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Channel Estimation Based on Complete Complementary Sequence for MIMO System

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Abstract. The novelty of complete complementary sequence (CC-S) is to improve the MIMO system performance utilizing complete orthogonality. What we proposed is the optimized design of implicit training sequence based on CC-S for MIMO wireless channel estimation. Channel estimation performance of MIMO system can be improved by resorting to the superior correlation of CC-S, including autocorrelation and cross-correlation. The method obtains the optimised implicit training sequences through an optimum training scheme with CC-S and the normalized mean square error (NMSE) of estimated channel matrix was analyzed. Simulation results demonstrate the performance improvements achieved by the proposed implicit training sequence algorithm. Furthermore, the feasibility of channel estimation based on CC-S for MIMO system is proved.

Keywords: complete complementary sequence (CC-S), MIMO system, NMSE, orthogonality

1 Introduction

The accuracy of channel estimation plays an important part in the bit error rate of MIMO system. Channel estimation based on training sequence for MIMO communication system has been widely investigated in fact system [1-2]. While the training sequence occupy some bandwidth and decrease the transmission rate. In order to trace the variable of the channel, the training sequence must be transmitted periodically. A new channel estimation algorithm based on implicit training sequence are invented in [3] is attractive. The training sequence is transmitted simultaneously along with the information sequence in this method. The advantage of the method is that the CC-S based training sequence no longer occupies the special time slot, so it has no loss of transmission rate, and can effectively improve the bandwidth utilization.

The precise channel state information (CSI) in the next generation wireless communication system represented by large-scale multiple-input multiple-output (MIMO) is crucial [4-5]. On account of this, a kind of optimal training sequence with low bandwidth occupancy is expected to design for channel estimation of MIMO system. Yuan and Fan proposes an implicit MIMO channel estimation by using the advantage of training sequence with zero correlation zone [6]. As for blind channel estimation of MIMO system, the scheme combining orthogonally space-time block coded is deduced [7]. In this paper, we applying complete complementary sequence to estimate CIR for MIMO system. The property of complete orthogonality of CCS-Ps are proved [8], as a result, it’s suitable that regard CC-S as optimal channel estimation training sequence. A kind of channel estimation algorithm is proposed under frequency-selective fading environments for Two-Way MIMO relay system [9]. A low complexity optimal estimator of MIMO ISI channels is provided [10]. Wang, Gao, Jiang, You and Hong succeed to study the normalized mean square error (NMSE) performance of CC-S-based MIMO channel estimation [11].

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In the scheme of MIMO channel estimation, the optimal training sequence should possess the superior correlation property including auto-correlation function (ACF) and cross-correlation function (CCF) [12]. However, the single training sequences with perfect correlation performance do not exist. While the CC-S is satisfied the perfect correlation performance for MIMO channel estimation. So far, many optimal training sequence sets have been constructed [13-14]. A framework of constructions of minimal storage regenerating codes with optimal access property was introduced [13]. A generic construction of generalized Chirp-like sequence sets with optimal zero correlation zone (ZCZ) property was described [14]. The construction [14] seems interesting because of two aspects: (1) The generic construction of Chirp-like ZCZ sequence sets with reduced alphabet size; (2) The construction of multiple quaternary ZCZ sequence sets with good properties. Enlightened by Ku and Huang [15], the optimization of implicit training sequences based on CC-S about NMSE analysis is researched in this paper.

This remainder of the paper is organized as follows. The basic concept of the CC-S and the expansion algorithm of CC-S as well as the channel estimation scheme are presented in Section II. The channel estimation model that is combined with CC-S is described in Section III. We will also derive the MMSE performance based MIMO communication system. The correlation function of CC-S and the MMSE performance are simulated in Section IV, followed by the conclusions of this paper.

