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Abstract. For the problem of weighting indexes not reasonable enough while we evaluate the index weight in multiple properties decision making by only subjective or objective weighting method, this paper comes up a combination weighting method based on maximizing deviations and normalized constraint condition methods. This method combines both subjective and objective information, deduces computational formula by double objective optimization model. The method also solves the problem of arbitrary choices problem, does quantitative experimental analysis on subjective and objective weighting methods respectively, then optimize decision-making. Then apply both subjective and objective methods on combination weighting method. Experimental data shows that this combination weighting method can cover relative importance of each index while also representing information, which came from index itself. By combining subjective and objective strategies, combination-weighting method is getting a more reasonable result.

Keywords: combination weighting, objective weighting method, subjective weighting method, weight

1 Introduction

In China, audiences are shunted into new media like Internet in nowadays. However, in the perspective of popularizing rate and exposed population, broadcasting and television are still the most powerful media, also the best transmitter of mass culture. At present, the most common method of broadcasting and television program evaluation is building an evaluation index system. In the process of evaluating, the weight of index is very important [1]. Meanwhile the determination of weight of index shall affect the accuracy of evaluating result in a certain extent. The current main method of weighting broadcasting and television depends on the long-term experience of practitioners. Relatively determining weight of each index by artificial is subject to decision makers' judgments, usually it will not violate common sense. However, due to boundedness of human being, it has low accuracy and reliability [2].

Based on the differences between the source of raw data and the calculation process when we calculate the weight coefficient, the evaluation index weighting determination method can be roughly divided into two types: subjective weighting method and objective weighting method. Subjective weighting method

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takes the method of qualitative. Experts make subjective judgment to get the weight according to the experience. Then they make a comprehensive evaluation to index. Common methods are analytic hierarchy process method, binomial coefficient method, sequential analysis method, least square method, and rank correlation analysis method. Analytic hierarchy process method separates elements, which related to decision into target, criteria, scheme and so on, then make qualitative and quantitative analysis decision based on it [3-5]. Binomial coefficient method is directly using the coefficients of binomial expansion as weight coefficients of indexes, and allocating indexes according to priority order [6]. Sequential analysis method compares targets by pairs. Get importance rate of each pair first, then threat last target as 1, compare it up sequentially, then calculate the importance rates between all targets, finally get weight index [7]. Least square method is a method based on analytic hierarchy process method. It treats complementary judgment matrix as judgment matrix, combines the best solution then gets result [8-9]. Rank correlation analysis method is also utilizing relative importance rate between two indexes to determine the weight of evaluation indexes [10-11].

According to the relationship of each index or the relationship between the index and evaluate results in the historical date, objective weighting method takes a comprehensive evaluate. It includes entropy evaluation method, mean square deviation method, coefficient of variation method [14-16], and maximizing deviations method. The rational of entropy evaluation method [3, 12-13] is: entropy is a measurement of information uncertainty. The less entropy, the more information. Therefore, when entropy of index is small, it means this index is more important in decision, which should have a bigger weight. Mean square method [6] does normalization processing to mean square coefficients of indexes, the results are weight vectors. Bigger mean square of indexes means bigger differences between decisions, which means more important in final decision. Coefficient of variation method [14]: if there is a big variation of index, then this index could differentiate evaluation objects in this aspect, so it should have a bigger weight. The empowerment strategy of maximizing deviations method [8, 17] is: deviations of values of different solutions under same index make the most important effect on evaluation results. Bigger deviation should have bigger weight. The ultimate goal is to improve the total deviation of all objects [18].

