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Robot High Precision Trajectory Tracking Variable Domain Fuzzy Control

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Abstract. In order to achieve the desired path of the robot, trajectory tracking of robot system is to follow a given trajectory change with each joint's controller output drive torque control of each joint position, velocity and other variables. However, the traditional design method of the controller was usually based on the controlled object model, but the various uncertain factors in the actual engineering will lead to the precise mathematical model of the robot can not be gained, therefore the traditional control method will be difficult to achieve the purpose of high precision control. The fuzzy controller does not need to know the exact mathematical, but the control precision of the fuzzy control is not high. In this paper, based on the fuzzy control of the robot, the paper introduces the variable universe theory, introduces the domain expansion factor in the input variables, and uses the genetic algorithm to optimize the expansion factor to achieve the purpose of the variable universe. The simulation results show that the variable universe fuzzy control method has good adaptability, robustness and anti-interference ability, and the control precision is very high, and it can solve the control difficulty caused by the coupling effect of each link, and has high application value for the uncertain robot system.

Keywords: fuzzy control, genetic algorithm, robot, trajectory tracking, variable universe

1 Introduction

In order to achieve the desired path of the robot, trajectory tracking of the robot system is to follow a given trajectory change with each link's controller output drive torque control of each link position, velocity and other variables. The traditional design method of the controller was usually based on the controlled object model, established the dynamic equation and designed the control rules through the traditional control theory. However, the various uncertain factors existed in the actual engineering, such as the load mass, the mass center of the link, the dynamic / static friction force, and the interference of the environment, it lead to the precise mathematical model of the robot was difficult to gained. In the process of the robot movement, the model and parameters of the robot were changed because of the changes of environment and loads, it also lead to the precise mathematical model of the robot was impossible to gained [1].

The fuzzy control was used to solve the control method of the uncertain system, the uncertain system composed of complex, nonlinear and unknown. The design of the fuzzy control was a summary of expert's experience, so there wasn't need certain mathematical model, and it wasn't sensitive to the parameter's variation, so the fuzzy control was used to solve the uncertain factors of the controlled object's parameter and structure [2]. However, due to the control rule's limitation, the control effect of

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the conventional fuzzy controller wasn't idea. The variable domain adaptive control [3] proposed by Li Hong Xing largely add the control rules through expand the universe of fuzzy variable, it significantly improved the accuracy of fuzzy control [4]. So in this paper, the variable universe control theory was used to design the controller of the robot, at the same time the genetic algorithm was used to optimize the domain expansion factor, in order to achieve the purpose of the robot's high precision trajectory tracking control.

2 Establish the Robot's Lagrange Equation Modeling

For a robot with N links, the dynamic performance can be described by the second order nonlinear differential equation. In this paper, as shown in Fig. 1, the typical double links rigid robotic arm was used to the research object, used the Lagrange equation established the mathematical model [5].

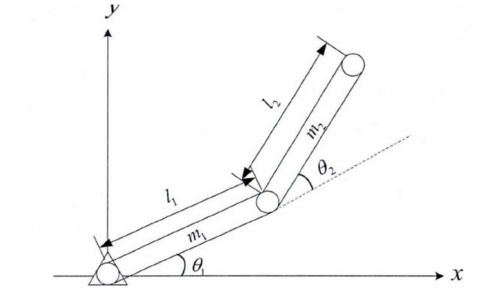

Fig. 1. The rigid robotic arm with double links

Among the picture, l_1 and l_2 are respectively express the length of the link one and the link two, m_1 and m_2 are respectively express the mass of the link one and the link two, θ_1 and θ_2 are respectively express the rotation angle of the link one and the link two.

