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Takagi-Sugeno-Kang Fuzzy Controller Design for Nonlinear Systems Using the Scaling Gain Adaptation

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Abstract. This paper proposes a Takagi-Sugeno-Kang (TSK) fuzzy controller design using scaling gain adaptation. The proposed adaptation law for the TSK fuzzy controller resulting from the direct adaptive approach is used to appropriately determine the control signal of the controller. The primary advantage of the proposed method is that the parameters of the fuzzy controller, including its rules and membership functions; do not need to be changed under control, particularly when only one adapted parameter is used. Usually, it is not easy to characterize the linguistic control rules and the membership function distribution without expert knowledge when the number of input variables of fuzzy controller is large. In this study, we first organize the TSK fuzzy controller without any specific experience information about the controlled system. Then, the proposed adaptation law is used to adapt the controller's output scaling gain so that the satisfactory system performance of the closed-loop control system can be achieved. Finally, we use the proposed fuzzy controllers to control a nonlinear system, verifying its effectiveness.

Keywords: adaptive fuzzy control, fuzzy control, fuzzy sets, membership functions, scaling gain

1 Introduction

Since fuzzy set theory and fuzzy control strategy were proposed, respectively, research for fuzzy controllers has become an important class in the field of automatic control [1-2]. Fuzzy controllers have been widely adopted in industrial applications, although in many situations, it is not easy to acquire the knowledge and the experience from a skilled operator. Thus, how to properly determine: (1) fuzzy control rules, (2) membership functions, and (3) scaling gains of the fuzzy controllers are all important issues in the design of a fuzzy control system [3-5]. Recently, various methods in the literature, such as genetic algorithms, neural networks and simulated annealing, are used to search for fuzzy controller parameters to improve the ability of a fuzzy controller [6-8]. However, the main disadvantage of these approaches is that it is difficult to tune the parameters of the controller in a real-time operating system. One effective tuning method is to adapt the interested parameter using an adaptive control scheme.

Broadly speaking, the adaptive fuzzy controller falls into one of three categories: the membership function adaptation, scaling gain adaptation, and rule adaptation. The membership function adaptation of fuzzy controller is most often used by engineers. It can be implemented through conventional control schemes of adaptive control, sliding mode control and optimal control. The rule adaptation of fuzzy controller concentrates on adapting the linguistic rules based on heuristic and non-heuristic adaptation schemes [9-11]. The scaling gain adaptation of fuzzy controllers according to heuristic tuning strategies changes the input and the output gain of the fuzzy controller.

This paper is organized as follows. Section 2 presents the fuzzy controller structure. In Section 3 we show how to derive the adaptation law for adapting the output scaling gain of the TSK fuzzy controller. The robust control derivation to guarantee system stability is given in Section 4. In Section 5 a simulation example with a nonlinear pendulum system is utilized to illustrate the performance of the proposed fuzzy controller. Finally, conclusions are given in Section 6.

2 Structure of Fuzzy Controller

Consider the following *n*th-order dynamical systems:

$$x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u \quad \mathbf{(1)}$$

$$y = x \quad \mathbf{(2)}$$

where f and g are unknown functions; u and y are, respectively, the control input and the output of the plant; and $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T = (x, \dot{x}, \cdots, x^{(n-1)})^T$ is the state vector, which is assumed to be available for measurement. Define the error e as, $e = r - y = r - x_1$, where r is the desired output. According to (1) and (2), the optimal controller is

$$u^* = \frac{1}{g(\mathbf{x})}[-f(\mathbf{x}) + r^{(n)} + \mathbf{c}^T\mathbf{e}] \quad \mathbf{(3)}$$

where $\mathbf{c} = (c_1, c_2, \cdots, c_n)^T$ is a positive constant vector, and $\mathbf{e} = (e_1, e_2, \cdots, e_n)^T = (e, \dot{e}, \cdots, e^{(n-1)})^T$ is the error vector. Substituting (3) into (1), we have

$$e^{(n)} + c_n e^{(n-1)} + \cdots + c_1 e = 0 \quad \mathbf{(4)}$$

Clearly, we could appropriately choose the element of vector **c** such that all the roots of (4) are in the open left-half complex plane. Thus we can say that the controlled system is stabilized if the optimal control (3) is employed. However, in many cases we cannot obtain the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ in advance; therefore, it is impossible to implement (3). To solve this problem, the TSK fuzzy controller is proposed to estimate (3).

