Analysis of MIMO SAR Imaging Algorithm Based on Complete Complementary Sequence and Phase Compensation

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Abstract. The CC-S (Complete Complementary Sequence) is composed of several complementary orthogonal sequences, which is satisfied with the requirement of the orthogonal Multiple-Input Multiple-Output (MIMO) radar signals. Aimed at the difficulty of high sidelobe in Synthetic Aperture Radar (SAR) imaging processing, an approach of depressing sidelobe output based on CC-S in MIMO SAR system was proposed. The model for orthogonal MIMO radar transmitting signals and the corresponding match filter were established in this paper. Equivalent phase compensation theory is used to carry out compensation for signal phase shift. The MIMO SAR imaging algorithm with CC-S was deduced. The simulation results demonstrate that the imaging performance of CC-S employed in orthogonal MIMO radar system is much better than that with traditional Linear Frequency Modulation (Chirp) signal and Barker signal, by which the feasibility and validity of CC-S applied in orthogonal MIMO SAR system are justified.

Keywords: complete complementary sequence, MIMO, SAR

1 Introduction

The Multiple-Input Multiple-Output (MIMO) radar improves the performance by multiple transmitting antennas transmitting signals to the target, multiple receiving antennas receiving signals from the target and carrying out the data process of signals received. Based on the current research on the MIMO radar, transmitting antennas of the orthogonal MIMO radar transmit the orthogonal waveform sets; the receiving ends recover transmitted signal components by matched filtering, in which the design of transmitted signals directly influences the orthogonal MIMO radar system performance. Time synchronization of the transceiver end is also an emphasis [1]. In order to suppress interference and improve multiple targets resolution, a good correlation function between transmitted signals is needed, which meets complete orthogonality; that is, the side-lobe of aperiodic auto-correlation function is zero and the main-lobe and side-lobe of aperiodic cross-correlation function is both zero. Currently the orthogonal MIMO radar mainly uses the poly-phase coding [2] and orthogonal frequency coding [3-4], which still cannot meet complete orthogonality between transmitted signals despite that the correlation functions of these two coding possess the rather low side-lobe performance. Theoretical researches showed that the sequence which meets complete orthogonality does not exist in the traditional single-code domain. Therefore the appearance of Complete Complementary Sequence (CCS) opened up a new direction for choosing the orthogonality MIMO radar signal.

Synthetic Aperture Radar (SAR) plays an important role in radar imagining due to all-weather, all-time and long-distance working [5-6]. Recently, with the MIMO radar development, applying it to target imaging is becoming an emphasis of MIMO radar [7-8]. Duan, Wang and Ma [7] dealt with movement compensation in ISAR imaging by deterministic relation between multiple MIMO radar matrix elements. Wang, Su and Zhu [8] mainly analyzed the relationship between MIMO radar transceiver array

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configuration, imaging performance and spatial spectral domain filling. These references both focused on
the air target imaging performance of ground MIMO radar. This paper mainly discussed the ground
target imaging of airborne MIMO SAR. Since the MIMO SAR system requires that transmitted signals
satisfy orthogonality, exploring orthogonal signals that apply well for MIMO SAR becomes a key
concentration in practical application.

This paper primarily analyzed a CCS-based MIMO SAR imaging method, utilizing the property of
CCS correlation function that it has no side-lobes, which meant the high peak side-lobe rate could be
easily achieved after range compression without suppressing the side-lobe. For the traditional SAR
system based on Chirp signal, the peak side-lobe rate is normally raised by window function, yet
resulting in main-lobe expansion and resolution reduction. The CCS-based MIMO SAR, however, due to
the deprivation of window function, promises the high peak side-lobe rate and good resolution
simultaneously, improving the SAR system performance.

2 MIMO SAR Imaging Analysis

2.1 CCS Signal Model

The CCS transmitting sequence: complementary pairs are transmitted in two pulse repetition periods,
which is shown in Fig. 1.