For a mechanical system, the system's subtraction between the total kinetic energy E_K and the total potential energy was defined by the Lagrange function L.

$$
\begin{aligned}
L = E_k - E_p = \frac{1}{2} m_1 (\frac{l_1}{2})^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left\{ l_1^2 \dot{\theta}_1^2 + (\frac{l_2}{2})^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) \cos\theta_2 \right\} \\
- m_1 g \frac{l_1}{2} \sin\theta_1 - m_2 g (l_1 \sin\theta_1 + \frac{l_2}{2} \sin(\theta_1 + \theta_2))
\end{aligned} . \tag{1}
$$

The Lagrange equation's form is,

$$\tau_i = \frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_i}) - \frac{\partial L}{\partial \theta_i} . \tag{2}$$

According to the Lagrange equation can get the dynamics equation of the double links robotic arm is,

$$\tau_1 = D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + D_{112}\dot{\theta}_1\dot{\theta}_2 + D_{122}\dot{\theta}_2^2 + D_1 . \tag{3}$$

$$\tau_2 = D_{21}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 + D_{212}\dot{\theta}_1\dot{\theta}_2 + D_{211}\dot{\theta}_1^2 + D_2 . \tag{4}$$

In the equation,

$$D_{11} = m_1(\frac{l_1}{2})^2 + m_2 l_1^2 + m_2(\frac{l_2}{2})^2 + 2 m_2 l_1 \frac{l_2}{2} \cos\theta_2 . \tag{5}$$

$$D_{12} = m_1(\frac{l_1}{2})^2 + m_2 l_1 \frac{l_2}{2} \cos\theta_2 . \tag{6}$$

$$D_{112} = -2m_2 l_1 \frac{l_2}{2} \sin\theta_2 . \quad \textbf{(7)}$$

$$D_{122} = -m_2 l_1 \frac{l_2}{2} \sin\theta_2 . \quad \textbf{(8)}$$

$$D_1 = m_1 g(\frac{l_1}{2}) \cos\theta_1 + m_2 g \left\{ l_1 \cos\theta_1 + \frac{l_2}{2} \cos(\theta_1 + \theta_2) \right\} . \quad \textbf{(9)}$$

$$D_{21} = m_2 (\frac{l_2}{2})^2 + m_2 l_1 \frac{l_2}{2} \cos\theta_2 . \quad \textbf{(10)}$$

$$D_{22} = m_2 (\frac{l_2}{2})^2 , D_{212} = 0 . \quad \textbf{(11)}$$

$$D_{211} = m_2 l_1 \frac{l_2}{2} \sin\theta_2 , D_2 = m_2 g(\frac{l_2}{2}) \cos(\theta_1 + \theta_2) . \quad \textbf{(12)}$$

The above equation written into matrix form,

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau . \quad \textbf{(13)}$$

In the equation,

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}, \ddot{q} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} . \quad \textbf{(14)}$$

$$M(q) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, G(q) = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} . \quad \textbf{(15)}$$

$$C(q,\dot{q}) = \begin{bmatrix} D_{112}\dot{\theta}_2 & D_{112}\dot{\theta}_2 \\ D_{211}\dot{\theta}_1 & D_{121}\dot{\theta}_1 \end{bmatrix} . \quad \textbf{(16)}$$

Just like the above double links robotic arm, the dynamic performance of the robot with N links can be described by the second order nonlinear differential equation, Written in matrix form is,

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) + \tau_d = \tau \quad \textbf{(17)}$$

In the equation, q was the angular displacement of the each link of robot, $M(q)$ was the inertia force coefficient matrix of the robot, $C(q,\dot{q})$ was the coriolis force item and the centripetal item, $G(q)$ was the gravity item, τ was the driving force which was added to each link by the controller of the robot, τ_d was the external disturbance of the robot.

In the process of establish the robot's mathematical model existed various uncertain factors, the factors composed of the link's mass center, High frequency un-modeled dynamics, the dynamic/static friction force, the link's flexibility, environment interference and measurement error, so the precision value of the $M(q)$, $C(q,\dot{q})$, $G(q)$ of the robot model is impossible to gained, and the parameters were also changed in the process of the robot's movement. Because of the above mentioned problems, the accurate mathematical model was difficult to established, the fuzzy controller was impossible designed or the control quality was very low, so the variable universe fuzzy controller was designed to solve the above mentioned problems.