Consider the following TSK fuzzy controller structure:

$$(j_1, j_2, \cdots, j_n) - th \ \ rule:$$

$$IF \ x_1 \ is \ A_{(1,j_1)} \ and \ x_2 \ is \ A_{(2,j_2)} \ and \cdots and \ x_n \ is \ A_{(n,j_n)} \ THEN \ u_f \ is \ u_{h(j_1,j_2,\cdots,j_n)} \quad \mathbf{(5)}$$

where

$$u_{h(j_1,j_2,\cdots,j_n)} = (j_1 + j_2 + \cdots + j_n) \cdot (k_1 x_1 + k_2 x_2 + \cdots + k_n x_n) \quad \mathbf{(6)}$$

The membership functions are shown in Fig. 1. Since the performance of a control system depends on the control signal, the scaling gain of output should be properly determined to yield a suitable control signal. The output of TSK fuzzy controller is given by

where

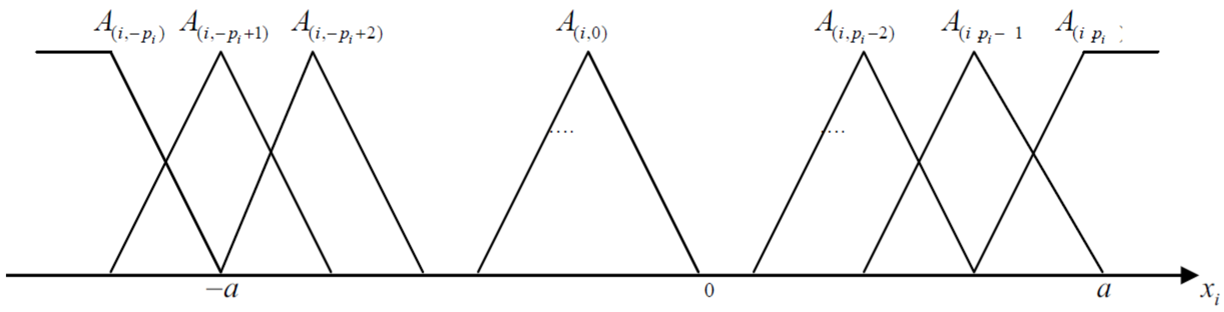


Fig. 1. Membership functions of the IF-part

$$u_f(\mathbf{x},k_o^{TS}) = k_o^{TS}\frac{\sum_{j_1=-p_1}^{p_1}\cdots\sum_{j_n=-p_n}^{p_n} u_{h(j_1,j_2,\cdots,j_n)}(\prod_{i=1}^{n}\mu_{A_{(i,j_i)}}(x_i'))}{\sum_{j_1=-p_1}^{p_1}\cdots\sum_{j_n=-p_n}^{p_n}(\prod_{i=1}^{n}\mu_{A_{(i,j_i)}}(x_i'))} = k_o^{TS}\eta(\mathbf{x}) \tag{7}$$

$$\eta(\mathbf{x}) = \frac{\sum_{j_1=-p_1}^{p_1}\cdots\sum_{j_n=-p_n}^{p_n} u_{h(j_1,j_2,\cdots,j_n)}(\prod_{i=1}^{n}\mu_{A_{(i,j_i)}}(x_i'))}{\sum_{j_1=-p_1}^{p_1}\cdots\sum_{j_n=-p_n}^{p_n}(\prod_{i=1}^{n}\mu_{A_{(i,j_i)}}(x_i'))} \tag{8}$$

The output scaling gain k_o^{TS} in (7) is allowed to be adjustable, and we find that different output scaling gain will result in a different control signal. Finally, the overall control laws for both fuzzy controllers are constructed as

$$u = u_f + u_r \tag{9}$$

where u_r is the robust control term, which will be given later.

