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The Shortest Path Selection Based on Adaptive Amoeba Algorithm and D Numbers Theory

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Abstract. Many methods have been applied to solve the shortest path problem in the transportation system when the evaluation of each period of road is precise. However, in practical applications, the information regarding the road usually changes over time, which results in the variability of traveling time for the same road segment. One critical issue is the quantification of the uncertainty and incompleteness during the information changes. D numbers theory is useful tool to characterize the uncertain and incomplete information. In this paper, we incorporate D numbers theory with adaptive amoeba algorithm to address the route optimization problem in the presence of uncertainty and incompleteness. Two numerical examples of transportation networks are used to illustrate the efficiency of the proposed method.

Keywords: adaptive amoeba algorithm, D numbers theory, incompleteness, shortest path selection, uncertainty

1 Introduction

In the transportation system, one of the fundamental problems is to find out the shortest path from the origin to the destination among the transportation networks [1-2]. The transportation system is usually described by a network. In a network consists of n nodes which is represented as N= (1,2,3…n), each ordered pair *(i, j)* represents as the arc from node i to node j in the network, where i and j belong to the set N. The shortest path selection is to find out the minimum sum value of ordered pair *(i, j)* from the origin to the destination. Due to its practical applications, it has been widely applied in many transportation systems, such as wireless networks [3], complex networks [4-6] and so on [7-9].

In transportation system, some information such as transportation lengths of each road segment is represented by the real numbers only if the attribute is precise. Most of the road values are usually imprecise because the information regarding the road often changes over time such as transportation time, transportation costs, transportation capacities, transportation demands, and so on [10-12]. As a result, much uncertainty are always involved in the road evaluations. Meanwhile, the weights cannot be quantized easily. Hence, it is the key issue that how to deal with uncertain information for solving the shortest path. Many methods are used for the representation and dealing with the uncertainty, such as probability theory [13], fuzzy sets theory [14-15], rough theory [16], Dempster-Shafer theory of evidence [17-18]. Dempster-Shafer theory of evidence is also called Dempster-Shafer theory or evidence theory, it is proposed by Dempster and Shafer [17-18]. The Dempster-Shafer theory has been used in many fields since it can combine multiple uncertain information. However, some strong hypotheses of the framework, such as the uncertainty must be independent and the sum of all the basic probability assessments (BPA) must equal 1, have limited its widely application in some fields especially in linguistic assessments [19-20]. Recently, D numbers theory is proposed [19], it can be regarded the extension of Dempster-Shafer

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theory of evidence. D numbers theory has not only the ability to deal with the uncertain information but also incomplete information. Meanwhile, D numbers theory can separate credibility to each element whether mutual exclusion or nonexclusive. The D numbers' combination rule can be used to combine multiple D numbers and the D numbers' integration has the ability to translate the information indicated by D numbers into real numbers. Because of the ability in representing uncertainty and incompleteness, D numbers theory has been used in environmental impact assessments [20-21], supplier selection [22], curtain grouting efficiency assessment [23], and so on [24-26].

Many approaches have been used to solve the shortest path selection, such as Dijkstra algorithm [27], A star algorithm [28], Bellman-Ford algorithm [29] and Floyd-Warshall algorithm [30]. Researchers have explored bio-inspired algorithm because of their flexibility and simplicity [31-33]. Genetic algorithm and ant colony algorithm are two common bio-inspired algorithms that both can deal with the shortest path selection problems [34-35]. In recent years, an amoebic organism of Physarum polycephalum has shown its ability in finding the shortest path in many theoretical problems [36-43]. In 2000, Nakagaki *et al.* found that placing the Physarum polycephalum in the maze with food resources placed in the entrance and exit separately, Physarum polycephalum can find out the shortest path from the entrance to the exit hours later [44]. As the ability in finding out the shortest path, the scientists established a mathematical model based on its working mechanism which can be named as adaptive amoeba algorithm [45]. The numerical examples in American high road navigation and route selection in complex systems have shown its effectiveness, Bonifaci has also proved the convergence properties of the mathematical model in solving the shortest path selection after that [46-48]. The adaptive amoeba algorithm can find out the entire shortest path when two or more shortest paths connecting the two nodes. Meanwhile, adaptive amoeba algorithm has the ability in shortest path selection in dynamic transportation networks. Researchers have applied adaptive amoeba algorithm in optimizing in graph theory area because of its efficiency [49-52]. Because of much uncertainty and incompleteness in the dynamic transportation networks, in this paper, the adaptive amoeba algorithm and D numbers theory are incorporated in the shortest path selection in uncertain environment. D numbers theory is efficiency in dealing with the uncertain and especially incomplete information while the adaptive amoeba algorithm is used to find out the shortest path.

The rest of the paper is organized as follows. Some basic theories of D numbers theory and the adaptive amoeba algorithm are presented in section 2. In section 3, the proposed method for shortest path selection is detail described. Illustrative numerical examples are presented in section 4 and some conclusions are given in section 5.