3 Designed the Variable Universe Fuzzy Controller

The accurate mathematical model of the controlled object wasn't used when designed the variable universe fuzzy controller, and it has many advantages, it composed of good robustness, good anti-interference, and has strong decoupling ability when many fuzzy controller respectively control different parameter, these characteristics were very import to design robot's controller. According to Li Hong Xing's proof, the commonly various fuzzy controller were able to concluded a kind of interpolation function, the fuzzy set expressed the fuzzy inference antecedent was used to as the interpolation's basis function in the significance of interpolation. However the control function got from the interpolation whether fully approach the real control function was depend on the distance of the fuzzy sets whether enough small, it mean the control rules were more enough [6], the fuzzy controller that its control rules are summarized by expert knowledge wasn't achieved, the effect of the fuzzy control wasn't idea and existed steady state error [7].

The variable universe fuzzy control system [8] was a kind of interpolation, and the dynamic of the interpolation was convergence with point by point, in the premise of the same rule, the universe was able to contract with the decrease of the input variable, it equivalently increased the quantity of the fuzzy control rule, it significantly enhance the nonlinear approximation capacity of the fuzzy system, so as to improve the output precision of the fuzzy system. Besides, the variable universe fuzzy system also had the characteristic of optimal approach error and local convergence. In the certain condition, the approach error optimum value of the nonlinear function can faster convergence to zero with the convergence of the input variable [9]. In this paper, the variable universe theory proposed by Li Hong Xing was introduced to the high precision trajectory tracking control of the robot, designed the adaptive fuzzy controller of the robotic each link, it can use the expansion factor automatically changed the domain expansion through the error and error variety- rate. The controller's structure of the multiple links robot had the same designed method, as shown in Fig. 2.

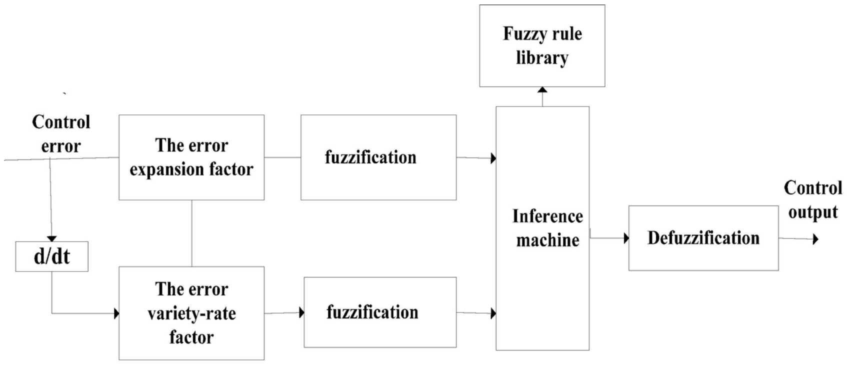

Fig. 2. The structure chart of the variable universe fuzzy controller

In this paper, the method that expand and contract the quantitative factor of the error and error variety-rate was used to realize the variable universe control. According to the expansion factor changed, the points fall in original universe were from the deviation proposed by Li Hong Xing, it were conformity with the fuzzy subset and membership function, it was known by the principle: when deviation reached maximum value, we should conduct "coarse" control in the original domain, let it quickly entered into stable area, at the moment deviation's factor was one. The deviation was gradually reduced, in this process, the universe constantly contracted with the deviation decrease, for the deviation, the universe contracted. In order to remain the universe was the constant value and the control effect was very idea, must accordingly increased deviation, relative to the changed deviation, the constant universe contracted, the moment the deviation expansion factor was exceed one. In other words, with the deviation constantly decreased, the expansion factor constantly increased from one [10], the design of the deviation variety-rate's expansion factor used the same theory, this paper used the expansion factor is,

$$a(x) = \frac{1}{1 - \lambda e^{(-kx^2)}} . \tag{18}$$

In the equation, $\lambda \in (0,1)$, $k > 0$, x was deviation or deviate variety-rate. Expansion speed of the error and error variety-rate was determined by k value, if the k value was bigger, the speed of domain expansion factor was faster, the respond speed was higher. λ was the minimum domain value range's coefficient, it reflected the controlled system's control precision [11-12].

