Journal of Computers Vol. 28, No. 4, 2017, pp. 1-12
doi:10.3966/199115592017082804001

Model and Algorithm: K-extended Constrain Independent Relay Node Placement Solution with Base Stations in Two-tiered Wireless Sensor Network

Zhi-Peng Gao[1], Qian Wang[1*], Kan-Chen[1], and Wei-jing Cheng[1]

[1] State Key Laboratory of Networking and Switching Technology
Beijing University of Posts and Telecommunications
Beijing, China
{gaozhipeng, wangqian1991}@bupt.edu.cn

Received 25 June 2015; Revised 8 December 2016; Accepted 8 December 2016

Abstract. In two-tiered wireless sensor network, relay nodes are responsible for delivering the sensed data from sensor nodes to base stations. From the perspective of business, it needs more relay nodes to ensure network connectivity. From the perspective of economy, relay nodes are relatively expensive, so people are intended to place a minimum number of relay nodes while guaranteeing the data collecting and delivering. In this paper, we firstly model fault tolerant relay node placement problem in the two-tiered wireless sensor network as a graphic problem, denoted by DBY-HCG. It distinguishes relay nodes from base stations. Secondly, we figure out keCi-RNPB algorithm. Moreover, an approximation algorithm aiming at finding the minimum length of k-vertex disjoint paths in connected sub-graph is raised up to support our algorithm's extensibility. Our algorithm is also the first solution for constrained version of fault tolerant relay node placement problem in the two-tiered wireless sensor network. Extensive experiments have been executed in both unconstrained and constrained situations and the numeric results show that our solution is close to optimal solution.

Keywords: 2tWSN, constrain independent, fault-tolerant, k-extended, relay node placement

1 Introduction

Two-tiered Wireless Sensor Network (2tWSN) includes base stations, relay nodes and sensor nodes as shown in Fig. 1. Base stations are called overlapping nodes which can bear damage and have excellent performance, high price. Communication can be conducted among base stations. Relay nodes are called cluster nodes which are responsible for collecting sensor data in the cluster and deliver these data to base station. They have restricted communication radius, denoted by R. Sensor nodes are members of cluster who are usually deployed in certain area [1-2]. They have low price relatively and restricted communication radius which is denoted by r. Relay nodes can gather data from sensor nodes and deliver data to base station acting as a connecting link between the preceding and the following in 2tWSN, so if relay nodes are broken, wireless sensor network business will be affected seriously. Relay node placement affects communication, router selection and topology generation, and then network fault tolerance will be affected. Wireless sensor network's fault tolerance can be assessed by ks and kr [3]. Fault tolerant relay node placement has been proved to be NP hard [4-8]. People expect to deploy relay nodes as few as we can so as to satisfy economic requests [9]. Recent researches in this field are focused on deployment algorithm and proof of its approximation proportion [10-12]. This paper studies relay node placement in 2tWSN considering their constrained location from the perspective of k-extension.

* Corresponding Author

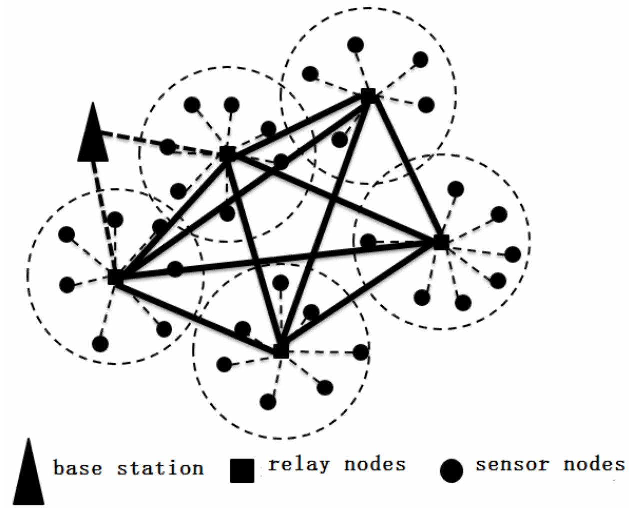


Fig. 1. Two-tiered wireless sensor network

The main innovations of this paper are described as follows. Firstly, we improve relay node placement model which is put forward by [10] and come up with Diff-B node and Y node hybrid communication graph, denoted by DBY-HCG. B node stands for base station while Y node stands for relay node. Base station, sensor node and relay node is called B, X and Y respectively. DBY-HCG can decrease the expense caused by keeping k-connected between relay nodes and all base stations through distinction of base station and relay nodes. Secondly, based on this model, we propose k-extended constrained independent relay node placement with base station, denoted by keCi-RNPB. Based on the knowledge of relay nodes' constrained location, this algorithm is realized by corresponding layout and approximate method. Finally, an approximate algorithm aiming at finding the minimum length of k-vertex disjoint paths in connected sub-graph is raised up to support our algorithm's extensibility. This algorithm can realize k-extensibility keeping time complication to an acceptable degree.

