effect possible.

[Example 2] an object of control under T-S model is denoted by two rules as follows:

T-S Model Plant:

$$R^1 : \mathit{If}\ x(k)\ \ is\ \ G_1, Then$$
$$x^1(k+1) = 2.18x(k) - 0.59x(k-1) - 0.603u(k)$$

$$R^2 : \mathit{If}\ x(k)\ \ is\ \ G_2, Then$$
$$x^2(k+1) = 2.26x(k) - 0.36x(k-1) - 1.12u(k)$$

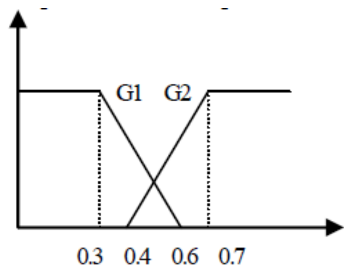
T-S Model Controller:

$$If\ x(k)\ is\ G_1, Then\quad u^1(k+1)=k_1^1x(k)+k_2^1x(k-1)$$

$$If\ x(k)\ is\ G_2, Then\quad u^2(k+1)=k_1^2x(k)+k_2^2x(k-1)$$

There, 4 parameters $c_1^1,\ c_2^1,\ c_1^2$, and c_2^2 were designed to stabilize the fuzzy closed-loop system.

Step1: Set up the membership function:



Step2: Design the state feedback controller and urge all rules to satisfy the criterion of asymptotical stability; then select

$$K^1=[k_1^1\ \ k_2^1]=[3.7813\ \ -0.1593]$$

$$K^2=[k_1^2\ \ k_2^2]=[2.0833\ \ 0.0611]$$

Step3: Find the positive and symmetric solutions for P_1 and P_2 in order to satisfy the Lyapunov equation under each rule

$$A_{rr}^TP_rA_{rr}-P_r+2I=0,\ \ r=1,2$$

If the positive solutions of P_1, P_2 cannot be found, then redesign K^1, K^2 again in order to find a set of P as follows:

$$P_1=\begin{bmatrix}5.3148 & 0.176\\ 0.176 & 3.2967\end{bmatrix}$$

$$\lambda_{\max}(Q_{12})=0.404,\ \ \lambda_{\max}(Q_{21})=1.221$$
$$\lambda_{\max}(R_{122})=0.266,\ \ \lambda_{\max}(R_{211})=0.308$$

Step4: Verify the global stability of the fuzzy system.

$$\sum_{i=1,i\neq r}^{m}\mu_i\lambda_{\max}(Q_{ri})+\sum_{i=1,i\neq r}^{m}\sum_{j=1,j\neq r}^{m}\mu_i\mu_j\lambda_{\max}(R_{rij})$$

(1) $\mu_1>\mu_2$:

$$\mu_2\lambda_{\max}(Q_{12})+\mu_2\mu_2\lambda_{\max}(R_{122})$$
$$=0.5*0.404+0.5*0.5*0.266<1$$

(2) $\mu_2>\mu_1$:

$$\mu_1\lambda_{\max}(Q_{21})+\mu_1\mu_1\lambda_{\max}(R_{211})$$
$$=0.5*1.221+0.5*0.5*0.308<1$$

Step5: Obtain simulation results (initial value x(k)=0.5, x(k-1)=0.5)

Fig.4 demonstrates that this system is a stable fuzzy system.

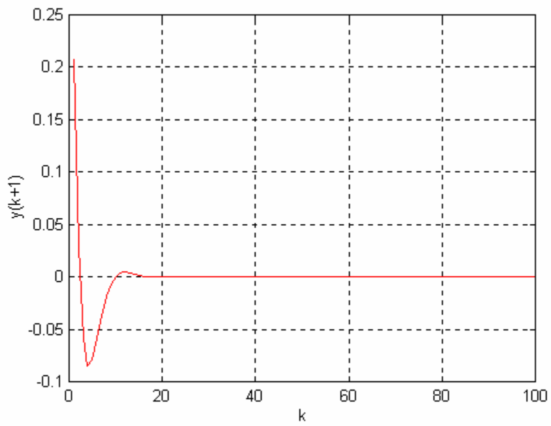


Fig. 4. System response

6 Conclusions

The fuzzy control architecture proposed in this paper is based on fuzzy modeling and has successfully incorporated the advantages of fuzzy dynamic model and fuzzy state feedback controller, and thus is beneficial to the tracking control of the reference model. The contributions made by this paper include: (1) Approximating an unknown system by constructing Takagi-Sugeno fuzzy systems model from input-output pairs, thereby building up the basic of theoretical analysis for fuzzy modeling; (2) Identifying the T-S model parameters by a learning-based algorithm contains four blocks: fuzzy C-Mean partition block, LS coarse tuning, fine turning by gradient descent, and emulation block.; (3) Meeting the requirements of both simplicity and accuracy for the input-output behavior by the proposed fuzzy design approach; (4) Implementing a discrete time full fuzzy system that is composed of a dynamic fuzzy model and a fuzzy state feedback controller, and finding a common positive-definite matrix P to satisfy the Lyapunov stability criterion; and (5) Avoiding the problem of determining the common P by establishing a global stability condition to guarantee the global stability of the closed loop system. Finally, simulation results for the trajectory tracking control of a mobile robot system show the effectiveness of the proposed control scheme of the TSK fuzzy controllers. In the future, researches may incorporate the powerful learning ability of the neural network to adapt the parameters of various fuzzy basis functions, thereby eliminating the approximation errors.

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