The input variable error e, the error variety-rate de, and the controller output U of the variable universe fuzzy controller used normalized conduction, the membership functions was designed triangles exception both sides were designed into gaussian function. The fuzzy set of the variable universe fuzzy controller input variable and output variable were {NB,NM,NS,ZO,PS,PM,PB}, included 49 fuzzy rules, as shown in Table 1.

Table 1. Rule table of the fuzzy control

U		E						
		NB	NM	NS	ZO	PS	PM	PB
EC	NB	PB	PB	PM	PM	PS	ZO	ZO
	NM	PB	PB	PM	PS	PS	ZO	NS
	NS	PM	PM	PM	PS	ZO	NS	NS
	ZE	PM	PM	PS	ZO	NS	NM	NM
	PS	PS	PS	ZO	NS	NS	NM	NM
	PM	PS	ZO	NS	NM	NM	NM	NB
	PB	ZO	ZO	NM	NM	NM	NB	NB

4 The Universe Expansion Factor was Optimize by Genetic Algorithm

The Genetic Algorithm (GA) [13] was a kind of evolutionary algorithm, it imitated the evaluative rule "Natural selection, the survival of the fittest". The genetic algorithm compiled problem parameter into chromosome, in order to exchange the population chromosome's information, the iterative approach was used to select, crossover and variate, and finally generated chromosome meet optimized goal. The genetic algorithm's basic steps shown as follows:

Step 1. Code: Before search, the GA expressed the solution space's data into hereditary space's genotype string structural data, and the different points was constituted by the string structure's different combination.

Step 2. Initial population's generation: randomly generated N kinds initial string structural data , each string structural data was called an individual, and N kinds individuals form into a group, with the string structural data as the initial point, GA began to evolution.

Step 3. Fitness evaluation: fitness indicated advantage and disadvantage for individual or solution.

Step 4. Choice: the purpose of the choice was selected the excellent individual from the current groups, and made them have opportunity breed descendants for next generation. Through select process, this idea was reflected by the genetic algorithm, and select's principle was adaptable individual has large probability of contributed one or more offspring for the next generation. Choice reflected thought of survival of the fittest proposed by Darwin.

Step 5. Cross: Crossover operation was the most important genetic operation in genetic algorithm. By the crossover operation, we were get new generational individual, and the new generational individual combined their father's characters, and cross reflected information exchange's thought.

Step 6. Variation： First randomly selected individual in population, and then randomly altered the string structural data with a certain probability. As well as biology, in GA, the variation's incidence was very low, usually the value was very small.

In order to optimize variable universe factor of the fuzzy controller, this paper used MATLAB optimization toolbox's genetic algorithm [14], used control system's comprehensive performance index $f = \int |e| t dt$ as the fitness evaluation function, the fitness was smaller, the control effect was better. Randomly generated the initial population, the population's was 100, the elite was 10, the cross ratio was 0.7; and the biggest evolution algebra was 500, we were obtained the parameter: $\lambda = 0.99$, $k = 0.1$ [11].