The paper is organized as follows. Section 2 introduces the classical solution to relay node placement problem. Section 3 builds relay node placement model, defines relative parameter, analyzes the method of layout in two versions: constrained and unconstrained. Section 4 comes up with keCi-RNPB algorithm in 2tWSN.Section 5 describes an approximation algorithm aiming at solving k-extended problem. Section 6 details our experimental results, verifies effectiveness of this algorithm. Section 7 concludes the paper.

2 Related Work

Relay node placement in wireless sensor network has been proved to be NP hard. The optical solution according with ILP function is usually gained by exhaustive method [4]. The consuming time of this method is long. So we need to find the approximate solution. Relay node placement approximate solution in wireless sensor network has two versions. One is unconstrained which is called unconstrained relay node placement strategy. The other is constrained which is called constrained relay node placement strategy. In general, solution is aimed at one of the two versions.

In the unconstrained location version, Cheng, Du, Wang and Xu puts forward solution to the relay node placement in the single tiered wireless sensor network [4]. They present that this problem is equal to STP-MSP problem in Steiner Tree when R is equal to r. According to the research about STP-MSP with approximate proportion 4 and 3 [13], Chen et al. improve the solution to3 and 2.5. The complexity of this algorithm is O(n3). Kashyap, Khuller and Shayman research the extended problem which increases the tolerance of network [5]. Considering the tolerance, they model this problem as minimum connected components in weighted graph. Approximate proportion of the solution is 2M. M is maximum node degree in minimum Steiner Tree. The complexity of 2-edge connected solution is O((knL)4). The above are relay node placement solutions taking no account of base station in the single tiered wireless sensor network. When we take base station into consideration in 2tWSN, the problem becomes more complicated. Lloyd and Xue put forward two tiered 2tRNP which can realize the connectivity of network

[7]. This algorithm is in the situation where $R \geq r$ and takes base station into consideration. They prove the approximate proportion to be 7. This research provides a basic solution and approximate proportion proved framework in the two tiered network. The complexity of this algorithm is up to cited coverage and the complexity of STP-MSP. Weiyi, Guoliang and Misra provide approximate proportion 16 solution to single tiered network and 20 solution to two tiered network when $R \geq r$ [10]. The two tiered solution is similar to [7], citing minimum weighted k-connected sub-graph algorithm [14]. Frank and Tardos put forward an approximate algorithm aiming at finding k-vertex disjoint paths in weighted graph [15]. The research provides a reference for minimum weighted k-connected sub-graph algorithm. Tang, Hao and Sen puts forward a polynomial complicated and approximate proportion 6 solution to 2-covered 2-connected problem through dividing area [11]. But this solution is limited to division and proof. So it is hard to extend to k-connected situation. Yu, Chen and Zhang, Chen, Li and Ding present "equal sex-angle" relay node to realize 3-connectivity [16-17]. This method is also hard to extend to k. Khuller and Vishkin, Frank and Tardos, Kashyap, Khuller and Shayman puts forward algorithm solving 2-connectivity [14, 15, 18]. This method can correspond to requirement of placement in fault tolerant situation, so it has been cited by many researchers.

In real life, because of the limitation in the environment, relay node should be set in appointed location. Therefore, relay node placement in constrained version has attracted more and more attention. Misra, Hong, Xue, and Tang put forward approximate algorithm with proportion 6.2 when $R \geq r$ and designs 2-connected approximate algorithm with performance proportion 10 [19]. Misra, Hong, Xue and Tang puts forward a O(1)-approximate framework first time, acting as an inspiring method for latter research [12]. The above researches are about node placement in single tiered network. Rajagopalan and Vazirani, Fleischer gives the solution to coverage and {0, 1, 2}-connected respectively [20-21]. Yang, Misra, Fang, Xue and Zhang puts forward O(1)-approximate proportion solution to 2-connectivity 2-coverage in 2tWSN [24]. Most of algorithms described above have a limitation: undistinguishing base station from relay node or restricting k value. Considering these limitations, this paper distinguishes base station from relay node. Relay node keep k-connected with no less than one base station. And the algorithm proposed in this paper can be extended to k.

