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The Controller Designs for the Continuous Time-Delay Singular Systems

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Abstract. The work develops transformation from regular pencil to standard pencil for a class of continuous singular systems with state and output time delays. It proposes a state feedback control to remove the impulsive modes from the system response. Finally, it presents a new technique for decomposing the time-delay singular system to an equivalent low-order regular system with a direct transmission term from input to output, with a view to developing a high-performance trajectory tracker for the sampled-data time-delay singular system.

Keywords: impulsive modes, singular system, time delays

1 Introduction

Singular systems naturally arise in the description of large-scale systems. Several examples can be found in power and interconnected systems. Over past decades, much research into singular systems has solved many complex problems concerning, for example the stability [1], impulsive modes [2], controllability, observability [3], and the sufficient and necessary conditions for the impulse controllability and observability of time-varying singular systems [4]. This paper proposes the transformation from regular pencil to standard pencil for a class of continuous singular systems with state and output time delays. It proposes a state feedback control to remove the impulsive modes from the system response. Subsequently, a new technique has been presented to decompose the time-delay singular system into an equivalent low-order regular system with a direct transmission from input to output. Then, to apply effectively the well-developed discrete-time optimal control theory to the regular time-delay system with the direct transmission term [5], the system is converted into a new extended discrete delay-free model with such a direct transmission term. The high-performance trajectory tracker for a class of time-delay singular systems is realized using the approach [5].

In the recent years, a large number of control systems are characterized by interconnected large scale subsystems, and many practical examples have been applied to decentralized control systems. The decen-

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tralized control of interconnected large-scale systems, have commonly appeared in our modern technologies, such as transportation systems, power systems, communication systems, etc. [6-8]. However, a survey of the literature indicates that the singular system issue has seldom been studied in such systems. Many research [9-10] results concerning about the singular/nonlinear system have successfully solved lots of complex problems.

2 Problem Description

Consider the following continuous time-delay singular system

$$E\dot{x}_c(t) = Ax_c(t) + \sum_{i=1}^{N_1} \hat{A}_i x_c(t-\tau_{s,i}) + Bu_c(t) \quad \textbf{(1a)}$$

$$y_c(t) = Cx_c(t-\tau_o) \quad \textbf{(1b)}$$

where $x_c(t) \in \Re^n$ is the state vector; $u_c(t) \in \Re^m$ is the control input vector, and $y_c(t) \in \Re^p$ is the output vector. E, A, $\hat{A}_i$, B, and C are known constant system matrices of appropriate dimensions and E is a singular matrix. The corresponding state time delays $\tau_{s,i}$, $i = 1, 2, \cdots, N_1$ and output time delay τ_o are assumed to be known. The transformation from regular pencil to standard pencil for the delay-free singular system has been presented [11]; however, due to the existed state delay terms, the approach is not directly applicable for the state-delay system [11]. Besides, how to remove the impulsive modes from the system response of a delay system has not yet been discussed in literature.

3 Main Result

3.1 Decomposition of a Class of Time-Delay Singular Systems

To decompose the time-delay singular system (1), it is firstly rewritten as

$$\sum_{i=1}^{N_1+1} \delta_i E\dot{x}_c(t) = Ax_c(t) + \sum_{i=1}^{N_1} \hat{A}_i x_c(t-\tau_{s,i}) + Bu_c(t) \quad \textbf{(2)}$$

where $\sum_{i=1}^{N_1} \delta_i = 1$, δ_i are scalars and $0 < \delta_i < 1$. The time-delay singular system (2) can be decomposed as the combination of the following subsystems as follows:

$$\delta_1 E\dot{x}_c(t) = Ax_c(t) + \left(\frac{1}{N_1+1}\right) Bu_c(t) \quad \textbf{(3a)}$$

$$\delta_2 E\dot{x}_c(t) = \hat{A}_1 x_c(t-\tau_{s,1}) + \left(\frac{1}{N_1+1}\right) Bu_c(t) \quad \textbf{(3b)}$$

$$\vdots$$

$$\delta_{N_1+1} E\dot{x}_c(t) = \hat{A}_{N_1} x_c(t-\tau_{s,N}) + \left(\frac{1}{N_1+1}\right) Bu_c(t) \quad \textbf{(3c)}$$

To transform the regular pencil into the standard pencil for the continuous time-delay singular system (1), the original regular pencil [11] and standard pencil [11] are modified as follows.