5 The Simulation Result and Analysis of the Robotic Variable Universe Fuzzy Control

This paper adopted the robot system of double links as the controlled object, its dynamic model is,

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau+d\,. \tag{19}$$

$$D(q)=\begin{bmatrix} v+q_{01}+2q_{02}\cos(q_2) & q_{01}+q_{02}\cos(q_2) \\ q_{01}+q_{02}\cos(q_2) & q_{01} \end{bmatrix}. \tag{20}$$

$$C(q,\dot{q})=\begin{bmatrix} -q_{02}\dot{q}_2\sin(q_2) & -q_{02}(\dot{q}_1+\dot{q}_2)\sin(q_2) \\ q_{02}\dot{q}_1\sin(q_2) & 0 \end{bmatrix}. \tag{21}$$

$$G(q)=\begin{bmatrix} 15g\cos q_1+8.75g\cos(q_1+q_2) \\ 8.75g\cos(q_1+q_2) \end{bmatrix}. \tag{22}$$

Among the equation, $v=13.33$, $q_{01}=8.98$, $q_{02}=8.75$, $g=9.8$, the controlled object's initial state was [0.9,0.3,0.6,0.5]. In order to verified ability and robustness of the variable universe fuzzy control, made the disturbance of two articular $d=20\sin(2t)$,meanwhile $D(q)$, $C(q,\dot{q})$, $G(q)$ all had the variation of 20%.

The given trajectory of double articular were,

$$qd1=1+0.2\sin(0.5\pi t)\,. \tag{23}$$

$$qd2=1-0.2\cos(0.5\pi t)\,. \tag{24}$$

Because of the controlled object was the double articular robot, so the double links were need control torque of controller provided, so used two controller with the same structure of double input and single output, controlled two links, it will ensured two links accurately tracked their given trajectory. The articular error e and the articular error variety-rate de as input variables of the variable universe fuzzy controller, and output variable of the controller was the articular control torque. The two variable universe fuzzy controller have something in common, it composed of control structure, input and output variable membership function, fuzzy control rule and the domain expansion factor, only compared to link two, the control torque of link one was smaller, so output universe of the two controller were different. The simulation model as shown in Fig. 3.

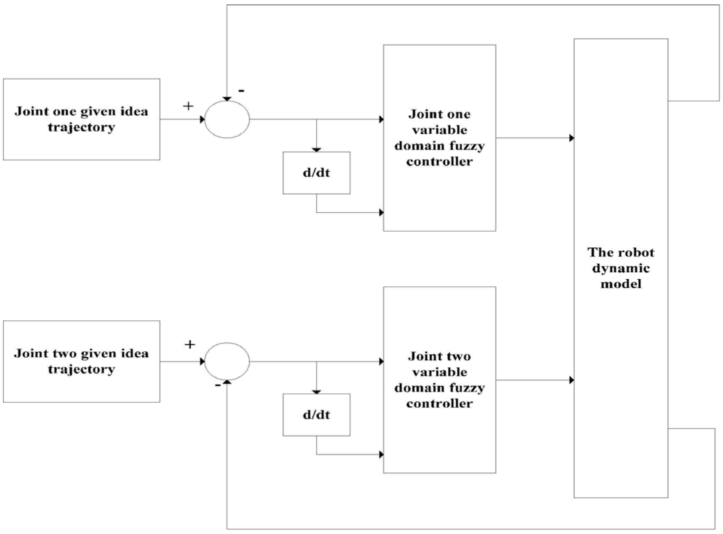

Fig. 3. The double links robotic variable domain fuzzy control block diagram

Under the control of the variable universe fuzzy controller, the robotic two links can quickly tracked the expected trajectory in different initial states. The control output curve of the double links robot was shown in Fig. 4 and Fig. 5, the articular tracking error was shown in Fig. 6 and Fig. 7. We can saw from the simulation results that two links can quickly tracked input curve within 10^{-5} magnitude, the steady-state error was very small.