3 HCG Model Distinguishing Base Station from Relay Node (DBY-HCG)

3.1 Definition

As mentioned before, two-tiered wireless sensor network includes three types of nodes. Sensor node which is denoted by SN is responsible for sensing and getting data. Its communication radius is r. Relay node which is denoted by RN is responsible for collecting data and delivering to base station. Its communication radius is R ($R \geq r$). Base station which is denoted by BS collects all data. BS can communicate with each other. The collection of SN, RN, BS is denoted by X, Y and B respectively. Sensor node only communicates with relay node or base station in its communicating area.

The original network composed by three types of nodes in 2tWSN is called hybrid communication graph (HCG). This paper models a HCG distinguishing base station from relay node, denoted by DBY-HCG(r, R, B, X, Y). It is defined as follows.

Definition 1: DBY-HCG (r, R, B, X, Y) is undirected weighted graph produced by (r, R, B, X, Y). Node collection $V=B\cup X\cup Y$. Initial edge collection E is defined as follows.

(1) For arbitrary base station bi, bj $\in$ B, edge (bi, bj) $\in$ E.

(2) For arbitrary relay node y and relay node or base station z, $y \in Y$, $z \in Y\cup B$, then(y, z) $\in$ E when $d(y, z) \leq R$.

(3) For arbitrary sensor node X and other node z, $x \in X$, $z \in B\cup X\cup Y$ then (x, z) $\in$ E when $d(x, z) \leq r$.

3.2 Design Graph

In unconstrained version, the weight of edge in HCG graph is initialized as w (zi, zj) = 0. Then, sub-graph composed of relay node and base station is Steiner which extend sub-graph to complete graph. The added edge is called Steiner edge. The weight defined to this edge is called Steiner of this edge which means the number of relay nodes to be placed in this direction so as to be connected.

For arbitrary yi $\in$Y, zj $\in$ Y$\cup$B, w is computed as equation (1).

$$w(y_i,z_j)=\begin{cases}0 & if\ d(y_i,z_j)\le R \\ \left\lceil \dfrac{d(y_i,z_j)}{R} \right\rceil -1 & otherwise\end{cases} \quad . \tag{1}$$

The graph having renewed is called Steiner Graph (Gs). The weight value means the number of relay nodes to be placed in this direction so as to be connected. Fig. 2 describes the process of Steiner to sub-graph including RN and BS in unconstrained version.

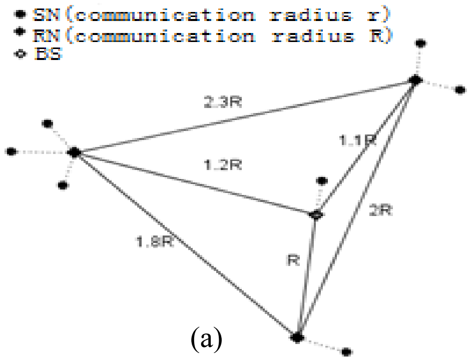


Fig. 2. Process of Steiner to sub-graph in unconstrained version

In constrained version, the weight of every edge e in the HCG is defined as equation (2) [12].

$$w(e) = \{ zi, zj \} \cap Y \tag{2}$$

Fig. 3 describes the process of Steiner to sub-graph including RN and BS in constrained version.

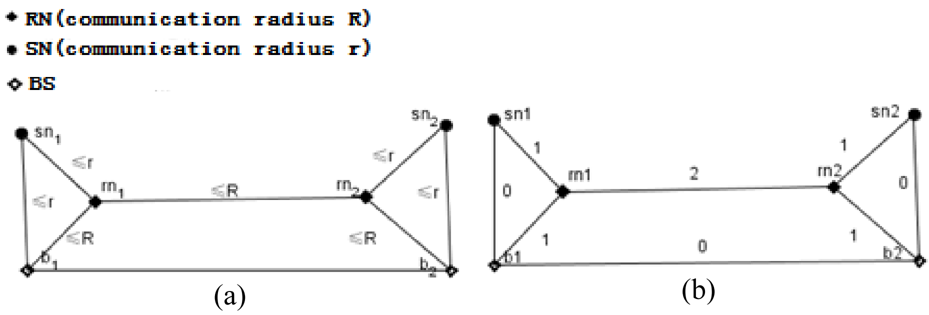


Fig. 3. Process of Steiner to sub-graph in constrained version

3.3 Model the Problem

Fault tolerance is assessed by *ks* and *kr* [3] which stands for the capacity of tolerating. *ks* and *kr* are defined as follows.

(1) If *ks*-1 relay nodes break, it will not affect sensor nodes in the corresponding cluster.