From Fig. 1, since the minimum time of receiving echo signal is $2\Delta W/C+\tau_p$, where $\Delta W$ is the
ground length of radar irradiation and $C$ is the wave speed, if the length of protection band is $3\tau_p$, the
maximum value of pulse repetition frequency is:

$$PRF_{\text{max}} = \left(\frac{2\Delta W}{C} + 5\tau_p\right)^{-1}$$  \hspace{1cm} (1)

Where $A_m$ and $B_m$ $(0 \leq m \leq M-1)$ are a pair of complementary sequences and $T$ represents the delay
time. The $M$ complementary sequences $\{A_m, B_m\}$ with the length of $L$ is defined as below:

$$\begin{cases}
A_m = (a_m^0, a_m^1, \cdots, a_m^{L-1}) \\
B_m = (b_m^0, b_m^1, \cdots, b_m^{L-1})
\end{cases} \hspace{1cm} (2)$$

Where $\{(A_0, B_0), (A_1, B_1), \cdots, (A_{M-1}, B_{M-1})\}$ meets the complete orthogonality as below:
$$R_{a_m a_n}(l) + R_{b_m b_n}(l) = \begin{cases} 2L & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \quad (3)$$

$$R_{a_m a_n}(l) + R_{b_m b_n}(l) = \begin{cases} 2L & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \quad (4)$$

When $0 \leq m \neq n < M$

$$R_{a_m a_n}(l) + R_{b_m b_n}(l) = 0 \quad \forall \tau \quad (5)$$

\{A_m, B_m\} and \{A_n, B_n\} make a pair of CCS, where $R_{a_m a_n}(\tau)$ is the auto-correlation function of sequence $A_m$, $R_{a_m a_n}(\tau)$ is the cross-correlation function between $A_m$ and $A_n$. Eq(3) and Eq(4) represent the auto-correlation function of CCS, Eq(5) represents the cross-correlation function of CCS and $l$ represents the discreet time offset. Therefore $A_m$ and $B_m$ make CCS sets.

From Fig. 1, signal $A_m, B_m$ are transmitted alternatively in the m antenna, which means the transmitted signal $s_m(\tau)$ of the m transmitting antenna is shown as below:

$$s_m(\tau) = \sum_{l=0}^{L-1} \left[ a_m \cdot \text{rect} \left( \frac{\tau - l \cdot T_p}{\tau_p} \right) + b_m \cdot \text{rect} \left( \frac{\tau - T - l \cdot T}{\tau_p} \right) \right] \cdot e^{j2\pi f_c t}$$

$$= [s_{a_m}(\tau) + s_{b_m}(\tau - T)] \cdot e^{j2\pi f_c t} \quad (6)$$

Where T is the pulse repetition period, $T_p$ represents the sub-pulse width, $\tau_p = L \cdot T_p$ is the transmitted pulse width, $f_c$ is the carrier wave frequency and rect$(t)$ is the rectangular window function, whose value is 1 when $0 \leq t \leq \tau_p$ and otherwise 0.

By utilizing CCS, the complete orthogonality is satisfied, making it possible for range-dimensional and azimuth-dimensional imaging to work independently. The imaging process refers to Chen and Zhang [9].

### 2.2 Equivalent Phase Center Compensation

In order to deal with the problem that the azimuth resolution and swath width cannot be raised simultaneously in MIMO SAR imaging mode, Currie [10] proposed a method that one transmitter is used in the transmitting end, and multiple receivers in the receiving end to realize Multiple Azimuth Beam (MAB). Its principle was to decrease time dimension samples by increasing azimuth spatial dimension samples, promising the azimuth resolution and improving the swath width simultaneously. Suess, Grafmueller and Zahn [11] came up with an azimuth single-transmitting multiple-receiving SAR mode, which meant multiple sub-apertures were used to transmit in the azimuth position and only single sub-aperture was used to transmit and multiple sub-apertures to receive echo signals. Subsequently MIMO SAR appeared [12], which resulted in spatial sampling rate increase, swath width expansion and azimuth resolution improvement. Unfortunately, the azimuth non-uniform sampling in this method caused Doppler ambiguity. In order to solve this problem, some relatively mature ambiguity resolution methods have been proposed and the following section analyzed uniform sampling.

MIMO SAR possesses the same principle with the regular single-transmitting multiple-receiving system. Along the azimuth the radar antenna consists of multiple sub-antennas; every sub-antenna has the same size and beam width and the radiation targets are from the same areas. The principle is shown in Fig. 2.
Where $M$ is the total number of antennas, $d$ is the distance between near antennas, $R(t)$ is the slant range of antenna 1 at the time $t$, $R(t + (m - 1)d/v_a)$ and $R(t + (n - 1)d/v_a)$ are the slant ranges of antenna $m$ and $n$ at the time $t$ respectively, $R(t + (m + n - 2)d/2v_a)$ is the equivalent slant range of antenna $m$ and $n$, $\theta$ is the angle between reference slant range $R(t)$ and the antenna normal direction, and $v_a$ is the speed.