3 Scaling Gain Adaptation of TSK Fuzzy Controller

Adding and subtracting gu^* on the right hand side of (1), and substituting (7) in (1), yields

$$e^{(n)} = -\mathbf{c}^T\mathbf{e} + g(-u_f + u^*) \tag{10}$$

or

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{b}[-u_f + u^*] \tag{11}$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -c_1 & -c_2 & \cdots & \cdots & \cdots & -c_n \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ g \end{pmatrix} \tag{12}$$

Suppose that an optimal input scaling gain vector k_o^{TS} is defined by

$$k_o^{TS*} = arg\min_{k\in R}\left[\sup_{\mathbf{x}\in R^n}\left|u_f(\mathbf{x},k_o^{TS}) - u^*\right|\right] \tag{13}$$

The minimum approximation error is

$$\varepsilon = k_o^{TS*}\eta(\mathbf{x}) - u^* \tag{14}$$

Using (1), (3), and (14), we can rewrite (11) as

$$\dot{\mathbf{e}} = \mathbf{Ae} + \mathbf{b}(k_o^{TS*} - k_o^{TS})\eta(\mathbf{x}) - \mathbf{b}\varepsilon \tag{15}$$

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{Pe} + \frac{g}{2\gamma}(k_o^{TS*} - k_o^{TS})^2 \tag{16}$$

where γ is a positive constant, and $\mathbf{P}$ is a $n \times n$ positive symmetric definite matrix satisfying the following Lyapunov equation.

$$\mathbf{A}^T\mathbf{P} + \mathbf{PA} = -\mathbf{Q}, \quad \mathbf{Q} > 0 \tag{17}$$

Differentiating (16) with respect to time, the adaptation law is chosen as

$$\dot{k}_o^{TS} = \gamma\mathbf{e}^T\mathbf{p}\eta(\mathbf{x}) \tag{18}$$

where $\mathbf{p}$ is the last column of $\mathbf{P}$. Then $\dot{V} = -\frac{1}{2}\mathbf{e}^T\mathbf{Qe} - \mathbf{e}^T\mathbf{Pb}\varepsilon$.

According to the conclusions of [12-13] the fuzzy system (5) is a universal approximator; that is, it can approximate any function to arbitrary accuracy if we provide enough rules. In other words, ε is small enough so that $\dot{V} < 0$. However, for practical implementation we can not give too many rules to describe the control action because (a) a heavily computational load is required; and (b) the system may be tend to be unstable if we can not provide the needed control signal in real time operation. Therefore, a necessary robust control can be incorporated to overcome this disadvantage. The control (9) is used to achieve this goal.

4 Robust Controller Derivation

A Lyapunov function candidate is given as

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{Pe} \tag{19}$$

So we have

$$\begin{aligned} \dot{V} &= -\frac{1}{2}\mathbf{e}^T\mathbf{Qe} + \mathbf{e}^T\mathbf{Pb}[u^* - u_r - u_f] \\ &\le \left|\mathbf{e}^T\mathbf{Pb}\right|(\left|u^*\right| + \left|u_f\right|) - \mathbf{e}^T\mathbf{Pb}u_r \end{aligned} \tag{20}$$

Thus, u_r is given as

$$u_r = sgn(\mathbf{e}^T\mathbf{Pb})[g_{min}^{-1}(f_{\max} + \left|\mathbf{c}^T\mathbf{e}\right| + \left|r^{(n)}\right|) + \left|u_f\right|] = K \cdot sgn(\mathbf{e}^T\mathbf{Pb}) \tag{21}$$

where $K = g_{min}^{-1}(f_{\max} + \left|\mathbf{c}^T\mathbf{e}\right| + \left|r^{(n)}\right|) + \left|u_f\right|$. Here, we assume that the bounds of system can be estimated, that is, $\left|f\right| \le f_{\max}$ and $0 < g_{\min} < g$.

Finally, the overall control (9) is

$$u = u_f + K \cdot sgn(\mathbf{e}^T\mathbf{Pb}) \tag{22}$$

The sign function is replaced by a saturation function to avoid a heavily chattering at $\mathbf{e}^T\mathbf{pb} = 0$, as

$$u = u_f + K \cdot sat(\frac{\mathbf{e}^T \mathbf{Pb}}{s}) \qquad \textbf{(23)}$$