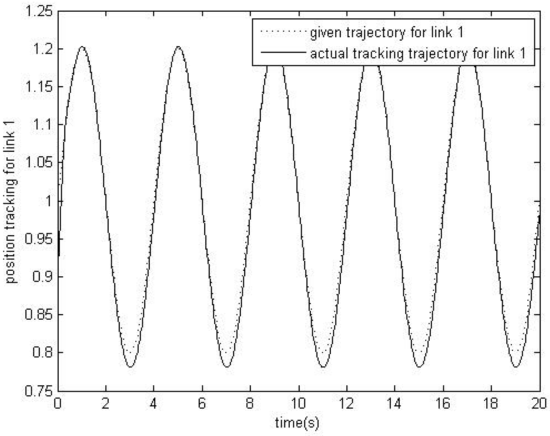

Fig. 4. The trajectory tracking variable domain fuzzy curve for link 1

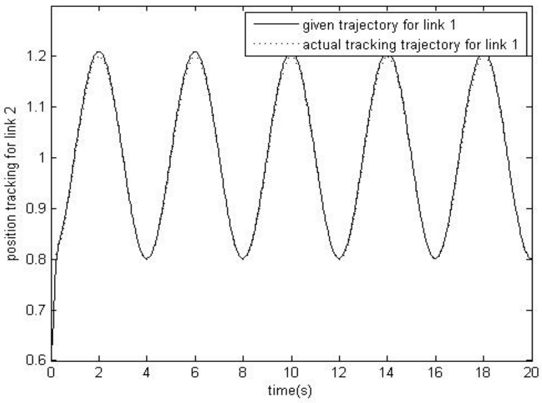

Fig. 5. The trajectory tracking variable domain fuzzy curve for link 2

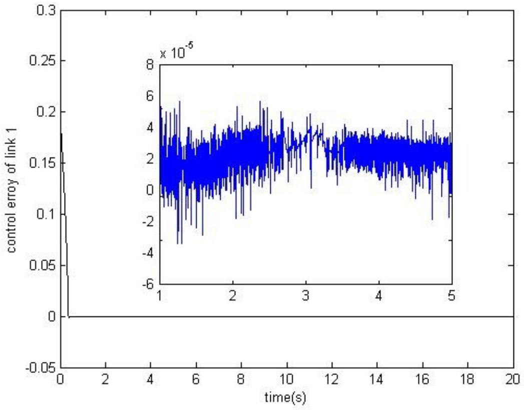

Fig. 6. The trajectory tracking error for link 1

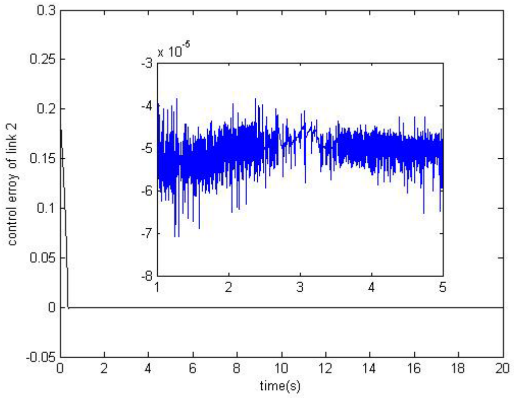

Fig. 7. The trajectory tracking error for link 2

Even controlled object model was uncertainty and existed disturbance, the variable universe fuzzy controller also achieved idea control target, insensitive for parameters change, strong robustness, and has a good solve method for the robotic coupling.

6 Conclusion

Due to independent of accurate mathematical model, the fuzzy control was suited to solve difficult control problem of the robot system. However, because of limited fuzzy control rules, the conventional fuzzy control was difficult achieved idea effect for the robot high precision trajectory. So on the basis of the conventional fuzzy control, this paper introduced variable universe theory, used domain expansion indirectly increased the fuzzy control rules, finally achieved robot high precision trajectory tracking control.

This paper designed domain expansion factor, then optimize the expansion factor through genetic algorithm, in order to achieved variable universe fuzzy control, introduced expansion factor in input variable of the error and error variety-rate, The simulation experiment of the double links robot shown that the designed variable universe fuzzy controller has more advantages, it composed of good flexibility, nice adaptability, stronger decoupling ability and high control precision, and there isn’t steady-state error, even more, each link variable domain controller has the same design, simple structure, easily realize, as well as adjust to many degrees of freedom robotic control, it has a good application value for robotic system.

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