Every transmitting antenna transmits different signals. After signals are reflected by the target, every receiver could receive echo signals transmitted by all transmitters, split different echo signals and arrange them along the azimuth direction, largely improving the spatial sampling rate. Besides, since the equivalent phase center is applied when time sampling is replaced by spatial sampling, the phase center error is introduced. Therefore the equivalent phase center error is in need of compensation, where the system could be equivalent to single-transmitting single-receiving mode, ranging data in a slow-time sequence according to array configuration [13] In this case, the MIMO SAR system of $M$ sub-apertures will produce $M^2$ sampling points in a Pulse Repetition Period (PRT), where the sampling point is considered as one when the azimuth positions coincide. The system can generate different efficient sampling points according to the different antenna configuration.

Fig. 3 shows the array configuration when the transmitting matrix element distance is different from the receiving matrix element distance. When the array configuration is $d_t = N d_r$ or $d_r = M d_t$, where $d_t$ is the transmitting matrix element distance and $d_r$ is the receiving matrix element distance, the configuration can make the equivalent virtual matrix element number reach $MN$ during one transceiver process of MIMO SAR.

![Fig. 3. Transceiver split configuration](Image)
The antenna array configuration is shown in Fig. 4. The transmitting and receiving antennas are in the same place when the aperture distance is even \(2M-1\) efficient sampling points are generated, increasing the sampling points to which \(2M-1\) times the single-transmitting single-receiving system and promising the azimuth resolution and broadening swath simultaneously. Expect for the configuration that the transmitting and receiving antennas are in the same place, different antenna configurations can result in different sampling points.

Fig. 4 demonstrates the equivalent phase center principle taking 4 transceiver antennas as the example.

If the bidirectional transceiver phase center is defined as the middle point of a line from the transmitting phase center to receiving phase center, as Fig. 4 shows, all 4 sub-antennas transmit and receive signals and they make 7 spatial samplings to the target during every PRT. In every PRT, the flying distance satisfies the following equation:

\[
v_a \cdot PRT = (2M - 1)d / 2
\]  

That is:

\[
PRF = \frac{v_a}{(2M - 1)d / 2}
\]

Although the said equivalent phase center approach keeps its position unchanged, there exists a certain phase difference between two echo signals because the equivalent phase center is from different transmitting and receiving antennas. Currie and Brown [14] showed a deduction method using the single-transmitting multiple-receiving phase difference.

As Fig. 3 shows, taking the first transmitting antenna as the example, the slant range from its phase center to the ground target is \(R(t)\), represented as the Taylor polynomial series:

\[
R(t) \approx R_0 - v_a \sin \theta (t - t_0) + \frac{v_a^2 \cos \theta}{2R_0} (t - t_0)^2
\]  

Where \(R_0\) is the slant range of where reference time \(t_0\) is, \(R_0 = h/\cos \phi\), \(h\) is the flying height, \(\phi\) is the elevation angle, \(T_s\) is time for synthesizing the aperture.

Taking the slant range of the first antenna as the reference slant range and deducting the phase course of echo signals, the self-transmitting self-receiving phase course is \(\phi = 2\pi \cdot 2R(t) / \lambda\), \(\lambda\) is the carrier
wave length. At time \( t \), the distance from the \( m \) antenna to the target \( R_m(t) \) is:

\[
R_m(t) = R(t + \frac{(m-1)d}{v_a})
\]  

(10)

Meanwhile, the distance \( R_n(t) \) from the target to the \( n \) antenna is:

\[
R_n(t) = R(t + \frac{(n-1)d}{v_a})
\]

(11)

Therefore the slant range of the \( m \) antenna both transmitting and receiving is:

\[
\begin{align*}
R_{mn} &= R_m(t) + R_n(t) \\
&= R(t + \frac{(m-1)d}{v_a}) + R(t + \frac{(n-1)d}{v_a}) \\
&= 2R(t) + \alpha \cdot \frac{(m+n-2)d}{v_a} + 2 \beta \cdot \frac{(m+n-2)d}{v_a} (t - t_0) \\
&+ \beta \left[ \frac{(m-1)^2 d^2}{v_a^2} + \frac{(n-1)^2 d^2}{v_a^2} \right]
\end{align*}
\]