2 Basic Theories

In this section, some basic theories including D numbers theory and the adaptive amoeba algorithm are introduced.

2.1 D numbers Theory

D numbers theory was proposed by Deng in 2012 [19-20]. It is often regarded as the extension of Dempster-Shafer theory of evidence. D numbers theory has overcome two strong hypotheses in Dempster-Shafer theory of evidence. Firstly, the basic concept frame of discernment in evidence theory requires that all the elements in evidence theory must be mutual exclusive and collectively exhaustive. In many applications, this hypothesis is difficult especially in the linguistic assessments, a lot of evaluation grades such as "Very good" "Good" "Average" "Poor" and "Very poor" are often used in the assessments. However, the exclusiveness hypothesis cannot be guaranteed precisely, therefore, the Dempster-Shafer theory of evidence is unreasonable in representing such a kind of assessments. Secondly, in Dempster-Shafer theory of evidence, the sum BPAs of all focal elements must be equal to 1, which means that a normal BPA must be with completeness constraint. However, in a lot of applications, the decision makers cannot provide a complete BPA because of the lack of professional knowledge or education background. Meanwhile, the experts might be hesitated when giving their evaluations. For example, ten experts are invited for the assessments, eight experts assess the object to be of good quality while the left two experts are hesitating, they do not agree or disagree the evaluation grades and neither do they give their own evaluations, so all the experts give a incomplete assessments. The incompleteness of frame of discernment may result in incomplete representation in an open world. To overcome the shortcomings in the previous evidence theory, a new methodology named as D numbers theory is proposed, the details are shown as follow.

Definition 1. For a finite nonempty set Ω, D numbers is a mapping from Ω to [0, 1],

$$D:\Omega \to [0,1],$$

with

$$\begin{cases} \sum_{B\subseteq\Omega} D(B) \le 1, \\ D(\phi) = 0 \end{cases}$$

where ϕ is the empty set and the elements in set Ω do not need to be mutual exclusive. Meanwhile, the sum of all the D numbers do not require to be equal to 1, which means that the incomplete information is reasonable in D numbers theory.

Definition 2 D numbers' simply representation. For a discrete set $\Omega = (b_1, b_2, b_3 \cdots b_n)$, where $b_i \in N^+$ and $b_i \ne b_j$ if $i \ne j$, for any $v_i \ge 0$ and $\sum_{i=1}^{n} v_i \le 1$, a special form of D numbers can be expressed as follows, $D(\{b_1\}) = v_1$;

$$D(\{b_2\}) = v_2;$$
$$D(\{b_3\}) = v_3;$$
$$....$$
$$D(\{b_n\}) = v_n;$$

or be represented simply as follows,

$$D = \{(b_1, v_1)(b_2, v_2)(b_3, v_3)...(b_n, v_n)\},$$

where b_i is the element in the set Ω, or most time it represents the assessments grades, v_i shows the confident or creditable degree to element b_i.

Definition 3 D numbers combination rule. Let D_1, D_2 be two D numbers with the simply representation,

$$\begin{aligned} D_1 &= \{(b_1^1, v_1^1)(b_2^1, v_2^1)(b_3^1, v_3^1)...(b_n^1, v_n^1)\} \\ D_2 &= \{(b_1^2, v_1^2)(b_2^2, v_2^2)(b_3^2, v_3^2)...(b_n^2, v_n^2)\} \end{aligned}.$$

The combine process of the two D numbers denoted as $D = D_1 \oplus D_2$ is defined as follows,

$$D(\{b\}) = v \cdot \tag{1}$$

where

$$b = \frac{b_i^1 + b_j^2}{2}; \quad v = \frac{v_i^1 + v_j^2}{2C},$$

$$C = \begin{cases} \sum_{j=1}^{m}\sum_{i=1}^{n} \frac{v_i^1 + v_j^2}{2} & \sum_{i=1}^{n} v_i^1 = 1; \sum_{j=1}^{m} v_j^2 = 1 \\ \sum_{j=1}^{m}\sum_{i=1}^{n} \frac{v_i^1 + v_j^2}{2} + \sum_{j=1}^{m} \frac{v_c^1 + v_j^2}{2} & \sum_{i=1}^{n} v_i^1 < 1; \sum_{j=1}^{m} v_j^2 = 1 \\ \sum_{j=1}^{m}\sum_{i=1}^{n} \frac{v_i^1 + v_j^2}{2} + \sum_{i=1}^{n} \frac{v_i^1 + v_c^2}{2} & \sum_{i=1}^{n} v_i^1 = 1; \sum_{j=1}^{m} v_j^2 < 1 \\ \sum_{j=1}^{m}\sum_{i=1}^{n} \frac{v_i^1 + v_j^2}{2} + \sum_{j=1}^{m} \frac{v_c^1 + v_j^2}{2} + \sum_{i=1}^{n} \frac{v_i^1 + v_c^2}{2} & \sum_{i=1}^{n} v_i^1 < 1; \sum_{j=1}^{m} v_j^2 < 1 \end{cases},$$

where $v_c^1 = 1 - \sum_{i=1}^{n} v_i^1$, $v_c^2 = 1 - \sum_{j=1}^{m} v_j^2$, the superscript in above equation is the order of the D numbers.

