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Describing the Emotional Model of PAD Based on Consistent Covering Granule

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Abstract. The basic concept of granule and consistent covering granule are introduced. The related theorem of consistent covering granule is deduced. The certainty and likelihood of the covering granule based on covering fuzzy theory is proposed. The method of qualitatively and quantitatively describing emotional model of the PAD is constructed using granular model based on consistent covering fuzzy theory. Finally, the examples demonstrate that the proposed methods can describe the emotional granules and non-typical emotions in the emotional model of the PAD effectively.

Keywords: consistent covering granule, covering fuzzy, emotional model of PAD, granule

1 Introduction

In the field of artificial intelligent, it is a focus how intelligent agents are endowed with emotion. Therefore, it is inevitable to research the affective computing model. Researchers are pursuing various methods and theories to quantize the emotions precisely so that the intelligent agents can perform in anthropoid behaviors. Therefore, it is important that the human emotions which are of abstract existence can be processed digitally and quantitatively to accomplish interaction between autonomous affective agents or robots and humans. A. Landowska comes up with conception that emotion recognition which is based on multimodel inputs including physiological sensors, video, depth sensors etc. takes into effect by classifier which maps the human emotions into the discrete model with six basic emotions including joy, sadness, fear, anger, disgust and surprise. The proposed model that is applied in the education and gaming performs well [1]. Hong proposes the emotional making-decision model called Emotional-Belief-Desire-Intention architecture in which the agent is endowed with six level emotions given to the set containing hate, dislike, unhappy, happy, like, love. The agent with EBDI model moved more effectively in the Tileworld domain [2]. Hommel presents a classifier which can recognize the 7 basic emotions including neutral, happy, sad, disgust, surprise, fear, anger, and map each image into an one-dimensional space. The classifier applied in service robotic can analyze the continual change of the humans emotional state in human-computer interaction [3]. Kim conceives an emotion generation model based on simplification of OCC model so as to activate the robots in believability and feasibility when deciding an emotional state using stochastic approach. And the robot can express six basic emotions smoothly and realistically [4]. Zhang et al. make use of the PAD psychological model to describe arbitrary emotional state in which a set of partial expression parameters is proposed to describe the typical facial expressive movement in regions. And in the PAD emotional space, there are 7 basic emotions introduced for annotating. The maps are built among PAD emotional space, the emotion expression parameters and facial animation parameters. So the talking avatar speaks vividly with the synthetic emotional facial expression integrated with speech [5]. Embgen designs a human-like robot which can express the emotions integrating facial expressions, sound, gestures and full body posture. When communicating with human, the robot conveys not only the basic primary emotions, but also the more complex

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secondary emotions, where the set of emotions are made of the following emotions: anger, disgust, fear, happiness, sadness and surprise, some emotions of which are primary emotions including happiness, sadness and fear, the others of which are secondary emotions including anger, disgust and surprise [6]. Kühnlenz et al. present the dimensional approaches used for evaluation of an emotion expressing robots and the PAD test based on a semantic differential approach is applied to depict the emotions of the robot [7]. Adams A proposes a classifier that can discern complex affective states with inputting the signals of facial expressions and head motions. The automated emotion classifiers used the categorical models to map the human behaviors into the discrete emotions based on the primary-process theory that a limited amount of emotions are embedded in our brain. The recognizing accuracy achieves 0.74 in human-computer interaction [8]. Boccanfuso et al. design a new and affective interface combining sound, color, movement and context to simulate the expression of emotions, with which a robot can interact with young children in the play environment autonomously and intelligently by expressing four basic emotions including happiness, fear, anger, sadness [9]. Kaiser designs an e-learning system enhanced with affective abilities when using an emotion recognition sensor system, where an affective component which can only recognize the human emotions between positive emotions and negative emotions plays an important role in affective communication and actions, in order to reduce the negative emotions of user and enhance the positive emotions [10]. As we learn above, in a large amount of emotional models proposed by researchers, there are common shortages that whatever theories the emotional models are designed based on, human affection is always mapped into the discrete space denoting the limited basic emotions simply. However, the human affection characterized by uncertainty and ambiguity presents subtle, complex and continuous, thereby it is obvious that the basic emotions in a discrete space can't reflect the actual human affection completely.