Finally, to improve the performance with faster convergence at large $\mathbf{e}^T\mathbf{Pb}$ and smoother (reduced chattering) at small $\mathbf{e}^T\mathbf{Pb}$, the fuzzy heuristic rule base, which is called as a fuzzy slider and shown in Table 1, is developed. Note that here the fuzzy sets in IF-part: PB, PM, PS, ZR, NS, NM and NB are denoted as Positive Big, Positive Medium, Positive Small, Zero, Negative Small, Negative Medium, and Negative Big, respectively. For convenience, we define

$$\sigma = \mathbf{e}^T \mathbf{Pb} \qquad \textbf{(24)}$$

Table 1. Fuzzy slider

σ	PB	PM	PS	ZR	NS	NM	NB
s	Small	Medium	Large	Very-Large	Large	Medium	Small

We derive the rules experimentally based on a typical state trajectory from the scheme of sliding mode control. Without loss of generality, the phase plane concerning two state variables is illustrated in Fig. 2, which shows a typical example of the state trajectory. At the beginning, around *a*, a small *s* is needed in order to achieve a small hitting time. Thus the rule around *a* is

$$IF \quad \sigma \text{ } is \text{ PB } THEN \text{ } s \text{ } is \text{ Small} \qquad \textbf{(25)}$$

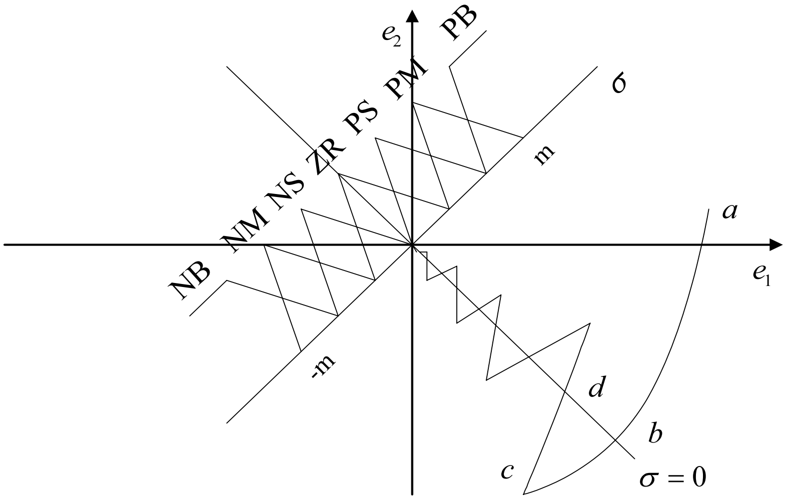


Fig. 2. Typical state trajectory of sliding mode control

Around *b* in Fig. 2, we expect a small robust control signal in order to avoid a large overshoot. So we need a large *s*, thus

$$IF \quad \sigma \text{ } is \text{ ZR } THEN \text{ } s \text{ } is \text{ Very-Large} \qquad \textbf{(26)}$$

In addition, the needed *s* around *c* and *d* are similar to those point *a* and *b*, respectively. We, therefore, can determine the remaining rules, which are shown in Table 1.

5 Case Study

In this section, a well-known nonlinear inverted pendulum system is popular employed to illustrate the effectiveness of the proposed fuzzy controller. The control task is to generate an appropriate actuator force, *u,* to control the motion of the cart such that the pole regulating in its upright position. The dynamic model of this pendulum system is characterized by

$$\dot{x}_1 = x_2 \tag{27}$$

$$\dot{x}_2 = f + gu \tag{28}$$

$$f = \frac{g_r \sin x_1 - \dfrac{mLx_2^2 \sin x_1 \cos x_1}{m+M}}{L\left(\dfrac{4}{3} - \dfrac{mcos^2 x_1}{m+M}\right)} \tag{29}$$

$$g = \frac{\dfrac{\cos x_1}{m+M}}{L\left(\dfrac{4}{3} - \dfrac{mcos^2 x_1}{m+M}\right)} \tag{30}$$

where the states x_1 and x_2 are the angle of pole and its angular velocity, respectively; g_r (acceleration due to the gravity) is 9.8 meter/sec^2; L (half-length of the pole) is 0.5 meters; M (mass of the cart) is 1.0 kg; and m (mass of the pole) is 0.1 kg.