2 Channel Estimation Architecture Based on CC-S

2.1 The Concept of CC-S

Assuming that $\left\{s_m^a, s_m^b\right\}$ are composed by M pairs of complementary sequence, the length of s_m^a and s_m^b is L_c. We can refer to $\left\{s_m^a, s_m^b\right\}$ as CC-S if the following conditions (1) and (2) are satisfied [15]:

For every $1 \le m, n \le M, m \ne n$, it holds that

$$R_{s_m^a s_m^a}(\tau) + R_{s_m^b s_m^b}(\tau) = \begin{cases} 2L_c & \text{if } \tau = 0 \\ 0 & \text{if } \tau \ne 0 \end{cases} \tag{1}$$

and

$$R_{s_m^a s_n^a}(\tau) + R_{s_m^b s_n^b}(\tau) = 0 \quad \text{if any } \tau \tag{2}$$

$R_{s_m^a s_n^a}(\tau)$ and $R_{s_m^b s_n^b}(\tau)$ denote the aperiodic correlation function between s_m^a and s_n^a, s_m^b and s_n^b respectively. Equations (1) and (2) denote the ACF and CCF of the CC-S respectively. τ denotes the time shift and D denotes the delay time.

2.2 Expansion Algorithm of CC-S

By using the zero property of CC-S, the performance of MIMO system can be improved. A simple and easy to implement method to generate sets of CC-S is essential either for calculation or hardware implementation.

In this paper, we utilizing a kind of expansion algorithm named 'generating tree', which can be achieved by matrix recursion. Under this pattern, the amount and length of CC-S are controllable by using different fundamental code block.

The pseudo-code of 'generating tree' algorithm can be expressed as followed in Fig. 1.

given fundamental code block X_i
for every iteration $i \in N^+$ **do**
    measure the size of matrix X_i and get l ,w
    define zero matrix, which size is $2l \times 2w$
    for every circulation $p = 1 : l/2$ **do**
        extract matrix X_p from X_i for each pair of lines
        select matrix $X_{p1} = X_p(1:1,1:w/2)$ $X_{p2} = X_p(1:1,w/2+1:w)$
        $X_{p3} = X_p(2:2,1:w/2)$ $X_{p4} = X_p(2:2,w/2+1:w)$
        respectively
        construct a new matrix
        $$Xlp = \begin{bmatrix} X_{p1} & X_{p3} & X_{p2} & X_{p4} \\ X_{p1} & \overline{X_{p3}} & X_{p2} & \overline{X_{p4}} \\ X_{p3} & X_{p1} & X_{p4} & X_{p2} \\ X_{p3} & \overline{X_{p1}} & X_{p4} & \overline{X_{p2}} \end{bmatrix}$$
        if the element x is real then
            $\overline{X_{p1}}$ represent the logic negation of X_{p1}
        else if the element x is complex number
            $\overline{X_{p1}}$ represent the conjugate operation of X_{p1}
        end if
        extract matrix $Z((4p-3):4p,:) = Xlp$
    end for
    update the set of CC-S code block X_i by using Z
end for

Fig. 1. Generating tree algorithm

2.3 Channel Estimation Scheme

The MIMO channel estimation model based on CC-S training sequence is described in Fig. 2. S/P denotes serial to parallel conversion and P/S denotes parallel to serial conversion. We assume MIMO system with M transmit and N receiver antennas. The transmitted signal from each antenna consists of data signal $\mathbf{c}(k)$ and pilot signal $\mathbf{s}(k)$. The signal $\mathbf{s}(k)$ describes the CC-S.

$$\mathbf{c}(k) = \left[c_1(k), c_2(k), \cdots, c_M(k)\right]^T \tag{3}$$

$$\mathbf{s}(k) = \begin{bmatrix} s_m^a(k) \\ s_m^b(k) \end{bmatrix} = \begin{bmatrix} s_1^a(k) & s_2^a(k) & \cdots & s_M^a(k) \\ s_1^b(k) & s_2^b(k) & \cdots & s_M^b(k) \end{bmatrix}^T \tag{4}$$
$$\underleftrightarrow{\text{pair}} \quad \underleftrightarrow{\text{pair}} \quad \cdots \quad \underleftrightarrow{\text{pair}}$$

At the output of the transmitter end, the *mth* symbols are represented as:

$$x_m(k) = c_m(k) + s_m^a(k) + c_m(k+D) + s_m^b(k+D) \tag{5.a}$$

At the receiver end, the symbols are decribed as:

$$\mathbf{x}(k) = \mathbf{c}(k) + \mathbf{s}(k) \tag{5.b}$$

m denotes the *mth* transmit antenna. Where $c(k)$ and $s(k)$ are the complex vectors of the information data sequence and the CC-S based training sequence respectively.