The emotion model of PAD initially proposed by Gebhard can describe the emotional states effectively by mapping the human abstract emotions into the space composed of the three dimensional of P (Pleasure), A(Arousal) and D (Dominance) and quantize the specific emotion using the corresponding point locating in the coordinate of PAD, in which the P(Pleasure) denotes whether the individual emotional state belongs to positive or negative character, the A(Arousal) denotes the individual neurophysiology activating level, and D(Dominance) denotes the dominating state. Due to the advantages above, the PAD plays an important role in combining the emotion and computation so that the abstract emotion can be processed as a mathematic model. But there are also some problems for the PAD model in nature, such as: (1) The points in the emotional coordinate space defined by the PAD model represent the emotions in discrete way while the emotions behave in continuous way in fact. (2) The emotions are compound and delicate, which are comprised by more than two basic emotions while the PAD model fails to describe the compound emotions above. (3) The PAD model cannot reflect the relationship among emotions accurately due to the fact that there are differences when one emotion transfers from another emotion with the relationships existing closely or distantly. (4) Person always makes use of knowledge about emotion in different hierarchy when recognize the emotion. For example, in the macroscopic rank, the emotions are divided into positive emotions and negative emotions, and in the next rank, the positive emotions are divided into happiness and easiness etc., while the negative emotions are divided into anger and nervousness etc. Hence, the PAD model should be improved to satisfy requirements for describing uncertainty of emotion effectively.

The rough set theory presented by Pawlak is suitable for handling the uncertainty, which generalize classical set theory to settle the uncertainty question and had demonstrated that it had advantages and effectiveness [11]. Pawlak's rough set is built on division of the universe, which the relation of equivalence corresponded with. However, it cannot afford to solve practical problems because of rigorous constraint. Thus, researchers extended the rough set to propose a covering rough set by replacing relation of equivalence with consistency relation [12-14]. Yao put forward the granular computing model based on rough set theory [15]. Zadeh presented the granular computing as a terminology firstly [16]. The rough set and covering rough set which was presented by Zakowski firstly is regarded as one of the most important granular computing, which is being researched as hot topics. Consistent covering granule based on covering rough set theory applies to construct the affective computing model, which can describe the emotion characterized by fuzzy and stochastic feature from granular viewpoint. On this basis, we combine the PAD model with consistent covering granule and propose an affective computing model for the first time so that the emotions can be described effectively.

In our model, besides the given 24 emotions in PAD space, arbitrary point also corresponds an emotion which locates vacant space among the 24 emotions. Firstly, we proposed a concept of the emotional covering fuzzy set to describe what emotional state any one point stands for. Then the consistent covering granule is introduced to classify the 24 emotions into different categories with different hierarchical consistency relation. For example, with the highest hierarchical consistency relation, the 24 emotions can be categorized into two emotions which are positive emotions and negative emotions respectively, as means there are two consistent covering granules to cover the emotions in the PAD space, while with a lower hierarchical consistency relation, the 24 emotions can be categorized into 24 emotions which are themselves, as means there are 24 consistent covering granules to cover the emotions in the PAD space. Subsequently, we defined the certainty and likelihood, which specify to what extent a given granule certainly/likely belong to the emotional covering fuzzy set which one point corresponds. As a result, any emotion mapped into the PAD space on conditions that the points locate in the PAD space with continuity and ambiguity can also be expressed digitally and mathematically. That will be beneficial to build the affection recognition model and affection expression model, where an agent or robot is required to recognize and express the emotions ambiguously and synthetically.

The rest of this paper is organized as fallows: Section 2 introduces the basic concept of granule. Section 3 introduces consistent covering granule based on consistent covering theory. In section 4, we proposed the fuzzy set based on covering granule. In section 5, we apply the fuzzy set based on covering granule in describing emotion in the emotional space of PAD. There are some examples presented to verify how the proposed model works. In the last section, there are some conclusions for the proposed model.