(12)

And the equivalent slant range between the \( m \) antenna and the \( n \) antenna is:

\[
R_{mn}^\text{eq} = 2R(t + \frac{(m+n-2)d}{2v_a})
\]

(13)

Combining Eq(12) and Eq(13), we can see that they only have a slight difference of the last item. As a result, the equivalent phase center difference is:

\[
\Delta \omega_{mn} = \frac{2\pi(R_{mn}^\text{eq} - R_{mm})}{\lambda} = \frac{2\pi}{\lambda} \frac{(m-n)^2 d^2}{2v_a^2}
\]

(14)

From the deduction above, we can know that the phase difference between two arbitrary antennas \( m \) and \( n \) is \( \Delta \omega_{mn} \) and \( \Delta \omega_{mn} = \Delta \omega_{nm} \). Under the condition of side-looking, \( \theta \) is the half beam-width angle.

Applying the equivalent phase principle, from Eq(14), we have \( \phi_{mn} = \exp(j\Delta \omega_{mn}) \) and \( \phi_{mn} = \phi_{nm} \). For the MIMO SAR system of \( M \) antennas, define \( y_{n,m} \) as the equivalent receiving signal and then the signal after phase compensation is \( E_{mn} = y_{n,m} \cdot \phi_{mn} \). According to the arrangement from Reference[11], arrange \( E_{mn} \) in the following way, where \( m,n \in \{1,\ldots,M\} \), and then the echo signal in a PRT can be described as below:

\[
P = \begin{bmatrix}
E_{1,1} & E_{1,2} & \cdots & E_{1,M} & 0 & \cdots & 0 \\
0 & E_{2,1} & \cdots & E_{2,M-1} & 0 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{M,1} & \cdots & E_{M,M-1} & E_{M,M}
\end{bmatrix}_{(M^2)}
\]

(15)

From the analysis above, first split the echo signals, then arrange them and take the average of data integration after phase error compensation and superposition, which can be equivalent to the echo signal phase course of a single-antenna system. The imaging flow is shown in Fig. 5.
3 Simulation Experiment

In order to testify the MIMO SAR imaging technology above, we carried out the following simulation experiment. Taking the double-transmitting double-receiving antenna system as the example, each transmitting matrix element of the radar system transmits the complementary sequence, in which the construction method refers to [15] and the code length is 40. The matrix element configuration is shown as Fig. 2, where the distance d between every antenna is 2m and the elevation angle is 24º. Due to the convenience of analysis, we took the side-looking as the simulation condition. Set the targets 9 ideal point scatters, the scattering amplitude of every scattering point is the same, and other simulation parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>simulation parameter</th>
<th>value</th>
</tr>
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<tbody>
<tr>
<td>wave length</td>
<td>0.018</td>
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<tr>
<td>sub-pulse length (μs)</td>
<td>0.025</td>
</tr>
<tr>
<td>signal band (MHz)</td>
<td>40</td>
</tr>
<tr>
<td>signal sampling rate (MHz)</td>
<td>50</td>
</tr>
<tr>
<td>Sub-array distance (m)</td>
<td>2</td>
</tr>
<tr>
<td>pulse repetition frequency (Hz)</td>
<td>220</td>
</tr>
<tr>
<td>flying height (km)</td>
<td>5</td>
</tr>
<tr>
<td>speed (m/s)</td>
<td>100</td>
</tr>
</tbody>
</table>

The CCS-based point target imaging result is shown in Fig. 6 (a). From Fig. 6, CCS can be applied to MIMO SAR system and generate the correct image of point target.
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Fig. 6. Ideal point target simulation result based on CCS

Pick 3 random point targets from the imaging result and carry out the quality evaluation. The result is shown in Table 2.

Table 2. Point target quality evaluation result based on CCS

<table>
<thead>
<tr>
<th>Target</th>
<th>Azimuth Resolution/m</th>
<th>Range Resolution/m</th>
<th>PSLR/dB Azimuth</th>
<th>ISLR/dB Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.888</td>
<td>3.33</td>
<td>-13.39</td>
<td>-34.337</td>
</tr>
<tr>
<td>2</td>
<td>0.897</td>
<td>3.34</td>
<td>-13.39</td>
<td>-34.337</td>
</tr>
<tr>
<td>3</td>
<td>0.890</td>
<td>3.33</td>
<td>-13.38</td>
<td>-34.337</td>
</tr>
</tbody>
</table>

Note. PSLR—peak side-lobe rate; ISLR—integral side-lobe rate.