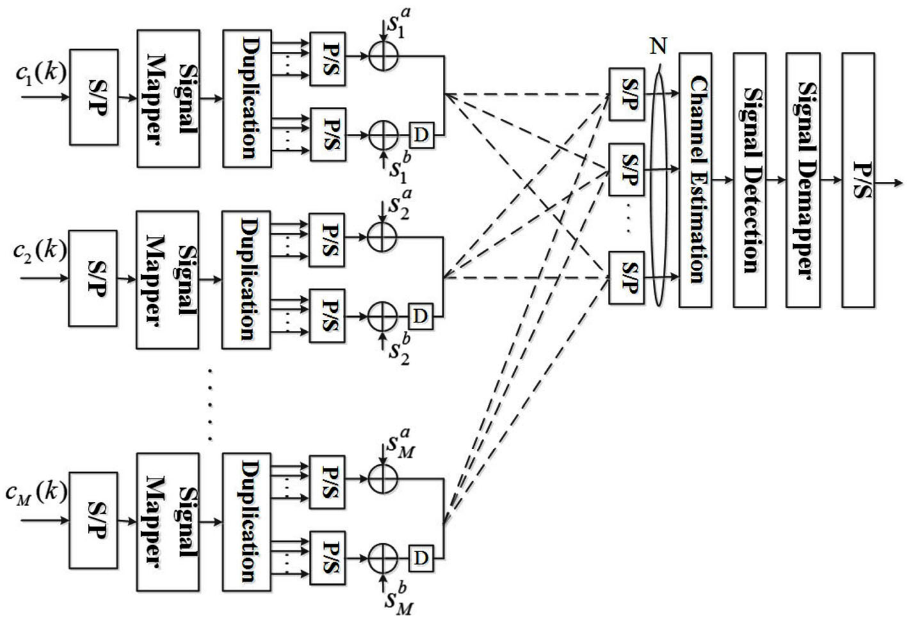


Fig. 2. The channel estimation scheme based on CC-S

The received signal $y(k) = \{y_n(k)\}_{n=1}^{N}$:

$$\begin{aligned}\mathbf{y}(k) &= \mathbf{H}(k)(\mathbf{c}(k) + \mathbf{s}(k)) + \mathbf{v}(k) \\ &= \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k)\end{aligned} \quad \textbf{(6)}$$

where **H** is the $N\times M$ complex channel matrix:

$$\mathbf{H}(k) = \begin{bmatrix} h_{11}(k) & h_{12}(k) & \cdots & h_{1M}(k) \\ h_{21}(k) & h_{22}(k) & \cdots & h_{2M}(k) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1}(k) & h_{N2}(k) & \cdots & h_{NM}(k) \end{bmatrix} \quad \textbf{(7)}$$

and

$$\mathbf{v}(k) = [v_1(k), v_2(k), \cdots, v_N(k)]^T \quad \textbf{(8)}$$

is the noise. We assume that the noise is white noise with variance of σ_v^2, and $(\cdot)^T$ is the operator for transpose.

3 Channel Estimation Model

The frame structure for MIMO system constructed by CC-S is shown in Fig. 3. The sequence A and sequence B are transmitted by turn. Gap denotes the Guard interval. The difference with complementary pair used in MIMO system is that, when the transmitter antenna number is odd, [12] will add virtual antenna to make it even. CC-S based training sequences can be exploited in MIMO system is free for transmitter antenna number.

Assuming the training sequence output is:

$$\mathbf{b}(k) = \hat{\mathbf{H}}(k)\mathbf{s}(k) \quad \textbf{(9)}$$

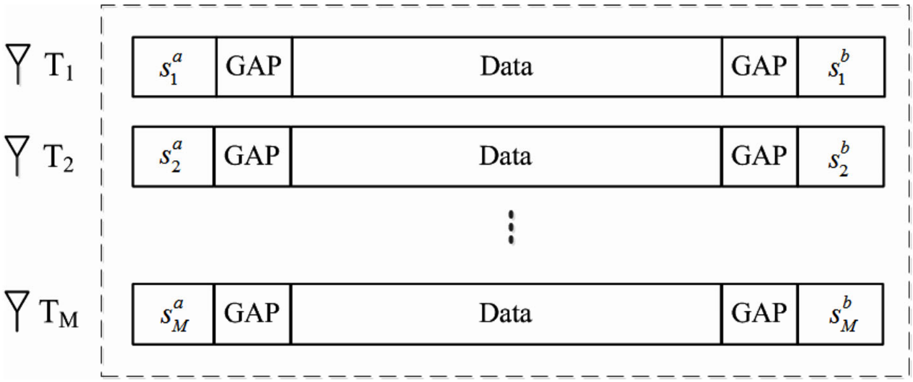


Fig. 3. Frame structure constructed by CC-S for MIMO system

where $\hat{\mathbf{H}}(k)$ denotes the estimation of $\mathbf{H}(k)$. The error state:

$$\begin{aligned}\mathbf{e}(k) &= \mathbf{y}(k) - \mathbf{b}(k) \\ &= \mathbf{H}(k)\mathbf{c}(k) + \mathbf{H}(k)\mathbf{s}(k) + \mathbf{v}(k) - \hat{\mathbf{H}}(k)\mathbf{s}(k)\end{aligned} \tag{10}$$

The cost function:

$$\zeta = E\left[\left|\mathbf{e}(k)\right|^2\right] = E\left[\left|\mathbf{y}(k) - \hat{\mathbf{H}}(k)\mathbf{s}(k)\right|^2\right] \tag{11}$$

ζ is minimized when the following condition is established:

$$E\left\{\left[\mathbf{y}(k) - \hat{\mathbf{H}}(k)\mathbf{s}(k)\right]\mathbf{s}^*(k)\right\} = 0 \tag{12}$$

where $(\cdot)^*$ denotes the conjugate operation. According to (12):

$$E\left[\hat{\mathbf{H}}(k)\mathbf{s}(k)\mathbf{s}^*(k)\right] = E\left[\mathbf{y}(k)\mathbf{s}^*(k)\right] \tag{13}$$

$$\hat{\mathbf{H}}(k)R_{ss}(k) = R_{ys}(k) \tag{14}$$

$$\hat{\mathbf{H}}(k) = R_{ys}(k)R_{ss}^{-1}(k) \tag{15}$$

$R_{ss}(k)$ and $R_{ys}(k)$ are the correlation function matrix of $s(k)$ and $s(k)$, $y(k)$ and $s(k)$ respectively.

4 Optimisation Training Sequence

From (6), we can conclude:

$$R_{ys}(k) = \mathbf{H}(k)R_{xs}(k) + R_{vs}(k) \tag{16}$$

Due to the uncorrelated variables $x(k)$ and $s(k)$, so (16) can be rewritten as:

$$R_{ys}(k) = \mathbf{H}(k)R_{ss}(k) + R_{vs}(k) \tag{17}$$

$$\mathbf{H}(k) = R_{ys}(k)R_{ss}^{-1}(k) - R_{vs}(k)R_{ss}^{-1}(k) \tag{18}$$

$R_{vs}(k)$ is the correlation function matrix between $v(k)$ and $s(k)$. The channel estimation error of MIMO system is:

$$\mathbf{e}_H = \hat{\mathbf{H}}(k) - \mathbf{H}(k) = R_{vs}(k)R_{ss}^{-1}(k) \tag{19}$$

As a result, the MSE of MIMO channel estimation is given by:

$$\begin{aligned} MSE &= trace\left[E\left(\mathbf{e}_H^H \mathbf{e}_H\right)\right] \\ &= trace\left[E\left(R_{ss}^{-H}(k) R_{vs}^H(k) R_{vs}(k) R_{ss}^{-1}(k)\right)\right] \end{aligned} \quad \textbf{(20)}$$

where $(\cdot)^H$ and $E(\cdot)$ are operators for Hermitian transpose and mathematic expectation.

$$\left[E\left(R_{vs}^H(k) R_{vs}(k)\right)\right] = N\sigma_v^2 R_{ss}(k) \quad \textbf{(21)}$$

According to (20) and (21), *MSE* can be formulated as:

$$MSE = N\sigma_v^2 trace\left[R_{ss}^{-H}(k)\right] \quad \textbf{(22)}$$

According to the optimisation theory, MSE is minimized by appropriately selecting training sequence $s(k)$. When the training sequence $s(k)$ is not satisfied the perfect orthogonality, the $R_{ss}(k)$ is not a diagonal matrix, the MSE attains its minimum by transferring $s(k)$ to $\varphi(k)s(k)$, which $\varphi(k)$ makes:

$$\begin{aligned} R_{(\varphi s)(\varphi s)}(k) &= \left(\varphi(k)s(k)\right)^H \left(\varphi(k)s(k)\right) = s^H(k)\left(\varphi^H(k)\varphi(k)\right)s(k) \\ &= \mathbf{\Lambda} \end{aligned} \quad \textbf{(23)}$$

$\mathbf{\Lambda}$ represents the diagonal matrix. The (23) opens an optimised direction for selecting the training sequence in MIMO system.