Definition 1: Modified regular pencil [11].

Let E_ℓ and A_ℓ be square constant matrices where $E_\ell = \delta_\ell E$, $A_\ell = \begin{cases} A, & \ell = 1 \\ \hat{A}_{\ell-1}, & \ell \neq 1 \end{cases}$, $\ell = 1, 2, \cdots, N_1 + 1$. If $\det(sE_\ell - A_\ell) \neq 0$, for all *s*, then $(sE_\ell - A_\ell)$ is called a regular pencil.

Definition 2: Modified standard pencil [11].

Let $(sE_{n,\ell} - A_{n,\ell})$ be a regular pencil. If there exists scalars α_ℓ and β_ℓ such that $\alpha_\ell E_{n,\ell} + \beta_\ell A_{n,\ell} = I_n$, then $(sE_{n,\ell} - A_{n,\ell})$ is called a standard pencil. The matrix coefficients of a standard pencil $(sE_{n,\ell} - A_{n,\ell})$ become are

$$E_{n,\ell} = (\alpha_\ell E_\ell + \beta_\ell A_\ell)^{-1} E_\ell \tag{4a}$$

$$A_{n,\ell} = \begin{cases} (\alpha_\ell E_\ell + \beta_\ell A_\ell)^{-1} A_\ell, & \ell = 1 \\ (\alpha_\ell E_\ell + \beta_\ell \hat{A}_{\ell-1})^{-1} \hat{A}_{\ell-1}, & \ell \neq 1 \end{cases}, \ell = 1, 2, \cdots, N_1 + 1 \tag{4b}$$

and the matrices $E_{n,\ell}$ and $A_{n,\ell}$ have the following nice properties.

Theorem 1.

(a) $E_{n,\ell} A_{n,\ell} = A_{n,\ell} E_{n,\ell}$, meaning that $E_{n,\ell}$ and $A_{n,\ell}$ commute each other.

(b) $E_{n,\ell}$ and $A_{n,\ell}$ have the same eigenspaces.

Based on Definitions 1 and 2, multiplying (3a) by $(\alpha_1 \delta_1 E + \beta_1 A)^{-1}$, and multiplying (3b) and (3c) by $(\alpha_2 \delta_2 E + \beta_2 \hat{A}_1)^{-1}$ and $(\alpha_{N_1+1} \delta_{N_1+1} E + \beta_{N_1+1} \hat{A}_{N_1})^{-1}$, respectively, transform the regular pencil into the standard pencil and yield the following equation.

$$E_{n,1} \dot{x}_c(t) = A_{n,1} x_c(t) + \psi_{n,1} B u_c(t) \tag{5a}$$

$$E_{n,2} \dot{x}_c(t) = \hat{A}_{n,1} x_c(t - \tau_{s,1}) + \psi_{n,2} B u_c(t) \tag{5b}$$

$$\vdots$$

$$E_{n,N_1+1} \dot{x}_c(t) = \hat{A}_{n,N_1} x_c(t - \tau_{s,N_1}) + \psi_{n,N_1+1} B u_c(t) \tag{5c}$$

where

$$E_{n,1} = (\alpha_1 \delta_1 E + \beta_1 A)^{-1} \delta_1 E \text{ , } E_{n,2} = (\alpha_2 \delta_2 E + \beta_2 \hat{A}_1)^{-1} \delta_2 E \text{ , } E_{n,N_1+1} = (\alpha_{N_1+1} \delta_{N_1+1} E + \beta_{N_1+1} \hat{A}_{N_1})^{-1} \delta_{N_1+1} E$$