Definition 4 D numbers' integration representation. For any simply representation formation of the D numbers, D numbers' integration representation is defined as follows,

$$I(D) = \sum_{i=1}^{n} b_i v_i \ . \tag{2}$$

Definition 5 modified D numbers combination rule. Note that the commutation property is not well satisfied in previous D numbers combination rule. Deng *et al.* made some attempts for multiple D numbers combination [20]. $D_1, D_2, D_3 ... D_n$ are n D numbers, u_j is the order variable of each D numbers which can be denoted as tuple (u_j, D_{u_j}) , then multiple D numbers combination is mapping f_D , which the following conditions are met,

$$f_D(D_1, D_2, D_3 ... D_n) = [\cdots[D_{\lambda_1} \oplus D_{\lambda_2}] \oplus \cdots D_{\lambda_n}]'$$

where D_{λ_i} is the D_{u_j} of the tuple (u_j, D_{u_j}) which has the ith lowest u_j .

When the D numbers' order variables are of the same value, 'Best-Worst combination' strategy is adopted. All possible combination sequences have been calculated, the highest and lowest D numbers' integration value of all the combination results is the best and worst combination separately.

2.2 The Adaptive Amoeba Algorithm and Its Mathematical Model

Many bio-inspired algorithms are obtained from the special organism, such as the birds or ants, so does adaptive amoeba algorithm, it is inspired by the Physarum polycephalum. Physarum polycephalum is a large, single-celled amoeboid organism. Its body consists of tubular network which is used for the transportation of the food resources and chemical signal. Meanwhile, its body is changeable, one single Physarum polycephalum can be divided into two independent alive ones and these two independent ones can also be mixed together. Nakagaki, Yamada and Tóth placed Physarum polycephalum in the maze with food resources at the entrance and the exit separately in 2000 [44]. Hours later, the Physarum polycephalum is overspread on the maze gradually. Then open-ended tubes that does not connect to the two food sources are gradually disappeared. Meanwhile, the longer tube is likely to disappear if two or more tubes connect the same two food sources while the shorter one become thicker and thicker as it absorbs much more energy [44]. With these feedback mechanisms, a mathematical model for maze solving problems has been constructed [45].

The mathematical model can be simplified as follows [45]. The whole maze can be described as a network, the entrance and the exit with food resources can be regarded as origin N_1 and destination N_2 , the other nodes are denoted as $N_i\ (i = 3,4,5...)$. The tubes can be viewed as edges where the edge connects node N_i and N_j is denoted as $e_{i.j}$. Q_{ij} is the variable which is used to express the energy through e_{ij} from node N_i to N_j as an approximately Poiseuille flow, the flux Q_{ij} can be denoted as follows,

$$Q_{ij} = \frac{\pi r_{ij}^4}{8k} \frac{p_i - p_j}{l_{ij}} , \tag{3}$$

where r_{ij} and l_{ij} are the radius and length of the tubes e_{ij} separately, k is the viscosity coefficient of the sol and p_i is the pressure at node N_i . Setting $D_{ij} = \pi r_{ij}^4 / 8k$, where can be called conductivity of edge e_{ij} , then the equation about flux Q_{ij} can be rewritten as follows,

$$Q_{ij} = \frac{D_{ij}(p_i - p_j)}{l_{ij}} , \tag{4}$$

Each node is assumed to be zero capacity. By considering the conservation law of sol, the following equation can be obtained,

$$\sum_i Q_{ij} = 0\,(j \neq 1,2)\cdot \tag{5}$$

For the entrance node N_1 the exit node N_2, the following two equations should be held as follows,

$$\sum_i Q_{i1} + I_0 = 0,\ \sum_i Q_{i1} - I_0 = 0, \tag{6}$$

where I_0 is the flux flowing from the entrance node (or into the exit node). Noted that I_0 is a constant, it means the total flux is fixed constant in the whole process.

Experimental observation shows that the tubes with lager resource flues are reinforced, it will become thicker and thicker while the smaller fluxes will disappear gradually. In order to describe such a mechanism, it is assumed that the conductivity Q_{ij} changes with time according to the resource flux flows through e_{ij}, the following equation for evolution of $Q_{ij}(t)$ is proposed as follows [44],

$$\frac{d}{dt} D_{ij} = f(|Q_{ij}|) - rD_{ij}, \tag{7}$$

where r is a decay rate of the tube. This equation implies that the tubes with small flux will disappear if there is no flux flows though the tube. f is monotonically increasing continuous function satisfying $f(0) = 0\cdot$

For example, a network consists of 6 nodes and 9 edges shown in Fig. 1 and the length of each edge are shown in Table 1 separately. Then the adaptive amoeba algorithm is used to find the shortest path from node *1* to node *6*.