2 The Basic Concept of Granule

Definition 2.1 Assuming U as a nonempty finite set, which is named as universe. Defining P(U) as a power set for U. $R: U \to P(U)$ denotes a relation of equivalence for U. U is classified by R to form the division $U/R = \{[u]_R / u \in U\} = \{U_1, U_2, U_i \cdots U_m\}$, which is named as equivalent class or basic set of R, and $U_1, U_2, U_i \cdots U_m$ is named as the granules of R [17]. There are three formulas deduced from the definition above:

(1) $U_i \subseteq U, U_i \neq \varnothing$,

(2) $U_i \bigcap U_j = \varnothing, i \neq j, i, j = 1, 2, 3 \cdots m$,

(3) $\bigcup_{i=1}^{m} U_i = U$.

Definition 2.2 Defining the granularity of R presented as follows [18]:

$$G(R) = \frac{|R|}{|U \times U|} = \frac{|R|}{|U|^2}.$$

Where $|A|$ denotes the cardinal number when U is discrete universe while $|A|$ equals to $\int_A dx$ when U is continuous universe.

Theorem 2.1 Assuming R as an equivalent relation for U, by which the division $U/R = \{U_1, U_2, \cdots U_m\}$ is classified, then:

$$G(R) = \frac{\sum_{i=1}^{m} |U_i|^2}{|U|^2}. \quad \textbf{(1)}$$

The process is omitted.

Property 2.1 Assuming R as an equivalent relation for U, then:

$$\frac{1}{|U|} \leq G(R) \leq 1. \quad \textbf{(2)}$$

Proof: the granularity of R is minimum, whose formula is $G(R)=\frac{\sum_{i=1}^{m}|U_i|^2}{|U|^2}=\frac{|U|}{|U|^2}=\frac{1}{|U|}$, when R is equivalent relation, while the granularity of R is maximum, whose formula is $G(R)=\frac{\sum_{i=1}^{m}|U_i|^2}{|U|^2}=\frac{|U|^2}{|U|^2}=1$, when R is universe relation.

Definition 2.3 Assuming R as an equivalent relation family for U, if $x,y\in U$, $xR_1y\Rightarrow xR_2y$, $R_1,R_2\in R$, then there is a conclusion that R_1 is thinner than R_2, as is expressed by $R_2\prec R_1$.

Property 2.2 Assuming R_1,R_2 as two equivalent relations for U, if $R_2\prec R_1$, then $G(R_1)<G(R_2)$.

3 Consistent Covering Granule

Definition 3.1 Assuming U as a universe and $C=\{C_1,C_2,C_i\cdots C_m\}$ as the subset family of U. If the subset is nonempty and satisfies $\bigcup C_i=U$, then C is regarded as a covering of U. It is self-evident truth that the concept of covering is generalized from the concept of division in Definition 2.1. But the formula 2) in Definition 2.1 is violated here, in other words, when $\exists i,j,i\neq j$, then $C_i\bigcap C_j\neq\varnothing$. And $C_1,C_2,C_i\cdots C_m$ also are defined as covering granules [19].

Definition 3.2 Assuming U as a universe and C as a covering of U, then the (U,C) is defined as a covering approximating space [19].

Definition 3.3 Assuming (U,C) as a covering approximating space, then $x\in U$, $Md(x)=\{K\in C/x\in K\wedge(\forall S\in C\wedge x\in S\wedge S\subseteq K\Rightarrow K=S)\}$ is regarded as minimum description [19].

Definition 3.4 Assuming (U,C) as a covering approximating space with $K\in C$, if K is the result of the union of several components which belong to $C-\{K\}$, then K is regarded as a reducible component of C, otherwise, a non-reducible component of C [19].

Definition 3.5 Assuming (U,C) as a covering approximating space, if any one component in C is non-reducible component, then C is reduced, it means that C is a minimum covering of U, otherwise C is reducible [19].

Proposition 3.1 Assuming (U,C) as a covering approximating space, if K is the reducible component of C, then $C-\{K\}$ is a covering of U yet [19].

Proposition 3.2 Assuming (U,C) as a covering approximating space and $K\in C$ as a reducible component of C, for any K_1 with $K_1\in C-\{K\}$, if and only if K_1 is the reducible component of $C-\{K\}$, then K_1 is regarded as a reducible component of C [19].

Definition 3.6 For a covering C in U, there is a new covering formed by reduction in methods of Proposition 3.1 and Proposition 3.2 [19].

With the computation of reduction of a covering, there will be a minimum covering with which the old covering can form the same covering upper and lower approximations.

Definition 3.7 Assuming TR as a binary relation for U, if TR satisfies the reflexivity and symmetry, then regarded as a consistency relation [20].