(2) If *kr*-1 relay nodes break, relay nodes will not be affected, so data can be delivered to base station.

This problem in the paper is called FT2tRNPB (fault-tolerant 2 tiered relay node placement with base stations). It is abstracted as follows. Given three nodes collection X, B and Y’ (if in unconstrained version, Y’ is null), we initialize DBY-HCG (r, R, B, X, Y’) using known nodes location and the number of nodes in the collection. We need to find a node collection Y to make |Y| minimum (if location is constrained, we need to find Y Y’). And DBY-HCG (r, R, B, X, Y’) should be subject to condition 1) and 2).

(1) For every x $\in$ X, there exist *ks* y $\in$ Y which is subject to d (x, y) $\le$ r. y is called cluster and x is member of cluster where y is head.

(2) For every cluster head y, there exist one or more b $\in$ B making *kr* disjoint paths between y and b.

Compared to traditional model, this paper distinguishes base station from relay node. So modeling to condition 2) is equal to finding *kr* disjoint paths between every cluster head y and one or more b. When *kr* = 1, the definition in this paper is the same as the traditional one. When *kr* > 1, model in this paper will decline redundancy of relay node placement. Fig. 4 shows result comparison in different constrained conditions when *k* = 2.

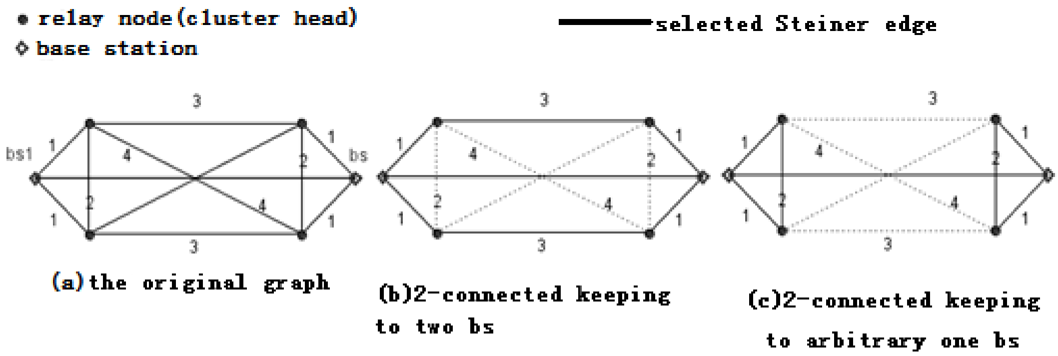


Fig. 4. Comparison in different constrained conditions

4 Relay Node Placement Algorithm

To satisfy with the need of the problem, this paper puts forward solution A. It is described as follows.

```
Algorithm A: relay node placement with fault tolerant in 2tWSN

Input:  collection X including n SNs;
        collection B including m BSs;
        communication radius R ≥ r;
        constrained relay nodes location collection Y'.
Output: collection Y including one relay node. (If Y' is not null, output is Y⊆Y')
Step:
S1. Based on X, we get a collection C={c_1,c_2,...,c_{u-k}} using approximate solution to coverage (constrained location and unconstrained location), so DBY-HCG(r, R, B, X, C) can satisfy with condition 1).
S2. Based on the layout described in section 3, HCG is Steiner to Gs( r, R, B, X, C).
S3. Adjust Gs graph and merge B to be a node b.
S4. Using approximate algorithm, we get a new collection C'={c_{u-k+1},c_{u-k+2},...,c_u} based on the Gs. C = C ∪ C'={c_1,c_2,...,c_u} .Then we create DBY-HCG(r, R, B, X, C) which satisfies with condition 2).
S5.Output Y=C∪R
```

This strategy can be applied in both unconstrained version and constrained version. We will describe this strategy in detail.

Step 1: This step realizes *ks* coverage which is the condition 1.

In unconstrained situation, this paper uses approximate solution *Dcover* [23]. The time complexity of *Dcover* is $O(2l^4 \cdot (2n)^{4l^2+1})$. In the situation of constrained candidate location, this paper uses approximate solution in [17] to realize node coverage. Ks-coverage can be realized by iterating *ks* times.

Step 2: This step renews HCG graph based on step 1. In unconstrained version, we use the layout-1 described in section 3-2. While in constrained version, we use the layout-2.

Step 3: Merge collection B to be a node b. The process is described by Fig. 5.