$$A_{n,1=} (\alpha_1 \delta_1 E + \beta_1 A)^{-1} A \text{ , } \hat{A}_{n,1} \triangleq A_{n,2} = (\alpha_2 \delta_2 E + \beta_2 \hat{A}_1)^{-1} \hat{A}_1 \text{ , } \hat{A}_{n,N_1} \triangleq A_{n,N_1+1} = (\alpha_{N_1+1} \delta_{N_1+1} E + \beta_{N_1+1} \hat{A}_{N_1})^{-1} \hat{A}_{N_1}$$

$$\psi_{n,1} = \left(\frac{1}{N_1 + 1}\right)(\alpha_1 \delta_1 E + \beta_1 A)^{-1} \text{ , } \psi_{n,2} = \left(\frac{1}{N_1 + 1}\right)(\alpha_2 \delta_2 E + \beta_2 \hat{A}_1)^{-1}$$

$$\psi_{n,N_1+1} = \left(\frac{1}{N_1 + 1}\right)(\alpha_{N_1+1} \delta_{N_1+1} E + \beta_{N_1+1} \hat{A}_{N_1})^{-1}$$

The time-delay singular system (1) can be decomposed as the equivalent low-order time-delay regular system, as follows [12].

$$\dot{\tilde{x}}_s(t) = \tilde{A}_s \tilde{x}_s(t) + \sum_{i=1}^{N_1} \tilde{A}_{ds,i} \tilde{x}_s(t - \tau_{s,i}) + \tilde{B}_s v_c(t) \tag{6a}$$

$$y_c(t) = \tilde{C}_s \tilde{x}_s(t - \tau_o) + \tilde{D} v_c(t) \tag{6b}$$

where $\tilde{A}_s = \overline{E}_{sk}^{-1} \overline{A}_{sk}$, $\tilde{A}_{ds,i} = \overline{E}_{sk}^{-1} \Lambda_{s,i}$, $\tilde{B}_s = \overline{E}_{sk}^{-1} \overline{B}_{sk}$ and $\tilde{D} = -\tilde{C}_f \overline{B}_{fk}$. The details of the parameters can be referred to [12]. Next, the equivalent low-order time-delay regular system is applied to the proposed approach [5] to realize the control of the time-delay singular system control.

2.2 Time-Delay Singular System Contol

For the equivalent low-order time-delay regular system (6), the following extended discrete delay-free regular system is obtained using that method [5].

$$\mathrm{X}_d((k+1)T) = \hat{G}_e \mathrm{X}_d(kT) + \hat{H}_e u_d(kT) \tag{7a}$$

$$y_d(kT) = \hat{C}_e \mathrm{X}_d(kT) + \hat{D}_e u_d(kT) \tag{7b}$$

where

$$\mathrm{X}_d(kT) = \left[\overline{x}_s(kT)\ \overline{x}_s(kT-T)\cdots\overline{x}_d(kT-h_1T)\ u_d(kT-T)\ \cdots u_d(kT-h_2T)\ r^*(kT)\right]^T$$

and $r^*(kT)$ is the reference input. Fig. 1 shows the implementation of the observer-based sub-optimal tracker for the time-delay singular system. The optimal control and digital redesign techniques are applied to the extended discrete delay-free regular system (7) to design the controller $\hat{K}(kT)$ and the observer $L_d(kT)$.

Fig. 1 shows the implementation of the observer-based sub-optimal tracker for the time-delay singular system.