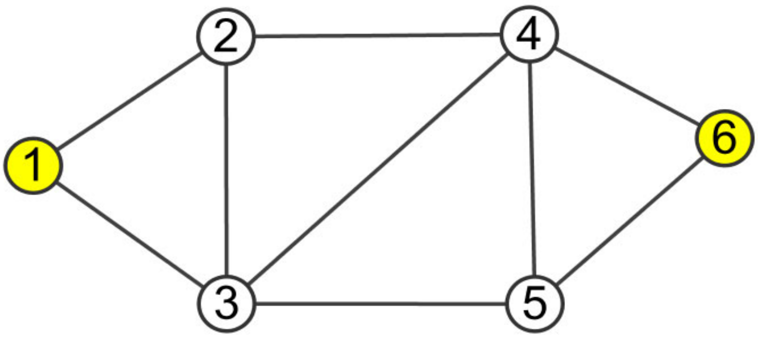

Fig. 1. A network consists of 6 nodes and 9 edges

Table 1. The length of each node in Fig. 1

Number	length	Number	length	Number	length	Number	length	Number	length
E1-2	4	E2-3	3	E3-4	2	E4-5	2	E5-6	3
E1-3	2	E2-4	4	E3-5	3	E4-6	2		

Step 1. Set conductivity of each node to be 0.5, it shows the Physarum polycephalum is overspread on the maze firstly.

Step 2. Calculate the pressure of each node by Equations (3-5). I_0 is set to be 1.

Step 3. With the pressure of each node calculated, then the flux of each edge can be obtained by Equation (3).

Step 4. Calculate the flux after the first iteration by Equation (6), here $D_{ij}^{n+1} = \frac{|Q_{ij}^n| + D_{ij}^n}{2}$ is often used for simplification.

Step 5. If the condition is satisfied, then end the iteration, or goes to step 2 again.

The network in Fig 1 is conducted for fifty times, the conductivity variation is shown in Fig. 2.

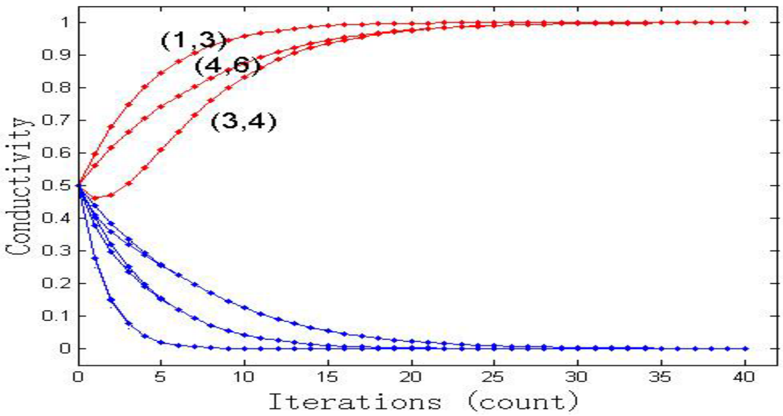

Fig. 2. The conductivity variation of each edge

As is shown in Fig. 2, the conductivity of three edges $e_{1,3}, e_{3,4}, e_{4,6}$ equal to 1 after about 25 times iteration, the conductivity of the other edges turns to 0 at end, meanwhile, the length of this path represented is 6.0 which is smaller than all the other paths, so the shortest path is 1→3→4→6.

3 Proposed Methods

In uncertain environment, two issues, namely the representation of the uncertain and incomplete weights of the roads and ranking of the weights should be solved. Based on the D numbers theory, the classical adaptive amoeba algorithm can be generalized to a fuzzy algorithm as follows.

In a transportation network, the lengths of the roads are assumed to be transportation cost or transportation time rather than the geographical distances, these kinds of weights of roads are assumed to be of uncertainty and incompleteness. For example, from the origin to the destination, half of the experts need five minutes while the left need six minutes because of some uncontrollable circumstance on the road. Then D numbers theory is capable in representing such a kind of uncertainty as *{(5, 0.5)*, (6, 0.5)}. Another ten experts may have different evaluation results, five experts need five minutes and three experts needs six minutes while the left two expert are hesitating, they do not give their opinions, neither do they agree five or six minutes are necessary or some other answers. So at this time, the information can be denoted by D numbers as *{(5, 0.5), (6, 0.3)},* the left 0.2 is the hesitating degree. Here the transportation time is the only transportation weight which is used to describe the road, then D numbers' integration representation can translate the information denoted by D numbers into numerical numbers which the adaptive amoeba algorithm can be used to find out the shortest path.