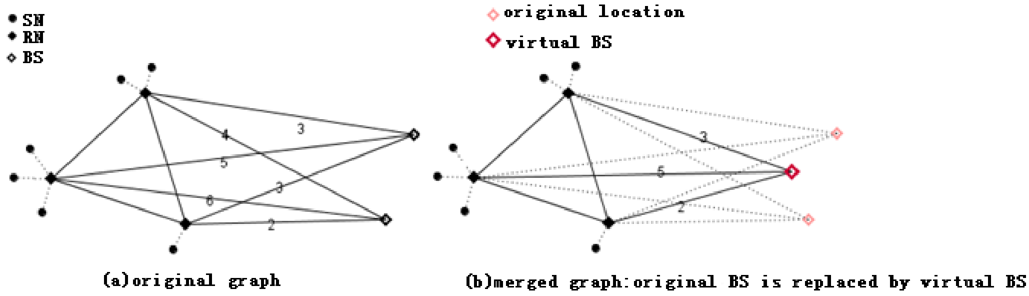


Fig. 5. Process of merging

Only if base stations keep connected with each other, the graph can be simplified as a collection Y1 and a node b. And the shortest path between b and other nodes can be kept. So we can decline the complexity of approximate algorithm aiming at finding k-vertex disjoint path between cluster collection C and collection B.

Step 4: This step finds *kr*-vertex disjoint paths which is condition 2.The algorithm aiming at finding minimum weighted sub-graph of disjoint paths between all cluster heads y and b node in current HCG will be described in next section.

Step 5: Output result.

We will analyze the time complexity of this algorithm. In unconstrained version, complexity of the algorithm is sum of covering time, merging time and node connecting time. |E| is edge created by RN Steiner after completion of covering. The time expense is mainly up to time of connection. While in constrained version, complexity of the algorithm is sum of covering time, merging time and node connecting time. In this situation, |E| is edge in communicating area. Similarly, all the expense is mainly up to time of connection.

We have analyzed researches about relay node placement under fault tolerance. And the summary is showed in Table 1 and Table 2.

Table 1. Comparison in unconstrained version

reference	single/two	B≠∅	R≠r	degree of connectivity	complexity	approximate proportion
[10]	single		R≥r	1	Polynomial	14
[10]	single	Y	R≥r	2	Polynomial	16
[10]	two		R≥r	1	Polynomial	10
[10]	two	Y	R≥r	2	Polynomial	20
[11]	two		R≥4r	1	Polynomial	8/4.5
[11]	two		R≥4r	2	Polynomial	6/4.5
this paper	two	Y	R≥r	K	O(k×|E|2×V2) E=|αV|2	O(1)

Table 2. Comparison in constrained version

reference	single/two	B≠∅	R≠r	degree of connectivity	complexity	approximate proportion
[19]	single		R ≥ r	1	Polynomial	5.5
[19]	single	Y	R ≥ r	1	Polynomial	6.2
[12]	single		R ≥ r	2	$O(V^3+\|E\|\times V\times\alpha(V))$	9
[12]	single	Y	R ≥ r	2	$O(\|V^3+\|E\|\times V\times\alpha(V))$	10
[24]	two	Y	R ≥ 2r	2	Polynomial	O(1)
this paper	two	Y	R ≥ r	K	$O(k\times\|E\|^2\times V^2)$	O(1)

5 Approximate Algorithm Aiming at Finding Minimum Weighted K-vertex Connected Sub-graph

Graph G is given which is the result of step 4 in section 4. Nodes collection V includes cluster head Y, base station collection B and Y'' (Y' is relay node constrained/candidate location collection or nodes collection of Steiner graph in unconstrained version). We need to find minimum weighted sub-G. This sub-G includes nodes collection Y and B which are satisfied with existing k-vertex disjoint paths from Y to B. Pseudo code of algorithm B is described as follows based on greedy algorithm.

```
Algorithm B: minimum weighted k-vertex connected sub-graph
 Input: k;
        G=(V,E);
        V={b} ∪ {y ∈ Y,Y is collection of cluster} ∪ {y ∈ Y',Y' is
candidate
          location};
        c(v, w)is weight of edge.
 Output: G' ←(V', E');
         V'= {b}∪{y ∈Y',Y' is collection of cluster}∪{y ∈Y',if∃(y,
z)∈E'}.
S1 Sort each edge in E in increasing order of c(v, w) ∈E
S2 G'←(V', E'), V'= {b}∪{y∈Y}, E'=Ø
S3 for every (v, w) ∈E in increasing order of c(v, w) do
S4     E'←E'∪ {(v, w)}
S5     V'=V'∪{v,w}
S6     if G' is k-vertex-connected then
S7         break;
S8     end if
S9 end for
S10 for (v, w) ∈E'in decreasing order of c(v, w) do
S11      G''← (V\({v, w}∩Y'), E'\{(v, w)})
S12      if G''is k-vertex-connected then
S13        G'←G'';
S14     end if
S15 end for
```