Subjective weighting method can reflect the experience judgment of policy makers. The relative important degree of attributes does not violate people's common sense generally. While its randomness is bigger, the decision-making accuracy and reliability is a bit poor. There is objective standard in the objective values. It can use certain mathematical model and get the coefficient weight of attribute by calculating. Ignorance of the subjective knowledge and experience of decision makers are its disadvantages. Sometimes the weight coefficient will be unreasonable. In view of the advantages and disadvantages of the subjective or objective weighting methods, there are many discussions about the combination weighting method. Wang, Gu and Yi [19] proposed an assignment method based on a linear combination of entropy. Wang, Mou and Li [20] according to optimization theory puts forward a kind of subjective preference and objective information linear comprehensive weighting method. Chen [21] constructs the comprehensive weighting method based on the sum of squared residuals. It uses the multiattribute decision-making schemes value as the basic idea. Zheng, Tagn and Shi [22] propose the comprehensive integration weighting method based on the sum of square residuals and the comprehensive integration weighting method based on normalized constraint condition respectively. Yang, Liang, Deng and Guo [23] present a combination weighting method based on AHP and variation coefficient. This method can assess supply chain risk by combination weighting, but because of the diversity of supply chain risk, the method will still ignore some information, makes weighting setting not reasonable. Wang, Huang and Li [24] proposes to use AHP and entropy method as an evaluation method of combination weighting. However, the completeness of the index and the calculation method need to be improved. Yang, Ju, Yan and Shan [25] propose a combination of subjective and objective methods according to the principle of maximizing deviation based on the combination of AHP and entropy method. Zhai, Lin, Wen and Huang [26] proposed a new combination evaluation method based on subjective and objective weighting, each assessment point to sort the data in order to obtain the maximum deviation, this method may ignore the attribute weights of evaluation index itself, easy to cause the evaluation results from the actual results. Most combination weighting theories are based on optimization theory, and establishes the single objective optimization model and solves it.

When we should ensure attribute index weights in the multiple attribute decision-making problem, calculating by subjective weighting method or objective weighting method singly will cause the problem

that the weight coefficient is unreasonable. Therefore, the paper puts forward a comprehensive combination of subjective and objective weight information weighting approach. To the problem of optional and non-theoretical foundation decision making, we do the experimental quantitative analysis on subjective weighting and objective weighting method, respectively. Through the analysis of the results and the optimization selection, reasonable combination weight could be got by the subjective and objective weighting method. In order to solve the disadvantages of various weighting methods, the key research of this paper is a combination weighting method based on maximizing deviations and normalized constraint condition methods. In this way, we could get a more reasonable result by considering both experience of decision maker and original data. This method has important scientific value for the construction of the evaluation system of radio and TV programs and the evaluation of specific programs. It also has important reference value to any projects that need to be evaluated by index.

2 Combination Weighting theory

There are two forms of the combination of the weighting method, multiplicative synthesis and linear weighted combination respectively. Multiplicative synthesis method is making each index weight from weighting methods multiplication. Then we should normalize processing combination weighting. This method is suitable for more index method and more uniform weight distribution. Linear weighted combination method is to weight each weight from each weighting methods, weight can be determined by decision makers have preference in different weighting methods, weight can be determined by decision makers' preference. When decision makers have no preference in different weighting methods, there it needs to use other methods to determine the relative importance of different weighting method. Therefore, there is a weight allocation problem in the linear weighted combination. The focus of this paper is to discuss the five common subjective weighting methods and four common objective weighting methods, which takes the optimization model to determine the weight allocation problem. Profile as shown in Fig. 1.

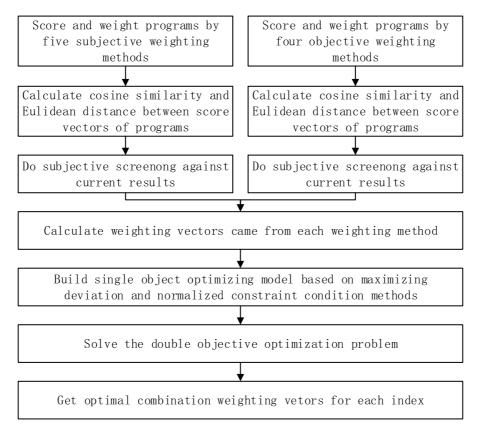


Fig. 1. Diagram for combination weighting

3 Select Weighting Methods

Use the 967 live program in 14 categories on one day of a province as evaluation object; the six evaluation indexes are audience rating, arrival rating, loyalty, the market share, the average viewing time and ratings section number. Here we use weighted assessment method to evaluate the programs.

3.1 Selection of Subjective Weighting Method

In this section, we will compare and analyze hierarchy process method, binomial coefficient method, sequential analysis method, least square method, and rank correlation analysis method these five major subjective weighting methods. Six indexes have same relative priority in these five weighting methods. Their significance ascending order: ratings section number, the market share, arrival rating, loyalty, the average viewing time, and audience rating.