In most cases, two or more transportation weights are used to describe the road condition, such as transportation cost and transportation time are two main factors in the shortest path selection problem. Because different weights have different assessment standards, the first step is to transform all the different weights under the same level. All the uncertain and incomplete information denoted by D numbers, b shows the evaluation grade while v shows the confidence degree about b, finding out the maximum value of all the D numbers when evaluating the same transportation weight, all the other value of b divide the maximum can change all the information into the same level. Let D_1 and D_2 be two D numbers which D_1 is used to evaluate one transportation weight and D_2 is used to assess the other as follows,

$$\begin{aligned} D_1 &= \{(b_1^1, v_1^1)(b_2^1, v_2^1)(b_3^1, v_3^1)\ldots(b_n^1, v_n^1)\} \\ D_2 &= \{(b_1^2, v_1^2)(b_2^2, v_2^2)(b_3^2, v_3^2)\ldots(b_m^2, v_m^2)\} \end{aligned}.$$

Firstly, calculate the maximum value of all the b, where

$$b_{1\max} = \max\{b_1^1, b_2^1 \ldots b_n^1\}$$
$$b_{2\max} = \max\{b_1^2, b_2^2 \ldots b_m^2\}.$$

Then D_1 and D_2 can be translated as follows,

$$D_1^{'} = \{(\frac{b_1^1}{b_{1\max}}, v_1^1)(\frac{b_2^1}{b_{1\max}}, v_2^1)(\frac{b_3^1}{b_{1\max}}, v_3^1)\ldots(\frac{b_n^1}{b_{1\max}}, v_n^1)\}$$
$$D_2^{'} = \{(\frac{b_1^2}{b_{2\max}}, v_1^2)(\frac{b_2^2}{b_{2\max}}, v_2^2)(\frac{b_3^2}{b_{2\max}}, v_3^2)\ldots(\frac{b_m^2}{b_{2\max}}, v_m^2)\}.$$

Then D numbers combination rule is used for the combination of different weights. Here if more than three transportation weights are considered, the modified D numbers' combination rule is adopted [20]. Then D numbers' integration can translate the information denoted by D numbers into real numbers. When all evaluations denoted by D numbers in uncertain environment have been translated into the real numbers, the classical adaptive amoeba algorithm is used to find out the shortest path. The shortest path selection process is shown in Fig. 3 as follows.

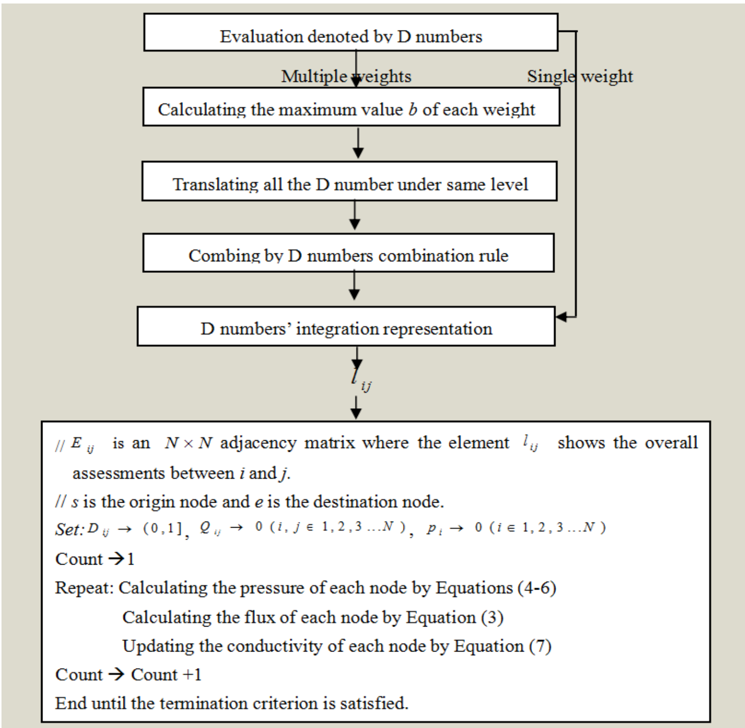

Fig. 3. The shortest path selection process

4 Numerical Examples

In this section, two numerical examples in uncertain environment are used to show the efficiency of the proposed method. Consider the transportation network with 23 nodes and 40 edges shown in Fig. 4 [2]. When considering the shortest path from node 1 to node 23, transportation time is the only transportation weight of each edge. All edge lengths denoted as D numbers with membership functions are shown in Table 2. For example, most people believe the transportation time from node 1 to node 3 is 10 minutes and the others consider 13 minutes should be enough, then the lengths from node 1 to node 3 can be represented by D numbers as *{(10,0.6)(13,0.4)}*. Another example is used to show the efficiency of D numbers theory. Ten experts are invited to evaluate the transportation time from node 19 to node 22, nine experts assess it to be 17 minutes while the left expert is hesitating because of lacking of information about the road. This kind of information is incomplete and can be represented by D numbers as *{(17, 0.9)}*.