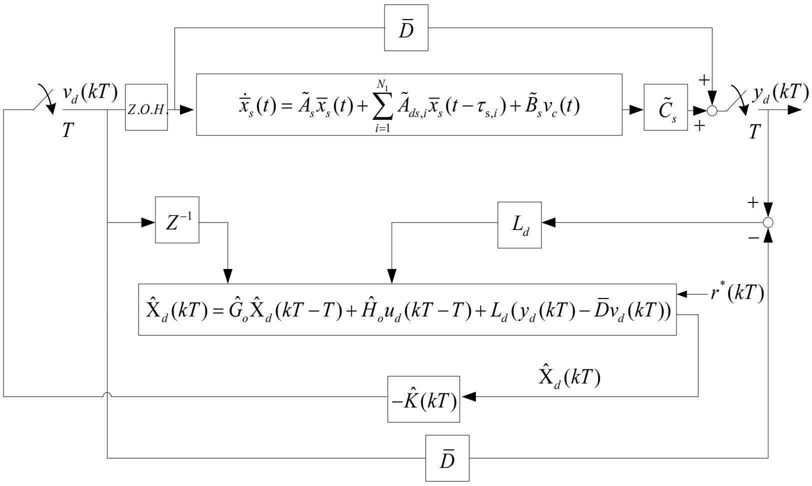


Fig. 1. Observer-based sub-optimal tracker for digitally redesigned time-delay singular system

3 An Illustrative Example

Consider a continuous time-delay singular system, described by (1), with

$$E = \begin{bmatrix} 1 & 4 & 4 & 4 & -9 & -8 \\ 4 & 12 & 8 & 8 & -20 & -16 \\ 2 & 6 & 5 & 7 & -14 & -12 \\ 1 & 16 & 11 & 14 & -26 & -25 \\ 1 & 10 & 7 & 7 & -15 & -14 \\ 2 & 4 & 3 & 2 & -7 & -5 \end{bmatrix},\ A = \begin{bmatrix} 1 & 2 & 1 & 1 & -3 & -2 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 2 & -3 & -2 \\ 1 & 3 & 1 & 3 & 0 & -4 \\ 2 & 2 & 0 & 1 & 0 & -2 \\ 1 & 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\hat{A}_1 = \begin{bmatrix} 0.51 & 2.83 & 2.13 & 1 & -3 & -2 \\ 0.51 & 2.83 & 3.13 & 1 & 0 & 0 \\ 0.51 & 2.83 & 2.13 & 2 & -3 & -2 \\ 0.71 & 4.24 & 2.45 & 3 & 0 & -4 \\ 0.62 & 2.83 & 1.63 & 1 & 0 & -2 \\ 0.31 & 1.41 & 1.82 & 0 & -1 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 1.24 & -1.7 \\ 3 & 3 \\ 1.5 & -1.4 \\ 2.5 & 0.45 \\ 1.8 & 1.7 \\ 1.4 & 0.9 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.03 & 3.5 & 2 & 0.2 & -2 & -1 \\ 0.5 & 3 & 1 & 0.7 & -2 & -0.2 \end{bmatrix},\ N_1 = 1,\ \tau_{s,1} = 1.5 \times T,\ \tau_o = 1.5 \times T$$

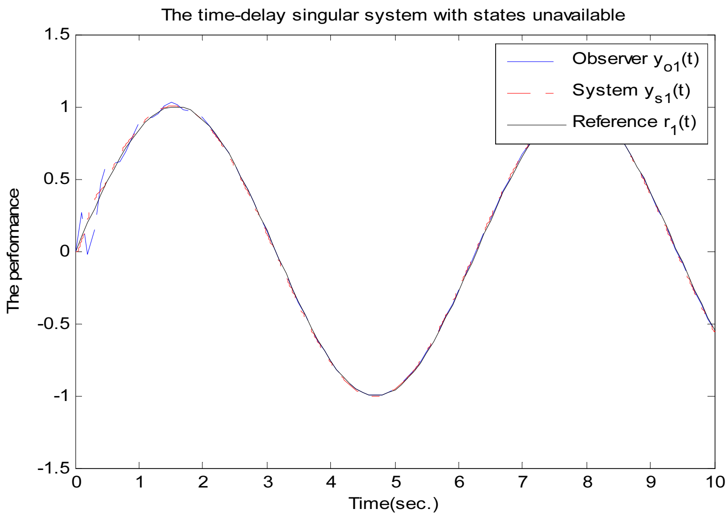
Let the sampling period $T = 0.1$ (s) and apply the reference input $r(t) = [\sin(t) \quad \cos(t)]^T$ to the system. According to Section 2, set $\delta_1 = 0.5$, $\delta_2 = 0.5$, $\alpha_1 = 0$, $\beta_1 = 1$ and $\alpha_2 = -1$, $\beta_2 = 1$. The time-delay singular system (1) can be decomposed as the equivalent low-order regular time-delay system (6), where