The rule base of the TSK fuzzy controller is constructed as follows:

$$IF\ x_1\ is\ A_{(1,j_1)}\ and\ x_2\ is\ A_{(2,j_2)}\ \ THEN\ \ u\ is\ u_{h(j_1,j_2)} \tag{31}$$

$$j_1, j_2 \in \{-3,-2,-1,0,1,2,3\} \tag{32}$$

The membership functions of IF-part are shown in Fig. 3. For the TSK fuzzy controller, the parameters are given as: $\mathbf{c} = (1,2)^T$, $k_1 = 1$, $k_2 = 2$, $k_o^{TS}(0) = 0$, and $\gamma = 100$. If we specify $\mathbf{Q} = diag(10,10)$, that yields

$$\mathbf{P} = \begin{pmatrix} 15 & 5 \\ 5 & 5 \end{pmatrix} \tag{33}$$

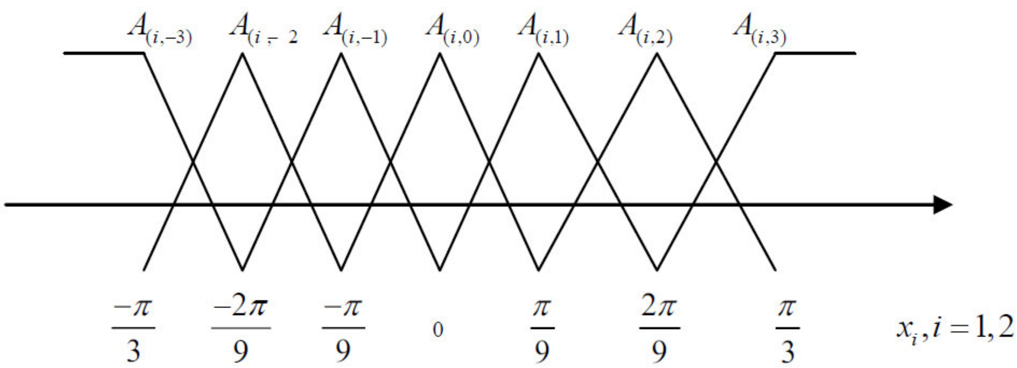


Fig. 3. Membership functions of the IF-part

We estimate the bounds of $f_{max} = 16 + 0.037x_2^2$ and $g_{min} = 0.638$ under the requirement of $|x_1| \le \pi/4$. The membership functions of fuzzy slider are respectively given in Fig. 4 and Fig. 5. Three initial conditions, $\mathbf{x} = (\pi/4, 0)^T$, $\mathbf{x} = (\pi/6, 0)^T$ and $\mathbf{x} = (\pi/18, 0)^T$, are used to verify the system's performance. We simulate two cases. (i) Using the TSK fuzzy controller to regulate the inverted pendulum system, the output trajectories are shown in Fig. 6. (ii) Appending the robust controller and adopting the fuzzy slider to the fuzzy controller, the output responses are also shown in Fig. 6. From the simulation results, we can find that the proposed control scheme for the scaling gain adaptation with TSK fuzzy controller, which does not add the robust controller, can balance the inverted pendulum with several different initial conditions. In contrast a faster convergence to the desired upright position can be obtained when the robust control is appended.