Definition 3.8 Assuming $TR:U\rightarrow P(U)$ as a consistency relation and P(U) as the power sets for U, there is a covering $U/TR=\{[x]_{TR}/x\in U\}=\{C_1,C_2,C_i\cdots C_m\}$, produced by consistency relation TR for U. And C_i is regarded as consistent covering granule satisfying the consistency relation TR and conforms to the following formula [20]:

$$\forall x,y\in C_i\quad TR(x,y)=1.$$

If the any other element z which is excluded from C_i for U is nonexistent and conforms to the following formula:

$$\forall x \in C_i \quad TR(x,z) = 1 .$$

then C_i is regarded as a maximum consistent covering granule with consistency relation TR for U. According to Definition 3.1, $\bigcup C_i = U$ and $C_i \bigcap C_j \neq \varnothing$ when $\exists i,j,i \neq j$ are deduced. As a result, U and $C = \{C_1, C_2, C_i \cdots C_m\}$ constitute a (U,C) which is a consistent covering approximation space.

Proposition 3.3 Assuming TR as a consistency relation for U, the consistent covering, which is composed of maximum consistent covering granule with traversing all elements in U, is regarded as the sole maximum consistent covering satisfying the consistency relation TR.

Theorem3.1 Assuming (U,C) as a consistent covering approximation space, in which C is a maximum consistent covering satisfying the consistency relation TR for U, then C is regarded as reduction.

Proof: if the maximum consistent covering C satisfying the consistent relation TR is not reduction, then

$$\exists C_i, \cdots C_j, C_k \subset C , C_i \bigcup \cdots \bigcup C_j = C_k, i, \cdots j, k \in I, i \neq \cdots \neq j \neq k .$$

in which I is set of subscript satisfying $C_i \bigcup \cdots \bigcup C_j = C_k, i, \cdots j, k \in I, i \neq \cdots \neq j \neq k$.

Then $\exists C_i, C_i \subseteq C_k, \cdots, \exists C_j, C_j \subseteq C_k \Rightarrow \exists x_i, x_i \in C_i \subset C_k, \cdots, \exists x_j, x_j \in C_j \subset C_k \Rightarrow TR(x_i, x_j) = 1, i, j \in I, i \neq j$.

Additionally, because of C_i being a maximum consistent granule under condition of TR, according to definition 3.8, there is no any other elements x_j satisfying $TR(x_i, x_j) = 1, i, j \in I, i \neq j$, with $x_i \in C_i$, which is excluded from C_i for U. It is contradictory for two conclusions above.

Therefore, for a consistent covering approximation space (U,C), if C is a maximum consistent covering satisfying the consistency relation TR for U, then C is regarded as reduction.

Definition 3.9 Assuming U as a universe with $\forall x, y, z \in U$, if function $d(x, y/\omega)$ satisfies the fallowing formulas [20]:

(1) $d(x, y/\omega) \geq 0$,

(2) $d(x, x/\omega) = 0$,

(3) $d(x, y/\omega) = d(y, x/\omega)$,

(4) $d(x, y/\omega) \leq d(x, z/\omega) + d(z, y/\omega)$.

then it is regarded as a function of distance for U, in which the elements constituting U is a vector with n-dimensions and ω is the weight of dimensions.

Theorem 3.2 Assuming U as a universe and $d(x, y/\omega)$ as a function of distance with $x, y, z \in U$, if $TR(x, y/r, \omega) = \begin{cases} 1 & d(x, y/\omega) \leq r \\ 0 & others \end{cases}$, in which $r \geq 0$, then $TR(x, y/r, \omega)$ is regarded as a consistent relation for U.

Proof: if $TR(x, x/r, \omega) = 1$, then $TR(x, y/r, \omega)$ is reflexivity. For $x, y \in U$, if $d(x, y/\omega) \leq r$ and $d(y, x/\omega) \leq r$, then $TR(x, y/r, \omega) = TR(y, x/r, \omega) = 1$. For $x, y \in U$, if $d(x, y/\omega) > r$ and $d(y, x/\omega) > r$, then $TR(x, y/r, \omega) = TR(y, x/r, \omega) = 0$. Namely, there is $TR(x, y/r, \omega) = TR(y, x/r, \omega)$ for $\forall x, y \in U$ and $TR(x, y/r, \omega)$ is symmetry. According Definition 3.7, $TR(x, y/r, \omega)$ is regarded as a consistency relation for U.