Pseudo code of algorithm C deciding whether G is k-connected in step 6 and 12 is described as follows.

```
Algorithm C

S1 For each vertex in Y (cluster) do
S2     if judge there exists k-vertex-disjoint paths between (y, b)
S3        continue; //Ek algorithm
S4     else
S5        break;
S6 end for
```

Given source node a and gathering node d, we execute Ek algorithm [25] to decide whether there exist k=2 paths from a to d. The process of seeking path is described in Fig. 6.

The main idea of this method is rooted in flow conservation theorem in network flow. The flow capacity in the edge means that this edge can be traversed only one time. Take nodes apart and assign the flow capacity of edge as 1. It means traversing this node no more than once.

Fig. 7 describes the process of executing Ek before taking nodes apart while Fig. 8 describes the process of executing Ek after taking nodes apart. In the Fig. 7, we can see the original execution of Ek which outputs two augmented paths shown in (a1) and (a2). Their flow capacities are both 1. The two paths intersect at V2 node. We take nodes of Fig. 7. (a) apart and get a new Fig. 8 (a'). Then we execute Ek in the (a') and get (a'1) which includes only one augmented path. The example above clarifies that we

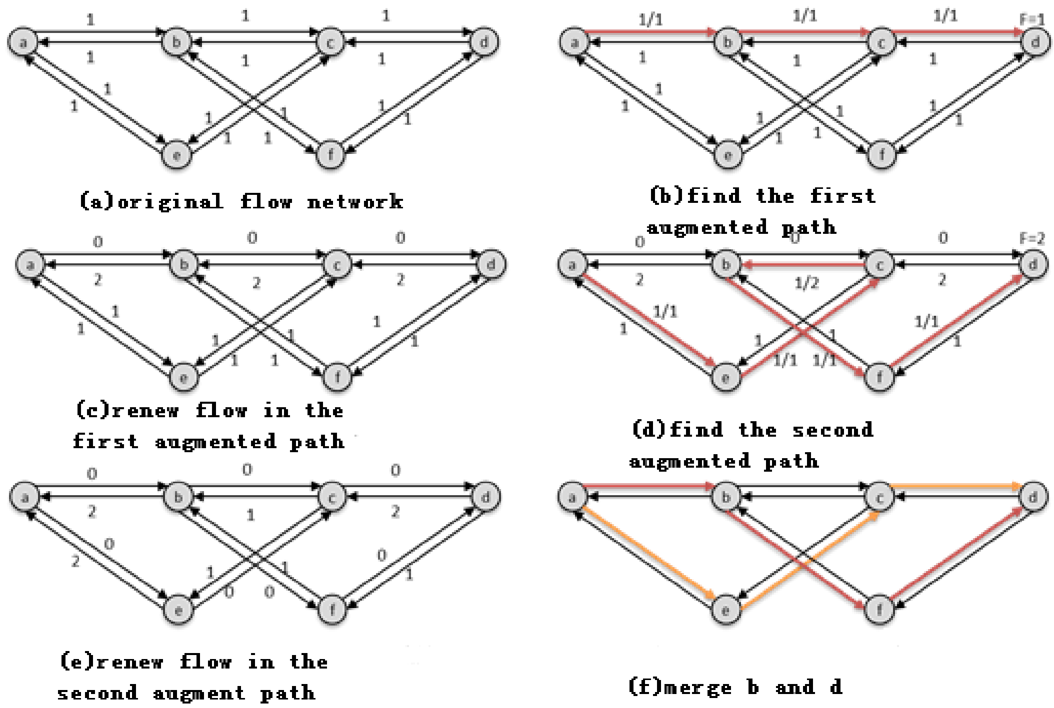


Fig. 6. Process of seeking path

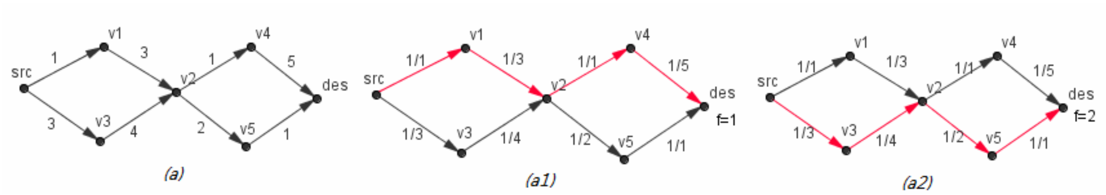


Fig. 7. Execution of Ek before taking nodes apart

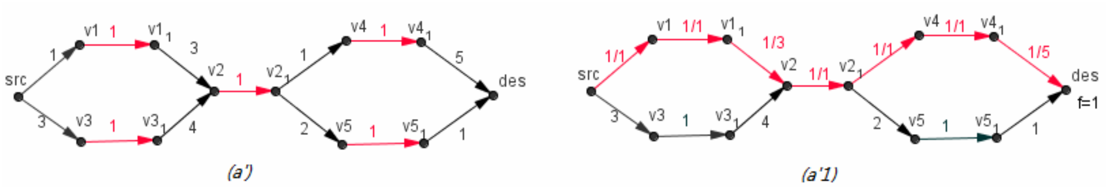


Fig. 8. Execution of Ek after taking nodes apart

can find only one augmented path from src to des after taking nodes of Fig. 8 (a') apart. This proves effectiveness of deciding nodes disjoint paths through taking nodes apart.

The time complexity of *Ek* is $O(k\cdot|E|)$.We can optimize the above algorithm. For example, in the step which is used to decide connectivity of G, we can give every couple of nodes a tab in the augmented path in the process of judgment. If some couple of nodes is found to be accorded with conditions, we will not execute Ek anymore. This optimization can save much running time to a large extent.