(1) In hierarchy process method, judgment matrix of indexes

- $A = \begin{pmatrix} 1 & 4 & 3 & 2 & 5 & 6 \\ \frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{3} & 2 & 3 \\ \frac{1}{3} & 2 & 1 & \frac{1}{2} & 3 & 4 \\ \frac{1}{2} & 3 & 2 & 1 & 4 & 5 \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 1 & 2 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{2} & 1 \end{pmatrix}$
- (2) In binomial coefficient method, significance order: ratings section number < the market share < arrival rating < loyalty < the average viewing time < audience rating.
- (3) In sequential analysis method, importance rate is two between any two indexes.
- (4) In least square method, corresponds to the reciprocal judgment matrix of hierarchy process method,

 $complementary judgment matrix B = \begin{pmatrix} \frac{1}{2} & \frac{4}{5} & \frac{3}{4} & \frac{2}{3} & \frac{5}{6} & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{2}{3} & \frac{3}{4} \\ \frac{1}{5} & \frac{2}{2} & \frac{1}{3} & \frac{1}{4} & \frac{2}{3} & \frac{3}{4} \\ \frac{1}{4} & \frac{2}{3} & \frac{1}{2} & \frac{1}{3} & \frac{3}{4} & \frac{4}{5} \\ \frac{1}{3} & \frac{3}{4} & \frac{2}{3} & \frac{1}{2} & \frac{4}{5} & \frac{5}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{7} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{pmatrix}$

(5) In rank correlation analysis method, significance order: ratings section number < the market share < arrival rating < loyalty < the average viewing time < audience rating. Relative importance rate

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between indexes
$$r_k = \frac{\omega_{k-1}}{\omega_k} = 1.2, k = 2, 3, ..., 6$$

The weighting result is as follow Table 1.

	audience rating	the average viewing time	loyalty	arrival rating	the market share	ratings section number
hierarchy process method	0.3794	0.2488	0.1604	0.1024	0.0655	0.0434
binomial coefficient method	0.3125	0.3125	0.15625	0.15625	0.03125	0.03125
sequential analysis method	0.5079	0.2540	0.1270	0.0635	0.0317	0.0159
least square method	0.4214	0.2351	0.142	0.0923	0.0635	0.0457
rank correlation analysis method	0.2506	0.2088	0.174	0.145	0.1208	0.1007

Table 1. Results of subjective weighting methods

From the Table 1, we can see that under the precondition of same relative significance order of six indexes, these five subjective weighting methods have same weighting trend. This is a reflection of how decision makers affect the weighting. However, there are also differences between each method according to the weights; these differences are determined by importance rate between indexes. Every method has different ways to calculate importance rate, these also makes difference results.

We rate all the programs on weights gotten from Table 1 by weighted assessment method. Calculate scores of programs in cosine similarity and Euclidean distance separately and compare these two results.

The calculation of cosine similarity of program scores is in Table 2. By analyzing the values of Table 2, any two programs' evaluation results have a similarity higher than 0.997, which is an extremely high similarity. Cosine similarity is using cosine value of two vectors to measure differences. That means cosine similarity pays more attentions on direction difference. Therefore, the result of one program will not have different rank by any of these five methods. That means if a program has a high rank by one method, then it will also get a high rank by other method.

	analytic hierarchy process method	binomial coefficient method	sequential analysis method	least square method	rank correlation analysis method
analytic hierarchy process method	1	0.9992	0.9979	0.9996	0.9987
binomial coefficient method	0.9992	1	0.9990	0.9993	0.9962
sequential analysis method	0.9979	0.9990	1	0.9991	0.9939
least square method	0.9996	0.9993	0.9991	1	0.9976
rank correlation analysis method	0.9987	0.9962	0.9939	0.9976	1

Table 2. Cosine similarity of program scores

The calculation of Euclidean distance of program scores is as follow Table 3. By analyzing the values of Table 3, the minimum distance of five methods is 473.6, the biggest reaches 1926. Euclidean distance focuses on absolute differences of single value. This result shows that although weighting methods could not change the rank of programs, there are still differences of each program.

Table 3	. Euclidean	distance	of p	rogram	scores
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	analytic hierarchy process method	binomial coefficient method	sequential analysis method	least square method	rank correlation analysis method
analytic hierarchy process method	0	707.8	1120.0	473.6	887.3
binomial coefficient method	707.8	0	772.4	635.0	1505.6
sequential analysis method	1120.0	772.4	0	746.3	1926.0
least square method	473.6	635.0	746.3	0	1207.6
rank correlation analysis method	887.3	1505.6	1926.0	1207.6	0

To sum up, Different subjective weighting methods will not cause obviously difference on ranking level, but there are still differences in detail places in certain scope. Therefore, these five methods all

have their special features. It is worth to keep them.