$$\tilde{A}_s = \begin{bmatrix} -0.51 & -1.82 & -0.84 & 1.96 \\ 0.22 & 0.71 & -0.15 & -0.6 \\ 0 & 0 & 0.19 & -0.4 \\ 0 & 0 & 0 & -0.83 \end{bmatrix}, \ \tilde{A}_{ds,1} = \begin{bmatrix} -1.51 & -1.82 & -0.84 & 1.96 \\ 0.22 & -0.29 & -0.15 & -0.6 \\ 0 & 0 & -0.81 & -0.4 \\ 0 & 0 & 0 & -1.83 \end{bmatrix}$$

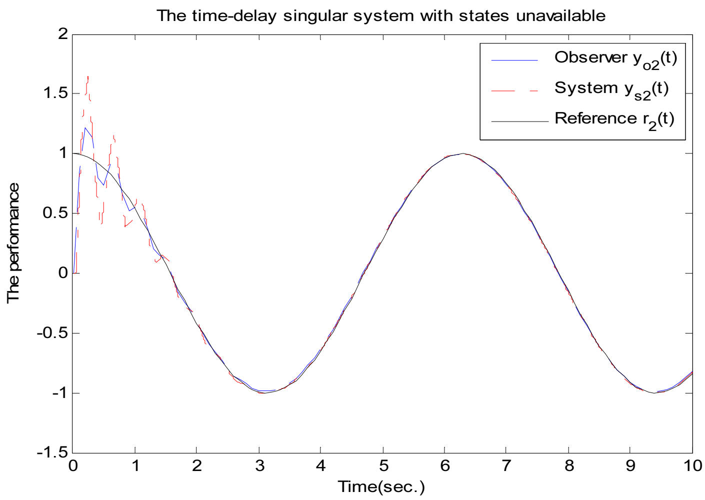
$$\tilde{B}_s = \begin{bmatrix} 0.06 & -3.31 \\ 0.02 & 1.4 \\ -0.08 & 0.36 \\ -0.1 & 1 \end{bmatrix}, \ \tilde{C}_s = \begin{bmatrix} 2.85 & 11 & 1 & -4.26 \\ 5.14 & 8.13 & 3 & -4.5 \end{bmatrix}, \ \tilde{D} = \begin{bmatrix} 0 & 0 \\ 0 & -0.01 \end{bmatrix}$$

and $\overline{x}_s(0) = [0.05 \quad 0.05 \quad 0.05 \quad 0.05]^T$.

For simplification, the numerical analysis (see [5]) is not presented; Fig. 2 shows the simulation results.



(a)



(b)

Fig. 2. Output responses of time-delay singular system with states unavailable

4 Conclusion

This paper presents a systematic methodology for transforming the regular pencil into the standard pencil and establishes equivalent low-order regular time-delay systems for a class of time-delay singular systems. The proposed approach in this paper facilitates the design of the controller or observer. In future works, this proposed approach will be applied to the decentralized control of interconnected large-scale time-delay singular systems, each of which can be controlled effectively with the decoupling property, such that when some unanticipated fault occurs in some subsystems, the tracking performance of other subsystems will not be affected.

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