Taking nodes apart increases the number of nodes. In the meanwhile, executing time of Ek algorithm decreases severalfold. Therefore, time complexity of Ek remains unchanged.

6 Experiment and Analysis

6.1 Configuration of Experimental Environment

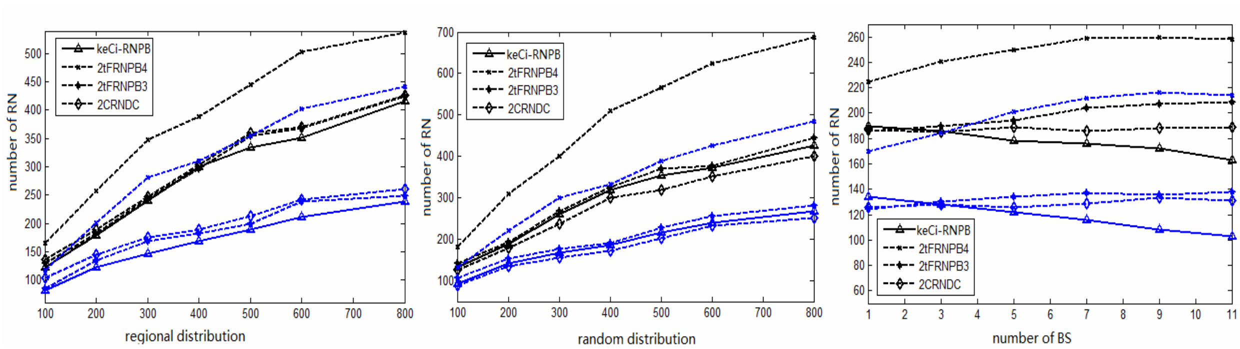
In order to verify the effectiveness of the algorithm in constrained and unconstrained version, experiment is divided into two parts. One describes RNP problem in unconstrained version while the other describes RNP problem in constrained version.

Based on nodes' distributed area in wireless sensor network, we set the network 800*800 m^2. The number of SNs in this network is from 100 to 800 which is distributed randomly in some regions or the whole region. Region distribution is emulated in this way: 500*500 m^2 network is divided into 9 regions and nodes are distributed into several regions among them. The communicating radius of sensors nodes is set as 20-50m. The number of base stations is set as 5/10 which can communicate ignoring distance. RNs use 802.11 protocol [11], so the communicating radius of RN is set as 200m.

6.2 Relay Node Placement in Unconstrained Version

Considering the fact that most of current researches define k to be 2, so we assign 2 to k in the experiment. The comparing algorithms are [10] and [11]. Algorithm 3 and 4 in [10] are denoted as 2tFTPB3, 2tFTPB4. Algorithm in [11] is denoted as 2CRNDC.

As shown in Fig. 9 (a), we can see that our algorithm has some advantages which is almost 0.02-0.03 comparing to 2tFTPB3 and 2CRNPC.The main advantage of this paper is merging BS nodes which makes placement satisfy 2-vertex disjoint path from SN to base station instead of the whole network. In Fig. 9 (b), sensor nodes distribute in the whole 2k*2k region equably. keCi-RNPB in this paper is better than 2tFTRNPB3 and 2tFTRNPB4, because definition of problem in modeling of keCi-RNPB is very explicit. In Fig. 9 (c), as BS increases, the need of relay nodes of keCi-RNPB decreases relatively. It is steady and better overall which dues to consider the particularity of BS.