3.2 Selection of objective weighting method

In this section, we will discuss four common objective weighting methods; they are entropy evaluation method, mean square deviation method, coefficient of variation method, and maximizing deviations method. First, calculate weighting vectors by data, we got from previous experiment and four objective weighting method, then score programs by weighting evaluation method. We will calculate the cosine similarity and Euclidean distance for programs and compare final results.

The cosine similarity of program scores is as follow Table 4. By analyzing the data from Table 4 we can find that the cosine similarity between maximizing deviations method and mean square deviation method is 0.9987, which reaches a high similarity. The cosine similarities with other methods against these two are lower than 0.86. This result shows level ranks of programs by maximizing deviations method and mean square deviation method are almost same.

Table 4. Cosine similarity of four objective weighting method
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	coefficient of variation method	mean square deviation method	maximizing deviations method	entropy evaluation method
coefficient of variation method	1	0.7476	0.7396	0.8414
mean square deviation method	0.7476	1	0.9987	0.8597
maximizing deviations method	0.7396	0.9987	1	0.8431
entropy evaluation method	0.8414	0.8597	0.8431	1

The Euclidean distance results are as follow Table 5. By analyzing the data from Table 5 we can see that Euclidean distance between maximizing deviations method and mean square deviation method is only 87.6, others are above 900. This means maximizing deviations method and mean square deviation method are getting same rank for programs.

	coefficient of	mean square deviation	maximizing	entropy evaluation
	variation method	method	deviations method	method
coefficient of variation method	0	1234.3	1253.7	978
mean square deviation method	1234.3	0	87.6	919.7
maximizing deviations method	1253.7	87.6	0	972.7
entropy evaluation method	978	919.7	972.7	0

To sum up, rank results that got from maximizing deviations method and mean square deviation method are very similar on either levels or detail values. That means we can get almost same results by these two methods. There are obvious differences between all the other methods. Therefore we should choose one of maximizing deviations method and mean square deviation method for further study. Here we pick mean square deviation method to continue our research.

4 Combination Weighting Method Based on Maximizing Deviations and Normalized Constraint Condition

In a multiple attribute decision making problem, the solution set is expressed as $S = \{S_1, S_2, \dots, S_m\}$, the attributes (or index) set is expressed as $P = \{P_1, P_2, \dots, P_n\}$, the attribute of the *i* scheme S_i accord to the *j* index P_j is expressed as $a_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n$, $\mathbf{A} = (a_{ij})_{m \times n}$ is called decision matrix.

After weighting decision problem by using subjective weighting method or objective weighting method respectively, we assume that there is l kind of subjective and objective weighting method to n indicators giving weight coefficient. The weight vector value of the k weighting method is

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$$\mathbf{W}_{\mathbf{k}} = \left(w_{1k}, w_{2k}, \cdots, w_{nk}\right)^{T}, k = 1, 2, \cdots, l, \ w_{jk} \ge 0, \ \sum_{j=1}^{n} w_{jk} = 1, k = 1, 2, \cdots, l, \ j = 1, 2, \cdots, n.$$

For synthesize the characteristics of each weighting methods, we consider the following combination weighting

$$\mathbf{W}_{c} = \theta_{1}\mathbf{W}_{1} + \theta_{2}\mathbf{W}_{2} + \dots + \theta_{l}\mathbf{W}_{l} .$$
(1)

 $\mathbf{W}_{\mathbf{c}} = (w_{c1}, w_{c2}, \dots, w_{cn})^T$ is called combination weighting coefficient vector. $\theta_1, \theta_2, \dots, \theta_l$ is called linear scale coefficient of the combination weighting coefficient vector. Satisfying $\theta_k \ge 0, k = 1, 2, \dots, l$, and normalized constraint condition

$$\sum_{k=1}^{l} \theta_k^2 = 1.$$
 (2)

If partitioned matrix $\mathbf{W} = (\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_l)$, $\mathbf{\Theta} = (\theta_1, \theta_2, \dots, \theta_l)^T$, \mathbf{W} is called weight coefficient vector matrix which made in *l* kinds of weighting methods. \mathbf{W} is the $n \times l$ matrix in fact, $\mathbf{\Theta}$ is *l* dimension column vector which made inlinear scale coefficient of the combination weighting coefficient vector. Formula (1), (2) can be expressed as

$$\mathbf{W}_{c} = \mathbf{W}\mathbf{\Theta} \,. \tag{3}$$

$$\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Theta} = \boldsymbol{1}\,. \tag{4}$$

According to the simple linear weighted combination method, the comprehensive index of the *i* program S_i can be expressed

$$Z_{i}(\mathbf{W}_{c}) = \sum_{j=1}^{n} b_{ij} w_{cj} , \ i = 1, 2, \cdots, m.$$
(5)

In the comprehensive evaluation problem, we always want to make the comprehensive evaluation values of each decision-making scheme space out the difference generally. In other word, we hope to make the comprehensive evaluation values of each decision-making scheme be decentralized as much as possible. We also hope to maximize the comprehensive evaluation value of each evaluation objects.