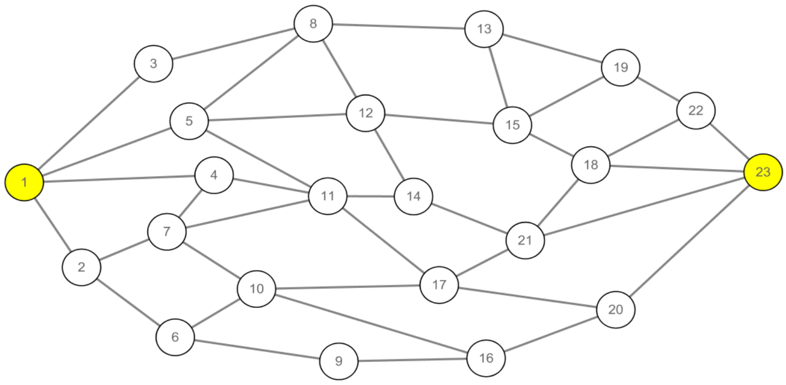

Fig. 4. A transportation network

Table 2. The assessments of all the edges in Fig. 3

Edge	Membership function	Edge	Membership function	Edge	Membership function
(1,2)	{(13,0.5)(15,0.5)}	(7,10)	{(9,0.4)(12,0.6)}	(15,18)	{(8,0.4)(11,0.6)}
(1,3)	{(10,0.6)(13,0.4)}	(7,11)	{(7,1)}	(15,19)	{(6,0.5)(10,0.5)}
(1,4)	{(10,0.5)(12,0.5)}	(8,12)	{(8,1)}	(16,20)	{(12,0.5)(14,0.5)}
(1,5)	{(8,1)}	(8,13)	{(5,0.2)(10,0.8)}	(17,20)	{(10,1)}
(2,6)	{(10,0.7)(15,0.3)}	(9,16)	{(7,0.5)(9,0.5)}	(17,21)	{(6,0.4)(8,0.6)}
(2,7)	{(11,1)}	(10,16)	{(12,0.4)(16,0.5)}	(18,21)	{(17,0.5)(18,0.5)}
(3,8)	{(10,0.2)(16,0.8)}	(10,17)	{(18,0.4)(20,0.6)}	(18,22)	{(5,0.4)(7,0.6)}
(4,7)	{(20,0.5)(22,0.5)}	(11,14)	{(10,1)}	(18,23)	{(7,0.6)(9,0.4)}
(4,11)	{(10,0.6)(16,0.4)}	(11,17)	{(9,0.6)(10,0.4)}	(19,22)	{(17,0.9)}
(5,8)	{(9,0.5)(11,0.5)}	(12,14)	{(13,0.4)(16,0.6)}	(20,23)	{(14,0.5)(16,0.5)}
(5,11)	{(10,0.4)(14,0.6)}	(12,15)	{(12,0.1)(15,0.9)}	(21,23)	{(16,1)}
(5,12)	{(13,0.5)(15,0.5)}	(13,15)	{(12,0.3)(15,0.7)}	(22,23)	{(4,0.3)(6,0.7)}
(6,9)	{(6,0.2)(10,0.8)}	(13,19)	{(17,0.5)(19,0.5)}		
(6,11)	{12,0.9}	(14,21)	{(11,0.5)(14,0.5)}		

Then D numbers integration is used to translate the fuzzy information into numerical numbers. For example, the information denoted as *{(10, 0.6) (13, 0.4)}* from node 1 to node 3 can be calculated according to Equation (2) as

$$I(D) = 10\times 0.6 + 13\times 0.4 = 11.2\ .$$

Then adaptive amoeba algorithm is used for selecting the shortest path, the conductivity variation of each edge is shown in Fig. 5.

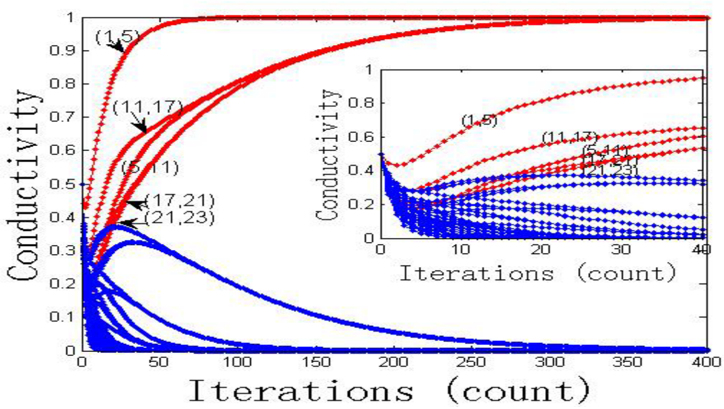

Fig. 5. Conductivity variation of each edge in Fig. 4

From Fig. 5, $e_{1,5}, e_{5,11}, e_{11,17}, e_{17,21}$ and $e_{21,23}$ turn to 1 for about 300 times iteration, it means that these roads are still connected. So the shortest path from node 1 to node 23 is 1→5→11→17→21→23. The total exact value of this path is 41.2 represented by D numbers integration representation, it is shorter than all the other paths.