Definition 3.10 Assuming S as a nonempty set and R as a relation for S, for $x, y, z \in S$, if it satisfies (1) xRx, (2) $xRy \Rightarrow y \not R x$, (3) $xRy, yRz \Rightarrow xRz$, then regarded as partially-order. The S and R constitute a set of partially ordered set, denoted by (S,R) [21].

Definition 3.11 Assuming (S,R) as a partially ordered set, for $x, y \in S$, if xRy or yRx, then x and y are comparable, otherwise noncomparable [21].

Definition 3.12 Assuming (S,R) as a partially ordered set, for $\forall x, y \in S$, if x and y are comparable, then (S,R) is regarded as a total-ordered set [21].

Definition 3.13 Assuming (S,R) as a total-ordered set, for $\forall x, y \in S$, if $\{x, y\}$ exists $\inf\{x, y\} \in S$ and

$\sup\{x,y\}\in S$, then (S,R) is regarded as total-ordered lattice. $\inf\{x,y\}\in S$ and $\sup\{x,y\}\in S$ denote the supremum and infimum of $\{x,y\}$ respectively [21].

Definition 3.14 Assuming TR as the consistency relation entirety specified in theorem 3.2, for $TR_1, TR_2 \in TR$, $x,y\in U$, if $TR_1(x,y)\Rightarrow TR_2(x,y)$, then it is called that TR_1 is thinner than TR_2, namely $[u]_{TR_1}\subseteq[u]_{TR_2}$, in which $\{[u]_{TR}\}$ with $u\in U$ is regarded as the maximum consistent covering satisfying consistency relation TR, as is denoted $TR_2\prec TR_1$.

Definition 3.15 If $[u]_{TR_1}\subseteq[u]_{TR_2}$, then it is called that $[u]_{TR_1}$ is paternal covering granule.

Proposition 3.4 Assuming D as the set of positive integer and $TR(x,y/r,\omega)$ as the consistency relation entirety for U specified in theorem 3.2, if $d_1\le d_2$ with $d_1,d_2\in D$, then $TR(x,y/d_2,\omega)\prec TR(x,y/d_1,\omega)$.

Proof: It is deduced from Theorem 3.2 naturally.

Theorem 3.3 Assuming $TR(x,y/r,\omega)$ as the consistency relation entirety for U specified in Theorem 3.2, $(TR(x,y/r,\omega),\prec)$ is regarded as a complete total-ordered lattice, in which $\prec$ denotes the relation defined in definition 3.14.

Proof: The $(D,\le)$ is a total-ordered set, as is defined in Definition 3.12, where $D\in[0,\infty)$. For $\forall d_1,d_2\in D$, there are always $\inf\{d_1,d_2\}=\max(d_1,d_2)\in D$ and $\sup\{d_1,d_2\}=\min(d_1,d_2)\in D$, then the $(D,\le)$ is total-ordered lattice defined in Definition 3.13. Thus the $(TR(x,y/r,\omega),\prec)$ is a total-ordered set defined by definition 3.12. The $f:d\to TR$ is deduced from Theorem 3.2, for $\forall d_1,d_2\in D$, the consistent covering granule $[u]_{TR(x,y/d_1,\omega)}$ and $[u]_{TR(x,y/d_2,\omega)}$ exists, then

$$\inf\{[u]_{TR(x,y/d_1,\omega)},[u]_{TR(x,y/d_2,\omega)}\}=[u]_{TR(x,y/d_1,\omega)}\bigcup[u]_{TR(x,y/d_2,\omega)}=[u]_{TR(x,y/\max(d_1,d_2),\omega)}\in\{[u]_{TR(x,y/r,\omega)}\}.$$

and $$\sup\{[u]_{TR(x,y/d_1,\omega)},[u]_{TR(x,y/d_2,\omega)}\}=[u]_{TR(x,y/d_1,\omega)}\bigcap[u]_{TR(x,y/d_2,\omega)}=[u]_{TR(x,y/nin(d_1,d_2),\omega)}\in\{[u]_{TR(x,y/r,\omega)}\}.$$

then $(TR(x,y/r,\omega),\prec)$ is regarded as a complete total-ordered lattice according to Definition 3.13.