4.1 Make the Comprehensive Evaluation Values of Each Decision-making Scheme be Decentralized As Much As Possible

In order to maximize to the total dispersion of all n indicators to all m decision-making plan, constructing the following objective function.

$$J_{1}(\mathbf{W}_{c}) = \sum_{j=1}^{n} \sum_{i_{1}=1}^{m} \sum_{i_{1}=1}^{m} \left| b_{i_{j}} - b_{i_{1}j} \right| w_{cj} .$$
(6)

n dimensional row vector \mathbf{B}_1 :

$$\mathbf{B}_{1} = \left[\sum_{i=1}^{m} \sum_{i_{1}=1}^{m} \left| b_{i_{1}} - b_{i_{1}1} \right|, \sum_{i=1}^{m} \sum_{i_{1}=1}^{m} \left| b_{i_{2}} - b_{i_{1}2} \right|, \cdots, \sum_{i=1}^{m} \sum_{i_{1}=1}^{m} \left| b_{i_{n}} - b_{i_{1}n} \right| \right].$$
(7)

The objective function can be expressed as

$$J_1(\mathbf{W}_c) = \mathbf{B}_1 \mathbf{W}_c = \mathbf{B}_1 \mathbf{W} \mathbf{\Theta} .$$
 (8)

So we can establish the optimization model based on maximizing deviation I:

$$\max F_{1}(\boldsymbol{\Theta}) = \mathbf{B}_{1} \mathbf{W} \boldsymbol{\Theta}$$
s.t.
$$\begin{cases} \boldsymbol{\Theta}^{T} \boldsymbol{\Theta} = 1 & \cdot \\ \boldsymbol{\Theta} \ge 0 \end{cases}$$
(9)

4.2 Maximize the Comprehensive Evaluation Value of Each Evaluation Objects

The comprehensive evaluation values of the *i* decision-making scheme S_i can be expressed

$$Z_{i}(\mathbf{W}_{c}) = \sum_{j=1}^{n} b_{ij} w_{cj} , \ i = 1, 2, \cdots, m.$$
(10)

In order to make the comprehensive evaluation value of each objects be bigger, we can establish the better model

$$\max F_{2}^{*}(\boldsymbol{\Theta}) = \left(Z_{1}(\mathbf{W}_{c}), Z_{2}(\mathbf{W}_{c}), \cdots, Z_{m}(\mathbf{W}_{c})\right)$$

s.t.
$$\begin{cases} \boldsymbol{\Theta}^{T} \boldsymbol{\Theta} = 1 \\ \boldsymbol{\Theta} \ge 0 \end{cases}$$
 (11)

As there is no preference relation between each decision scheme, we can get the answer by the weighted linear summation method that the multi-objective decision model convert into equivalent single objective optimization model. The objective function, in other word, the comprehensive evaluation values of all the decision is

$$J_{2}(\mathbf{W}_{c}) = \sum_{i=1}^{m} Z_{i}(\mathbf{W}_{c}) = \sum_{i=1}^{m} \sum_{j=1}^{n} b_{ij} w_{cj} .$$
(12)

n dimensional row vector \mathbf{B}_2

$$\mathbf{B}_{2} = \left[\sum_{i=1}^{m} b_{i1}, \sum_{i=1}^{m} b_{i2}, \cdots, \sum_{i=1}^{m} b_{in}\right].$$
 (13)

The objective function can be expressed

$$J_2(\mathbf{W}_c) = \mathbf{B}_2 \mathbf{W}_c = \mathbf{B}_2 \mathbf{W} \mathbf{\Theta} .$$
 (14)

So we can establish the optimization model based on the normalized constraint condition II:

$$\max F_{2}(\boldsymbol{\Theta}) = \mathbf{B}_{2} \mathbf{W} \boldsymbol{\Theta}$$
s.t.
$$\begin{cases} \boldsymbol{\Theta}^{T} \boldsymbol{\Theta} = 1 & . \\ \boldsymbol{\Theta} \ge 0 \end{cases}$$
(15)