Another network of 9 nodes and 12 edges are illustrated to show the effectiveness of the proposed methods under multiple different transportation weights where the network is shown in Fig. 6. In the first numerical example, transportation time is the only transportation weight when evaluating the roads. However, in the real application, transportation time and transportation cost or more than three transportation weights are often considered in the transportation network. Here when considering the shortest path from node 1 to node 9, the evaluations of transportation time and transportation cost denoted as D numbers are shown in Table 3 as follows.

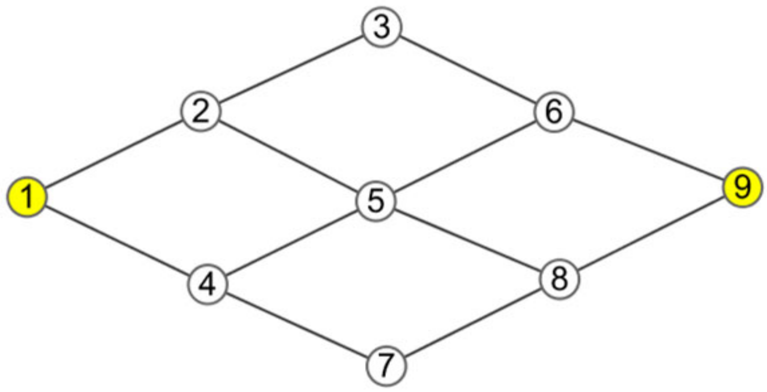

Fig. 6. A transportation network with 9 nodes and 12 edges

Table 3. The evaluations of transportation time and transportation cost in Fig. 6

Edge	Membership functions		Edge	Membership functions	
(1,2)	{(6,0.7)(9,0.3)}	{(15,0.6)(16,0.4)}	(4,7)	{(4,1)}	{(7,0.5)(8,0.5)}
(1,4)	{(5,0.4)(8,0.6)}	{(15,1)}	(5,6)	{(8,0.5)(10,0.5)}	{(15,0.4)(20,0.6)}
(2,3)	{(3,1)}	{(5,0.4)(7,0.6)}	(5,8)	{(4,0.9)}	{(6,0.5)(7,0.5)}
(2,5)	{(10,0.9)(15,0.1)}	{(18,0.4)(22,0.6)}	(6,9)	{(12,0.6)(15,0.4)}	{(20,0.6)(25,0.4)}
(3,6)	{(5,1)}	{(14,0.4)(16,0.6)}	(7,8)	{(8,0.5)(11,0.5)}	{(18,0.5)(21,0.5)}
(4,5)	{(5,0.2)(8,0.8)}	{(9,0.4)(12,0.6)}	(8,9)	{(12,0.4)(15,0.6)}	{(25,1)}

As transportation time and transportation cost are of different dimension relations. So transforming the different edges' weights into the same evaluation standard first and D numbers combination rule is used to combine the weights by the proposed methods. Here if more than three transportation weights are considered for the assessments of the roads, the modified D numbers' combination rule [20] is adopted. Then D numbers' integration representation translates the information denoted by D numbers into real numbers by Equation (2) and the final results are shown in Table 4.

Table 4. The combined results of the two weights

Edge	Combined results	Edge	Combined results
(1,2)	0.5490	(4,7)	0.4167
(1,4)	0.5198	(5,6)	0.6774
(2,3)	0.2213	(5,8)	0.1843
(2,5)	0.7874	(6,9)	0.8875
(3,6)	0.5920	(7,8)	0.7070
(4,5)	0.4445	(8,9)	0.9533

Then adaptive amoeba algorithm is used for selecting the shortest path, the conductivity variation of each edge is shown in Fig. 7. As is shown in Fig. 7, the continuity of $e_{1.4}, e_{4.5}, e_{5.8}$ and $e_{8.9}$ turns to 1 at last, it means the shortest path from node 1 to node 9 is 1→4→5→8→9 and its length is 2.098 when considering both transportation time and transportation cost, here the two different road weights have been translated into the same level and the D numbers' combination rule has been used to combine these two factors.

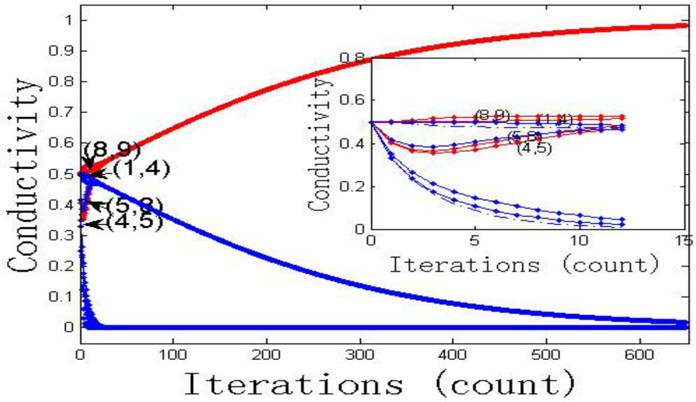

Fig. 7. Conductivity variation of each edge in Fig. 6

For further comparison, the single transportation weight for the shortest path selection in Fig. 6 is considered. D numbers' integration representation is used to translate the transportation time and transportation cost denoted by D numbers in Table 3 into numerical numbers which are shown in Table 5 separately.