According to Theorem 3.3, given a universe U, the $\{[u]_{TR(x,y/r,\omega)}\}$ classified by $TR(x,y/r,\omega)$ can constitute a complete total-ordered lattice based on inclusion relation so that the operation of synthesis and decomposition can be carried out in the lattice in multi-granularity with freedom.

4 The Fuzzy Set Based on Covering Granule

Definition 4.1 Assuming U as a universe, A is a fuzzy subset when $\forall x\in U$ and $\exists\mu_A\in[0,1]$ that describes to what extent x is subordinate to A. Mapping $\mu_A:U\to[0,1]$, $x\to\mu_A(x)$ is regarded as membership function of A. The fuzzy subset also is abbreviated as fuzzy set. $F(U)$ denotes fuzzy power set which is fuzzy subset entirety for U [19].

Definition 4.2 Given a normalized distance function $d(x,y/\omega)$ defined in Definition 3.9 for U, there is a corresponding fuzzy set $\tilde{A}$ for any one set A for U, then the membership function is $A(x)=\sup\{1-d(x,y/\omega)/y\in A\}$.

The Definition 4.2 expands the general set A to fuzzy set $\tilde{A}$ which takes A as core by the normalized distance function.

Definition 4.3 Assuming $A\in F(U)$, then the $\mathrm{supp}(A)=\{x/x\in U,\mu_A(x)>0\}$ denotes the support of A and $\mathrm{kernel}(A)=\{x/x\in U,\mu_A(x)=1\}$ denotes the core of A.

Definition 4.4 Assuming (U,C) as a covering approximation space with $X\subseteq U$ and $\forall x\in U$, then $D(x,X)=\dfrac{|(\bigcup Md(x))\bigcap X|}{|\bigcup Md(x)|}$ is regarded as inclusion degree which describes to what extent the x is included in X.

Definition 4.5 Assuming U as universe and C as a covering for U, then $\mu_A^{'}(x)$ with $A\in F(U)$ and $x\in U$ is regarded as fuzzy covering membership which describes to what extent x is subordinate to A, when $\mu_A^{'}(x)=\frac{\sum_{y\in\bigcup Md(x)}A(y)}{\left|\bigcup Md(x)\right|}$.

The fuzzy covering membership reflects the relationship between the elements and their minimum description, and considers the membership of elements in given fuzzy set and in their minimum description, which is a synthesized indication describing to what extent every element in universe is subordinate to A.

Definition 4.6 Assuming U as universe and C as a covering for U, then the membership functions of upper and lower approximations with A related to the covering approximation space (U,C) are as follows:

$$\overline{\mu}_A(x)=\max\{\mu_A(x),\mu_A^{'}(x)\}\,,\ \underline{\mu}_A(x)=\min\{\mu_A(x),\mu_A^{'}(x)\}\,.$$

then $\overline{\mu}_A(x)$ and $\underline{\mu}_A(x)$ are regarded as covering fuzzy set with A related to covering C.

Definition 4.7 Assuming U as universe and $C=\{C_1,C_2,C_i\cdots C_m\}$ as the maximum consistent covering, then the certainty is defined, which describes to what extent the consistent covering granule C_i is definitively included in A with $A\in F(U)$ and in the covering approximation space (U,C), likewise the likelihood is defined, which describes to what extent the consistent covering granule C_i is probably included in A with $A\in F(U)$ and in the covering approximation space (U,C). The certainty and likelihood are presented as follows:

$$Bel_A(C_i)=\min_{x\in C_i}\{\underline{\mu}_A(x)\}\,,\ PI_A(C_i)=\max_{x\in C_i}\{\overline{\mu}_A(x)\}\,.$$

5 The Consistent Covering Granule Description for Emotional Space of PAD

The coordinate axis for the emotional space of PAD stand for implications as fallows: +P indicates pleasure while –P indicates non-pleasure, +A indicates arousal while –A indicates non-arousal, +D indicates dominance while –D indicates non-dominance. The number range on the coordinate axis is [－1, ＋1], where -1 means that the value is lowest for corresponding dimension while +1 means that the value is highest for corresponding dimension. For example, the point locating at (0.2, 0.2, -0.1) in the coordinate denotes hope. With the method of the quantization above, the relationship between emotions in OCC and the points in the PAD exists one-to-one correspondence [22], as is shown in Table 1.