We measure the performance of the MIMO channel estimation algorithm using the normalized mean square error (NMSE) defined as:

$$NMSE = E\left(\frac{\left\|\hat{\mathbf{H}}(k) - \mathbf{H}(k)\right\|^2}{\left\|\mathbf{H}(k)\right\|^2}\right) \quad \textbf{(24)}$$

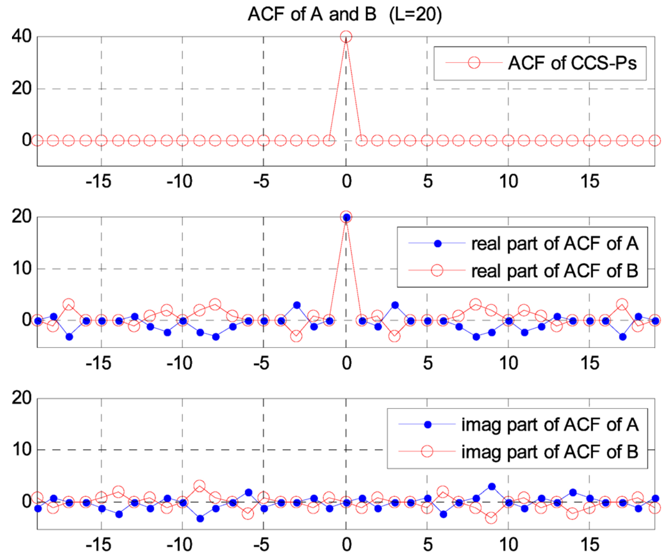
5 Simulation Results

In our simulation example, we have considered different scenarios with different CC-S and different numbers of transmit and receive antennas. Furthermore, different length of CC-S is chosen to demonstrate the system performance. The simulation parameters are exhibited in Table 1.

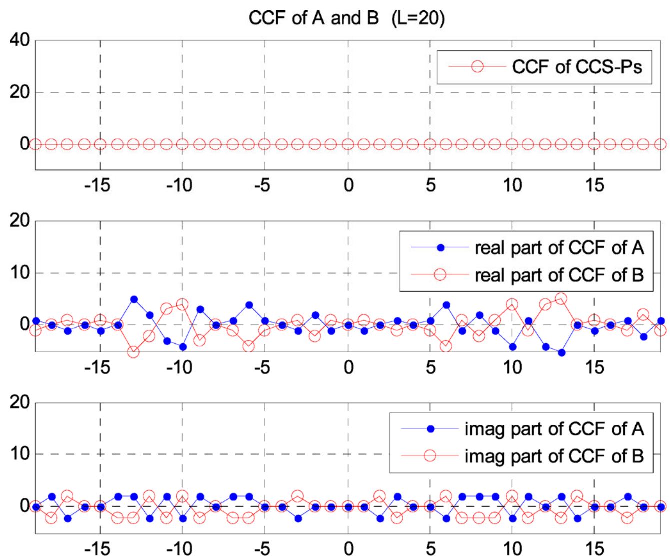
Table 1. Simulation parameters

Parameters	Value
Length of CC-S	20
	40
Length of m sequence	31
Carrier frequency	2.5GHz
Numbers of transmit antennas	4,2,1
Numbers of receive antennas	4,2,1
Baud rate of transmitted signal	1.28MBaud/s
Modulation mode	QPSK

Fig. 4 shows the correlation function of CC-S with the length of 20. Fig. 4a describes the ACF performance and Fig. 4b demonstrates the CCF performance of CC-S. In order to analyze orthogonality of CC-S, The real part of correlation function with sequence A and B, the imaginary part of correlation function are also displayed in Fig. 4. From Fig. 4, we can see that the side-lobe between correlation of sequence **A** and correlation of sequence **B** is complementary, so the complete orthogonality of the CC-S is satisfied.



a) Auto-Correlation Function Performance



b) Cross-Correlation Function Performance

Fig. 4. Correlation function of CCS-Ps between **A** and **B**

Simulation has been carried out for the new implicit estimation scheme using CC-S and m sequence for the SISO system. In the proposed implicit scheme, the CC-S is constructed in [4] and equipped with L=40. For each simulated SNR, the NMSE is obtained by averaging over 3000 independent simulation runs.