Table 5. The evaluations of the edges

Edge	Time	Cost	Edge	Time	Cost
(1,2)	6.9	15.4	(4,7)	4.0	7.5
(1,4)	6.8	15.0	(5,6)	9.0	18.0
(2,3)	3.0	6.2	(5,8)	3.6	6.5
(2,5)	10.5	20.4	(6,9)	13.2	22.0
(3,6)	5.0	15.2	(7,8)	9.5	19.5
(4,5)	7.4	10.8	(8,9)	13.8	25.0

When considering the transportation time, the adaptive amoeba algorithm finds out the shortest path from node *1* to node *9* where the conductivity variation of edges are shown in Fig. 8. As can be seen, the variation of $e_{1,2}, e_{2,3}, e_{3,6}$ and $e_{6,9}$ turn to 1 while the other edges turn to 0 at last, it shows the shortest path is 1→2→3→6→9 when transportation time is the only evaluation weight considered. With the same method, when only consider the transportation cost, the edges' conductivity variation is shown in Fig. 9 where we can find out the shortest path is 1→4→5→8→9.

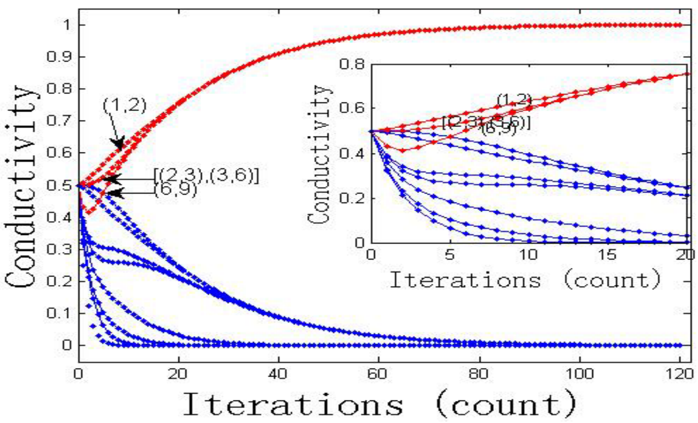

Fig. 8. Conductivity variation considering transportation time only

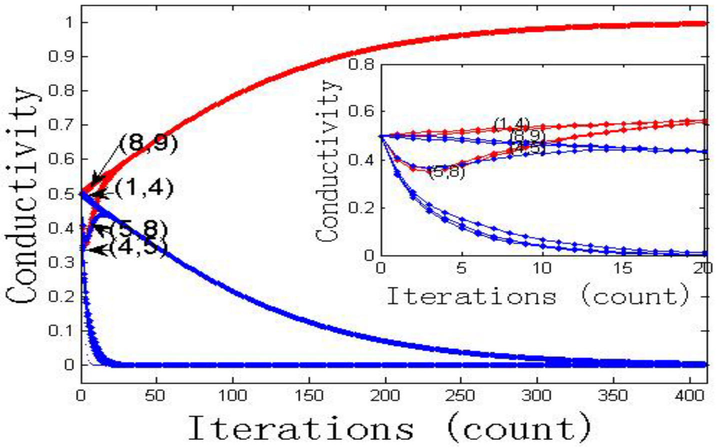

Fig. 9. Conductivity variation considering transportation cost only

Comparing the shortest path by considering different transportation weights, the length of 1→2→3→6→9 is 28.1 and the length of 1→4→5→8→9 is 31.6 when transportation time is the only factor considered. Meanwhile, the total exact value of 1→2→3→6→9 is smaller than all the other road when transportation time is the only factor to assess the road, so it is the shortest path. Similarly, the length of 1→4→5→8→9 is 57.3 and the length of 1→2→3→6→9 is 58.8 when only transportation cost is considered, the value of path 1→4→5→8→9 is the smallest in the transportation network from node 1 to node 9 when only consider transportation cost. But when consider both the transportation weights, the shortest path is 1→4→5→8→9 which is different with the result when only consider transportation time. However, it is the same with the result when only consider transportation cost.

5 Conclusion

It is important to observe the shortest path from the origin the destination in the transportation management system especially in uncertain environment. In this paper, the classical adaptive amoeba algorithm is extended to solve the shortest path selection with uncertain and incomplete edge weights. D numbers theory is used for the representation of the information of uncertainty and incompleteness. Meanwhile, the proposed method has solved the problem when the same road is assessed under multiple different weights based on D numbers theory. The two examples are used to illustrate the efficiency of the proposed method. The proposed method can be applied to real applications in transportation systems or many other network optimization problems that can be regarded as shortest path selection problems.

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