Fig. 6 analyzes the CCS-based MIMO SAR imaging result and testifies the feasibility of CCS imaging. In order to compare with the traditional Chirp signal imaging result, Fig. 7 demonstrates the imaging result where, in the traditional single-transmitting single-receiving mode, the Chirp signal, under the same simulation condition, is used as the transmitting signal.

Fig. 7 (a) shows the point target imaging result based on Chirp signal, and Fig. 7 (b) and Fig. 7 (c) give the azimuth and range profile based on Chirp signal respectively.

Fig. 7. Ideal point target simulation result based on Chirp signal

Similarly, pick 3 random point targets from the imaging result and carry out the quality evaluation. The result is shown in Table 3.
Table 3. Point target quality evaluation result based on Chirp signal

<table>
<thead>
<tr>
<th>Target</th>
<th>Azimuth Resolution/m</th>
<th>Azimuth PSLR/dB</th>
<th>ISLR/dB</th>
<th>Azimuth Azimuth Azimuth Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.888</td>
<td>-13.366</td>
<td>10.217</td>
<td>10.411</td>
</tr>
<tr>
<td>2</td>
<td>0.897</td>
<td>-13.365</td>
<td>10.217</td>
<td>10.411</td>
</tr>
<tr>
<td>3</td>
<td>0.890</td>
<td>-13.366</td>
<td>10.217</td>
<td>10.411</td>
</tr>
</tbody>
</table>

Fig. 8 shows the Barker mode-based imaging result in the traditional single-transmitting single-receiving mode and the Barker mode takes the length of 13. Fig. 8 (a) represents the Barker mode-based point target imaging result, and Fig. 8 (b) and Fig. 8 (c) give the azimuth and range profile based on Barker mode.

![Fig. 8. Ideal point target simulation result based on Barker sequence](image)

Similarly, pick 3 random point targets from the imaging result based on Barker sequence and carry out the quality evaluation. The result is shown in Table 4.

Table 4. Point target quality evaluation result based on Barker sequence

| Target | Azimuth Resolution/m | Range Resolution/m | PSLR/dB Azimuth Azimuth Range Azimuth ISLR/dB Range |
|--------|----------------------|-------------------|--------|-----------------|--------|
| 1      | 0.888                | 3.328             | 13.288 | 21.809          | 10.143 | 14.234 |
| 2      | 0.890                | 3.333             | 13.289 | 21.817          | 10.146 | 14.233 |
| 3      | 0.890                | 3.33              | 13.290 | 21.766          | 10.142 | 14.233 |

Combine the index analysis from Fig. 6 to Fig. 8 and from Table2 to Table 4. From the range dimension, the point target satisfies the ideal impulse response function; from the range and azimuth profile, the compression property of complementary sequence (-34.3dB) is better than the value of Chirp signal (-13.2dB) and Barker code (-21.8dB), without the main-lobe expansion and increasing the peak side-lobe rate and promising the range resolution simultaneously; different channels are free from co-channel interference and the anti-interference ability is improved due to the orthogonality of CCS. By applying CCS to the orthogonal MIMO SAR system, side-lobe output can be well suppressed, the peak side-lobe rate is improved on the basis of unchanged resolution and the anti-interference ability is enhanced.

Table 5 shows the comparison of PSLR and ISLR between this paper and other references [16-17]. From Table 5, the imaging result based on CCS is better than the other two. Therefore we can conclude that CCS-based MIMO radar system can greatly improve the target performance.
Table 5. Comparison between this paper and other documents

<table>
<thead>
<tr>
<th></th>
<th>Reference[16]</th>
<th>Reference[17]</th>
<th>This paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSLR/dB</td>
<td>-18.2</td>
<td>-25.6</td>
<td>-34.3</td>
</tr>
<tr>
<td>ISLR/dB</td>
<td>-5.7</td>
<td>-16.2</td>
<td>-24.4</td>
</tr>
</tbody>
</table>

4 Conclusion

According to the CCS side-lobe cancellation characteristic, this paper primarily discussed the MIMO SAR imaging problem based on CCS and phase compensation. The simulation result demonstrated that CCS-based MIMO SAR has better point target index comparing to Chirp signal and Barker code.

Acknowledgements

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References