4.3 Solve the Double Objective Optimization Problem

This paper will consider the two constraints mentioned above at the same time. On the one hand, it can make the comprehensive evaluation values of each decision-making scheme space out the difference. On the other hand, it can maximize it. This is a double objective plan problem. Using the linear weighted sum method solves the double objective optimization problem.

$$\max F(\mathbf{\Theta}) = \alpha \mathbf{B}_1 \mathbf{W} \mathbf{\Theta} + \beta \mathbf{B}_2 \mathbf{W} \mathbf{\Theta}$$

s.t.
$$\begin{cases} \mathbf{\Theta}^T \mathbf{\Theta} = 1 & . \\ \mathbf{\Theta} \ge 0 \end{cases}$$
 (16)

Among then $\alpha + \beta = 1$.

The corresponding weights can be given with different values according to different situations. There is a calculating in the situation that the two optimized conditions have the same importance. After such a right assignment, we can establish double objective optimization model.

$$\max F(\mathbf{\Theta}) = 0.5\mathbf{B}_{1}\mathbf{W}\mathbf{\Theta} + 0.5\mathbf{B}_{2}\mathbf{W}\mathbf{\Theta}$$

$$s.t.\begin{cases} \mathbf{\Theta}^{T}\mathbf{\Theta} = 1 & . \\ \mathbf{\Theta} \ge 0 \end{cases}$$
(17)

If $0.5(\mathbf{B}_1 + \mathbf{B}_2) = \mathbf{B}_3$, the optimization model is

$$\max F(\mathbf{\Theta}) = \mathbf{B}_3 \mathbf{W} \mathbf{\Theta} \,. \tag{18}$$

$$s.t.\begin{cases} \mathbf{\Theta}^T \mathbf{\Theta} = 1\\ \mathbf{\Theta} \ge 0 \end{cases}.$$
 (19)

Use the Lagrange multiplier method to solve the model, the optimal solution of the double objective optimization model is

$$\boldsymbol{\theta}_{k}^{*} = \mathbf{B}_{3} \mathbf{W}_{k} / \sqrt{\sum_{k=1}^{l} (\mathbf{B}_{3} \mathbf{W}_{k})^{2}}, k = 1, 2, \cdots, l.$$
(20)

Then take formula (20) into formula (1), we can get the optimization combination weighting vector of multiple attribute decision making based on the combination of maximizing deviations and normalized constraint condition.

$$\mathbf{W}_{\mathbf{c}}^{*} = \boldsymbol{\theta}_{1}^{*} \mathbf{W}_{1} + \boldsymbol{\theta}_{2}^{*} \mathbf{W}_{2} + \dots + \boldsymbol{\theta}_{1}^{*} \mathbf{W}_{1}.$$
⁽²¹⁾

Because of the normalized processing for weight vector, we need to take the normalized processing for $\mathbf{W}_{c}^{*} = \left(w_{c1}^{*}, w_{c2}^{*}, \cdots, w_{cn}^{*}\right)^{T}$. We only need to take the normalized processing for the $\theta_{k}^{*}, k = 1, 2, \cdots, l$.

$$\theta_{k}^{**} = \theta_{k}^{*} / \sum_{k=1}^{l} \theta_{k}^{*} = \mathbf{B}_{3} \mathbf{W}_{k} / \sum_{k=1}^{l} (\mathbf{B}_{3} \mathbf{W}_{k}), k = 1, 2, \cdots, l.$$
(22)

Obviously, $\sum_{k=1}^{l} \theta_k^{**} = 1, \theta_k^{**} \ge 0, k = 1, 2, \dots, l$. Take formula (22) into formula (1). Get the optimization

normalized combination weighting vector of multiple attribute decision making based on the combination of maximizing deviations and normalized constraint condition.

$$\mathbf{W}_{c}^{**} = \mathbf{\theta}_{1}^{**} \mathbf{W}_{1} + \mathbf{\theta}_{2}^{**} \mathbf{W}_{2} + \dots + \mathbf{\theta}_{1}^{**} \mathbf{W}_{1}.$$
(23)

5 Experimental Analysis

We use five kinds of subjective weighting methods and three kinds of objective weighting methods which is screened before to calculate.