(a) (b) (c)

Fig. 9. Different sensor node density

All in all, keCi-RNPB is better and flexible in k=2 situation. It can extend to k and will not be affected by environment. In the meantime, we find 2CRNPC has good performance if nodes in the whole region distribute equably. However, 2CRNPC is hard to extend to k-connectivity. The performance of 2tFTRNPB3 is good but its covering strategy is not appropriate to practical application in hostile environment. 2tFTRNPB4 need to pay extra expense for proof of the algorithm.

6.3 Relay Node Placement in Constrained Version

In constrained version, current researches solve relay node placement in single wireless sensor network or 2-conncetion in 2tWSN. So we select optimal algorithm to compare with algorithm in this paper directly. This problem is NP hard so there are no algorithms solving kr-covered ks-connected relay node placement in 2tWSN in polynomial time. We use exhaustive method to computer optimal solution. The algorithm in this paper is evaluated by approximate proportion of real performance of placement and optimal solution. Based on the fact that time consumption of exhaustive method is enormous, we diminish experimental scale to 500*500m^2.The number of SN is diminished to 40-140. Communicating radius of SN is 30m and |BS|=3.

As shown in Fig. 10 (a), number of RN will increase with the increasing of sensor nodes. From the Fig. 10 (b), we can see that the result of our algorithm is similar to optimal solution as the increase of connecting degree k.

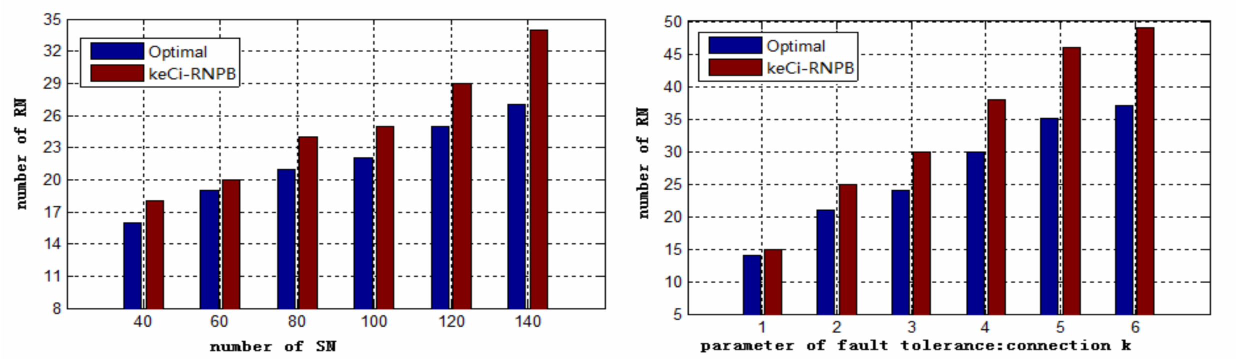


Fig. 10. (a) different density of SN and k=2 (b) different degree of connectivity and |SN|=80

The comparison between our algorithm and optimal results in different fault tolerant parameter k is described as Table 3. We can see distance between them is no more than 2 when degree of connectivity reaches 6.

Table 3. Proportion of executing performance (our algorithm/optimal result)

(number of SN, radius of communication)	K=1	K=2	K=3	K=4	K=5	K=6
(80,30m)	1.0714	1.1905	1.25	1.2667	1.3143	1.324
(140,30m)	1.1176	1.3123	1.275	1.7667	1.2683	1.5023
(80,50m)	1.0833	1.1333	1.1980	1.15	1.1471	1.1707

7 Inclusion

We put forward DBY-HCG model in this paper. Based on this model, we propose keCi-RNPB algorithm. Besides, we present solution to k-vertex connected minimum weighted sub-graph problem which makes keCi-RNPB able to extend to cover k-connectivity. This solution ensures time complexity in constant direct proportion to k. keCi-RNPB is appropriate in unconstrained and constrained version. It solves relay node placement problem which degree of connectivity is bigger than 2 in constrained version of 2tWSN.In the future, we will research relay node placement in three-dimensional wireless sensor network.

Acknowledgement

This paper is partly supported by the National Natural Science Foundation of China (61272515, 61121061), and National Science & Technology Pillar Program (2015BAH03F02).

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