It is necessary that the space composed of PAD is regarded as universe of the researched object to describe the emotional states in space of PAD by means of consistent covering granule model. The universe is a set of emotional states as the table 1 shows.

Definition 5.1 Assuming TR as a consistency relation for $U:P\times A\times D$, by which the $U:P\times A\times D$ is classified into a maximum consistent covering $U/TR=\{[x]_{TR}/x\in U\}=\{C_1,C_2,C_i\cdots C_m\}$, then C_i is regarded as emotional granule for $U:P\times A\times D$ satisfying consistency relation TR.

Theorem 5.1 Assuming TR as a consistency relation for $U:P\times A\times D$ and $d(x,y/\omega)$ as a normalized distance function defined in Definition 3.9, in which $x=(x_P,x_A,x_D)\in U:P\times A\times D$, $y=(y_P,y_A,y_D)\in U:P\times A\times D$. If $TR(x,y/r,w)\Leftrightarrow d(x,y/\omega)\le r$ with $r\ge 0$, then $TR(x,y/r,w)$ is regarded as a consistency relation for $U:P\times A\times D$.

Proof: The conclusion can be deduced from Theorem 2.2 naturally.

Definition 5.2 The $U:P\times A\times D$ is classified into a maximum consistent covering $U/TR=\{[x]_{TR}/x\in U\}=\{C_1,C_2,C_i\cdots C_m\}$ by the consistency relation $TR(x,y/r,\omega)$, which is specified in theorem 3.2 for $\forall d\in[0,\infty)$, then C_i is regarded as emotional granule of $TR(x,y/d,\omega)$, and the corresponding emotional granularity defined as follows:

Table 1. The relationship between emotions in OCC and the points in the PAD

NO.	EMOTION	P	A	D
1	admiration	0.5	0.3	-0.2
2	angry	-0.51	0.59	0.25
3	disgust	-0.4	0.2	0.1
4	disappoint	-0.3	0.1	-0.4
5	pain	-0.4	-0.2	-0.5
6	afraid	-0.64	0.6	-0.43
7	fear	-0.5	0.3	-0.7
8	gloat	0.3	-0.3	-0.1
9	satisfaction	0.6	0.5	0.4
10	grateful	0.4	0.2	-0.3
11	happy	0.4	0.2	0.2
12	hate	-0.6	0.6	0.3
13	hope	0.2	0.2	-0.1
14	delight	0.4	0.2	0.1
15	like	0.4	0.16	-0.24
16	love	0.3	0.1	0.2
17	sympathy	-0.4	-0.2	-0.5
18	arrogance	0.4	0.3	0.3
19	relaxed	0.2	-0.3	0.4
20	remorse	-0.3	0.1	-0.6
21	reproach	-0.3	-0.1	0.4
22	resentment	-0.2	-0.3	-0.2
23	contentment	0.3	-0.2	0.4
24	shame	-0.3	0.1	-0.6

$$G(TR(x, y/d, \omega)) = \frac{\sum_{i=1}^{m} |C_i|^2}{|U|^2}.$$

Theorem 5.2 Assuming TR as a consistency covering for $U: P \times A \times D$, then there is $d_1 \le d_2 \Leftrightarrow G(TR(x, y/d_1, \omega)) \le G(TR(x, y/d_2, \omega))$ for $\forall d_1, d_2 \in [0, \infty)$.