We choose Gehua viewing date in Beijing in June 2014 as the date source. Types in 14 shows as evaluation object set, select 6 viewing index as a set of properties, $P_1, P_2, P_3, P_4, P_5, P_6$. They are audience rating (%), arrival rating (%), loyalty (%), the market share (%), the average viewing time (minutes) and ratings section number (period). P_1, P_2, P_3, P_4, P_5 is quality-benefit type attributes, P_6 is cost properties. Table 6 for the attribute value.

Set up original decision matrix by the data in table 6. Get normalization formula standardization of decision matrix according to the data in the literature [27]. The standardized data is shown in Table 7.

PROGRAM type	P_1	P_2	P_3	P_4	P_5	P_6
finance	0.0013	0.2839	0.0048	0.0282	1076.4846	3.1349
tv series	0.0021	0.5080	0.0041	0.2626	5604.5420	9.0750
moive	0.0033	0.2849	0.0117	0.0406	1545.5377	2.9174
law	0.0042	0.3020	0.0137	0.0321	1154.0796	2.9143
education	0.0015	0.0669	0.0219	0.0030	481.2764	1.4299
other	0.0015	0.5823	0.0025	0.0944	1757.7659	21.0363
teenagers	0.0013	0.1868	0.0068	0.0189	1097.0800	3.4919
Life service	0.0025	0.5578	0.0044	0.0874	1699.2490	13.8613
sports	0.0032	0.3090	0.0105	0.0550	1929.8000	4.1186
drama	0.0025	0.1647	0.0151	0.0113	743.1830	1.8426
news	0.0036	0.4839	0.0075	0.1487	3331.5970	6.5669
mucis	0.0018	0.2239	0.0079	0.0186	898.9922	2.1484
special	0.0020	0.4606	0.0042	0.0942	2219.0070	5.4914
variety	0.0029	0.4396	0.0067	0.1051	2592.4540	5.2894

Table 6. Gehua viewing date in Beijing in March 2014

Table 7. The standardized viewing data

prOGRAM type	P_1	P_2	P_3	P_4	P_5	P_6
finance	0.0000	0.4210	0.1186	0.0971	0.1162	0.9130
tv series	0.2759	0.8558	0.0825	1.0000	1.0000	0.6101
moive	0.6897	0.4230	0.4742	0.1448	0.2077	0.9241
law	1.0000	0.4562	0.5773	0.1121	0.1313	0.9243
education	0.0690	0.0000	1.0000	0.0000	0.0000	1.0000
other	0.0690	1.0000	0.0000	0.3521	0.2492	0.0000
teenagers	0.0000	0.2326	0.2216	0.0612	0.1202	0.8948
Life service	0.4138	0.9525	0.0979	0.3251	0.2377	0.3660
sports	0.6552	0.4697	0.4124	0.2003	0.2827	0.8629
drama	0.4138	0.1898	0.6495	0.0320	0.0511	0.9790
news	0.7931	0.8091	0.2577	0.5612	0.5563	0.7380
mucis	0.1724	0.3046	0.2784	0.0601	0.0815	0.9634
special	0.2414	0.7639	0.0876	0.3513	0.3392	0.7928
variety	0.5517	0.7231	0.2165	0.3933	0.4121	0.8032

Calculate weight by five kinds of subjective weighting methods and three kinds of objective weighting methods. According to the formula (7) and formula (13) to calculate the *n* dimension row vector \mathbf{B}_1 , \mathbf{B}_2 , respectively

B1 = (67.9310,66.3163,55.9381,51.6448,48.5792,50.4893) **B2** = (5.3448,7.6013,4.4742,3.6907,3.7853,10.7715)

So,

The weight vector from each weighting into formula (22) is obtained $\theta_1^{**}=0.1703$, $\theta_2^{**}=0.1682$, $\theta_3^{**}=0.1698$, $\theta_4^{**}=0.1625$, $\theta_5^{**}=0.1624$, $\theta_6^{**}=0.1668$. So, combination weight vector is

$$\mathbf{W}_{c}^{**} = 0.1703 \mathbf{W}_{1} + 0.1682 \mathbf{W}_{2} + 0.1698 \mathbf{W}_{3} + 0.1625 \mathbf{W}_{4} + 0.1624 \mathbf{W}_{5} + 0.1668 \mathbf{W}_{6}$$
$$= (0.2753, 0.1111, 0.1997, 0.1761, 0.1649, 0.0730)^{T}$$

The evaluation results and rankings from each weighting are shown in Table 8 and Table 9.