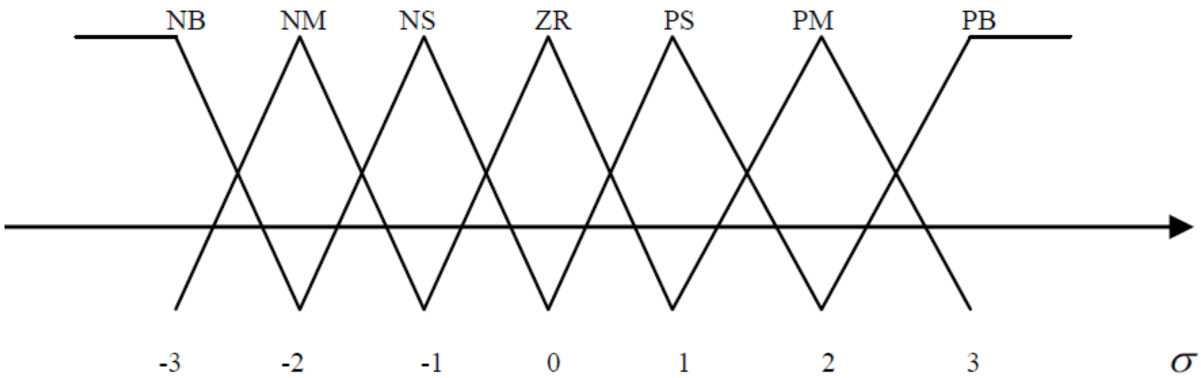


Fig. 4. Membership functions of the IF-part

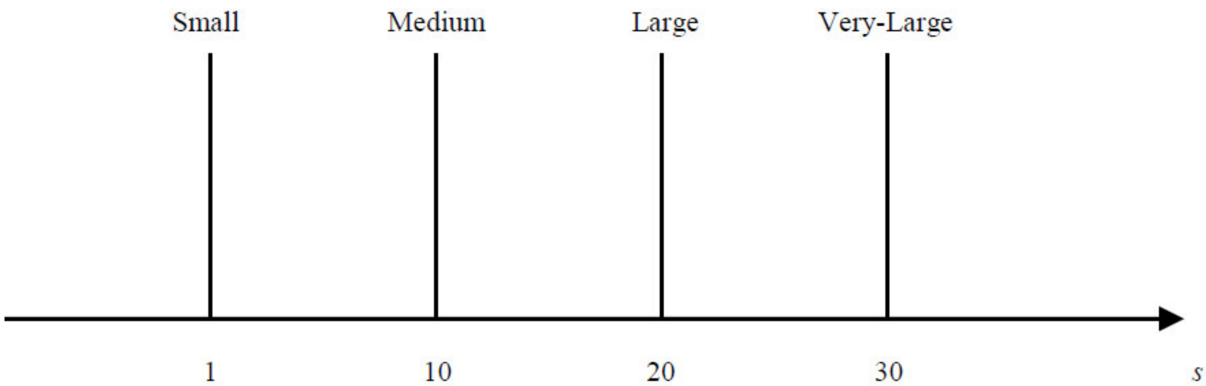
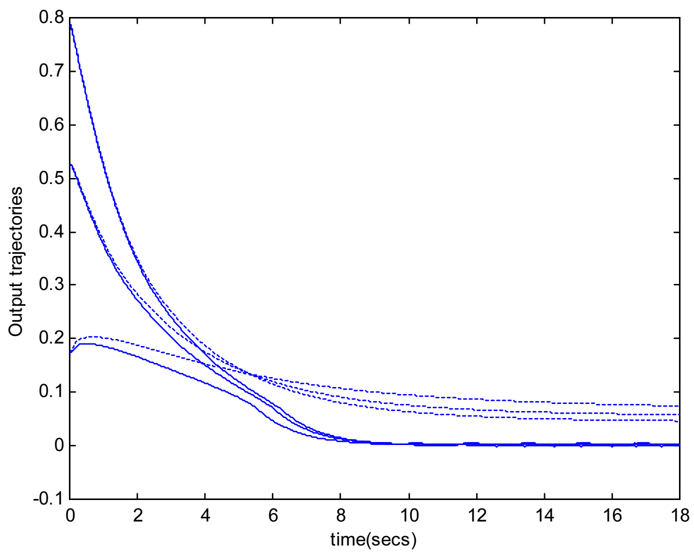


Fig. 5. Membership functions of the THEN-part



--: without adding robust control
- : appending robust control and fuzzy slider

Fig. 6. Output trajectories using the modified TSK fuzzy controller

6 Conclusions

Scaling gain adaptation for the designing of a TSK fuzzy controller is proposed in this paper, which is unlike the existed conventional approaches based on the heuristic tuning rule. An adaptation mechanism for TSK controller is given to approximate a well-known optimal control. The primary advantage is that only one adapted parameter is used. Thus, we can design the original fuzzy controller using intuitive control common sense, and then adapt the output scaling gain without any knowledge of the controlled plant. For the system stability, since TSK is a universal approximator, if we provide enough rules to describe the control action results in $\varepsilon \rightarrow 0$, then the designed control system will be stable. Unfortunately, in physical implementation, using too many rules causes an excessive computation load; or even worse system may be trapped or unstable. Therefore, the simplest robust control is appended to guarantee the stability of the fuzzy control system. In addition, a fuzzy slider is employed to variegate the slope of the saturation function to improve the system performance. Computer simulation results for the inverted pendulum system show the effectiveness of the proposed control scheme of the TSK fuzzy controllers.

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