The NMSE performance is described in Fig. 5. What we can see From Fig. 5 is that the NMSE performance with CC-S is better than either m sequence or optimsed m sequence under the circumstance of single antenna. While the same channel estimation performance could be obtained for using CC-S training sequence and optimised m sequence which is obtained by the method proposed by this paper.

In the Fig. 6, MIMO systems with $M=4, N=4$ and $M=2, N=2$ are considered for comparison purpose, as well as the SISO system. Since the number constraint of pairs of m sequence cannot be satisfied in MIMO system, therefore, only the proposed scheme using the CC-S based training sequence is simulated. From Fig. 6, it is clear that the simulated curves with different transmit and receive antenna number. These curves describe the CC-S training sequence-based channel estimation algorithm works very well.

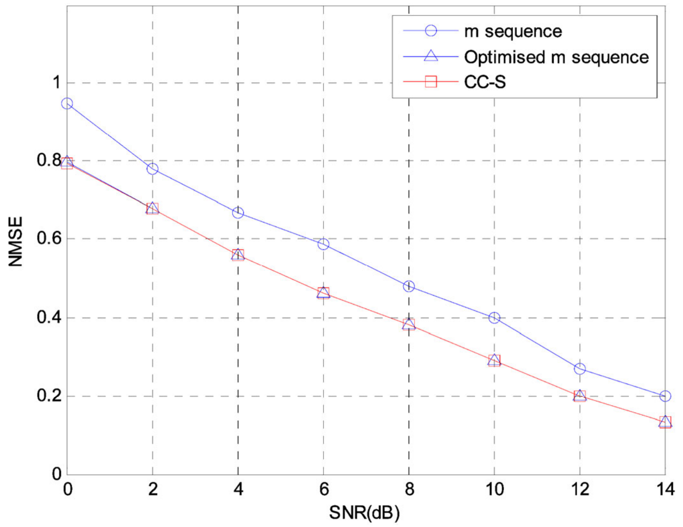


Fig. 5. NMSE performance versus different training sequence

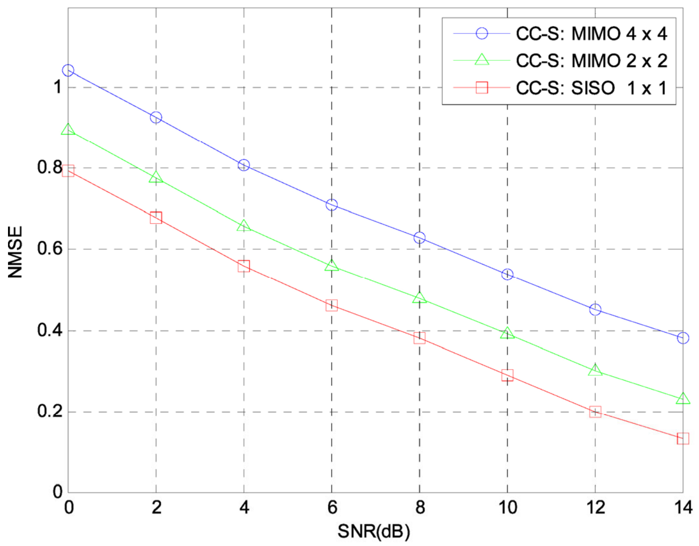


Fig. 6. NMSE performance versus different number of antennas

6 Conclusion

In this paper, we have established a channel estimation scheme based on CC-S and designed an optimisation method for training sequence in MIMO system. Through simulation, we have compared the algorithm performance of CC-S training sequence based channel estimation with that an algorithm based on m sequence. The proposed algorithm scheme is shown. The simulation results also demonstrate the efficiency and validity of CC-S applied in MIMO system. This Letter tends to express the application based on CC-S training sequence in MIMO channel estimation. Further researched work may be considered is algorithm complexity and how to combine the compressive sensing and CC-S to reduce the computation quantity and so on.

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