Program type	AHP	Rank	binomial coefficient method	Rank	sequential analysis method	Rank	least square method	Rank	rank correlation analysis method	Rank
finance	0.1442	13	0.1555	13	0.0992	13	0.1396	13	0.2147	13
TV series	0.5455	3	0.5949	2	0.4997	5	0.5327	3	0.5978	2
Movie	0.4805	5	0.4443	6	0.5086	3	0.4973	4	0.4713	6
Law	0.5954	2	0.5421	3	0.6576	2	0.6225	2	0.5496	3
Education	0.2306	11	0.2096	11	0.1787	11	0.2175	11	0.2924	10
Other	0.2341	10	0.297	10	0.198	10	0.2217	10	0.267	11
Teenager	0.1208	14	0.1213	14	0.0761	14	0.1154	14	0.1891	14
Life service	0.3836	7	0.4148	7	0.3805	7	0.3857	7	0.393	7
Sports	0.4787	6	0.4492	5	0.5023	4	0.4938	6	0.4733	5
Drama	0.3386	8	0.3061	9	0.3357	8	0.3443	8	0.3584	9
News	0.6397	1	0.6356	1	0.6679	1	0.6538	1	0.6236	1
Music	0.2048	12	0.1975	12	0.1774	12	0.2052	12	0.2561	12
Special	0.3271	9	0.3531	8	0.2937	9	0.3242	9	0.3802	8
Variety	0.4869	4	0.4875	4	0.4919	6	0.4956	5	0.4993	4

Table 8. The evaluation results and rankings (1)

Table 9. The evaluation results and rankings (2)

Program type	coefficient of variation method	rank	mean square deviation method	rank	entropy evaluation method	rank	combination weighting	rank
finance	188.1654	11	0.2812	13	0.1694	13	0.1885	13
TV series	977.9327	1	0.6254	2	0.6423	1	0.589	2
Movie	269.784	8	0.4913	6	0.4058	6	0.4713	6
Law	201.6298	9	0.5511	3	0.4621	3	0.5608	3
Education	84.1257	14	0.3378	10	0.2621	10	0.2625	10
Other	310.8658	6	0.2868	12	0.2549	11	0.2505	11
Teenager	191.8188	10	0.2505	14	0.1552	14	0.164	14
Life service	299.0552	7	0.412	8	0.3481	7	0.3855	7
Sports	336.9619	5	0.4934	5	0.4175	5	0.473	5
Drama	129.8295	13	0.391	9	0.3014	9	0.3464	9
News	581.6123	2	0.6311	1	0.5858	2	0.631	1
Music	157.0325	12	0.3113	11	0.2045	12	0.2359	12
Special	387.6488	4	0.432	7	0.3426	8	0.3611	8
Variety	452.6219	3	0.5275	4	0.4554	4	0.495	4

It can be concluded from the Table 8 and Table 9 that various ways of weighting result in various evaluations. The combination weighting method based on the maximizing deviations and normalized constraint condition assembles the advantages of other methods, which renders the methods of combination weighting more completed and accurate. In the Table 8 and Table 9, the combination weighting based on the maximizing deviations and normalized constraint condition can not only reflect subjective decision, but also simultaneously reflect the objective decision, and can effectively integrate other different methods. Experimental results indicates that weights calculated by different methods are not equal to each other, in which case it's tough to generate a final evaluation of the projects. The way of combination weighting proposed in this paper maximized the final evaluation value and the deviation between value of our method and that of other methods, which makes the final weight coefficient of evaluation stable and reasonable.

6 Conclusion

The confirmation of weight of the evaluation index is a vital link of multiple attribute decision making, whether the evaluation is reasonable plays a pivotal role on the scientificity of evaluation result. The change of a certain weight will influence the overall judgment. Therefore, it must be scientific and objective to settle a weight. This paper primarily summarizes the confirmation of weights available and

analyzes the merits and demerits of all the methods, and subsequently puts forward a new method towards the combination weighting that conducts a combination of the objective and subjective weighting information. On the one hand, this method considers the deviation of weight vectors ascertained by different methods of the weights, makes the overall deviation up to the maximum and scatters the degree of each integrated assessment of objects as much as possible. On the other hand, the method also considers the integrated assessment of objects, and generally the bigger the value is, the better the plan performs. Thus, in order to maximize the sum evaluation, an optimization model of double targets is built. The paper then solves the model and gives the formula of weights. The experimental analysis in the end indicates the rationality of the method. Finally, quantitative analysis is carried out on objective and subjective weighting methods, and screening for weighting methods by optimal selection according to the results of analysis. Experimental analysis shows the rationality of this method.

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