Proof: (1) Proof of sufficiency: $d_1 \le d_2 \Rightarrow TR(x, y/d_2, \omega) \prec TR(x, y/d_1, \omega)$ is deduced from Proposition 3.4, then $[u]_{TR(x,y/d_1,\omega)} \subseteq [u]_{TR(x,y/d_2,\omega)}$, $\sum \left|[u]_{TR(x,y/d_1,\omega)}\right|^2 \le \sum \left|[u]_{TR(x,y/d_2,\omega)}\right|^2$ with $\forall u \in U$, as a result, $d_1 \le d_2 \Rightarrow G(TR(x, y/d_1, \omega)) \le G(TR(x, y/d_2, \omega))$. (2) Proof of necessity: Assuming $G(TR(x, y/d_1, \omega)) \le G(TR(x, y/d_2, \omega))$, if $d_1 > d_2$, then $TR(x, y/d_2, \omega)) \Rightarrow G(TR(x, y/d_1, \omega)$ is deduced from Theorem 3.2. $[u]_{TR(x,y/d_2,\omega))} \subset [u]_{TR(x,y/d_1,\omega))}$, $\forall u \in U$ is deduced, then $\sum \left|[u]_{TR(x,y/d_2,\omega)}\right|^2 < \sum \left|[u]_{TR(x,y/d_1,\omega)}\right|^2$, $G(TR(x, y/d_1, \omega)) >$ $G(TR(x, y/d_2, \omega))$. This result is contradictory to the assumption.

Deduction 5.1 If $G(TR(x, y/d_1, \omega)) \le G(TR(x, y/d_2, \omega))$, then $TR(x, y/d_2, \omega) \prec TR(x, y/d_1, \omega)$.

Example 5.1 When $d_1 = 0.15$ and $w = [1,1,1]$, please solve the emotional granule and granularity of $TR(x, y/r, w)$ in emotional space of PAD.

According to the Theorem 5.1, the consistency relation matrix constructed by $TR(x, y/r, w)$ is as the follow:

$$TR(x,y/0.15,w)=\begin{bmatrix} 1 & 0.47 & 0.36 & \cdots & 0.53 \\ 0.47 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & 1 & \cdots & \vdots \\ 0.22 & 0.39 & 0.26 & \cdots & 1 \end{bmatrix}.$$

According to Proposition 3.3 and Theorem 3.1, the emotional granules of $TR(x,y/r,w)$ are given as follows:

{{admiration, grateful, happy, hope, delight, like, love}, {angry, disgust, hate}, {disgust, disappoint}, {disappoint, pain, fear, sympathy, remorse, shame}, { afraid, fear}, { gloat, grateful, hope, delight, like, love}, { content, happy, delight, love, arrogance }, {admiration, happy, hope, delight, like, love, arrogance }, { gloat, love, relax, content}, {disgust, reproach}, {disappoint, pain, sympathy, resentment }, {gloat, happy, love, content}}.

As we see above, the emotional granules divided by $TR(x,y/r,w)$ describe the emotional states on the high level, which are abstracted and classified from the viewpoint of coarse-granularity. For example, {admiration, grateful, happy, hope, delight, like, love} is an emotional granule filled with positive emotions. {disgust, disappoint} is an emotional granule showing discontented faces to someone else. {disappoint, pain, fear, sympathy, remorse, shame} has intense psychological activities. The reason why the disappointment is in two different emotional granules is that it can be regarded as emotion showing discontent outwards when belonging to {disgust, disappoint}, while it can be regarded as a mental activity when belonging to {disappoint, pain, fear, sympathy, remorse, shame}. The overlapping phenomenon just reflects the feature of covering granule.

According to definition 5.2, the emotional granularity is given as follows:

$$G(TR(x,y/0.15,\omega))=\frac{\sum_{i=1}^{m}|C_i|^2}{|U|^2}=0.4583\,.$$

Example 5.2 When $d_1=0.3$ and $w=[1,1,1]$, please solve the emotional granule and granularity of $TR(x,y/r,w)$ in emotional space of PAD, compare with $d_1=0.15$ and explain the relationship between the two granules.

According to the Theorem 5.1, the consistency relation matrix constructed by $TR(x,y/r,w)$ is as the follow:

$$TR(x,y/0.3,w)=\begin{bmatrix} 1 & 0.47 & 0.36 & \cdots & 0.53 \\ 0.47 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & 1 & \cdots & \vdots \\ 1 & 0.39 & 1 & \cdots & 1 \end{bmatrix}.$$

According to Proposition 3.3 and Theorem 3.1, the emotional granules of $TR(x,y/r,w)$ are given as follows:

{{admiration, gloat, grateful, happy, hope, delight, like, love, arrogance, content}, {angry, disgust, disappoint, afraid}, {disgust, disappoint, pain, afraid, fear, sympathy, remorse, shame}, {admiration, content, grateful, happy, hope, delight, like, love, arrogance, satisfaction}, {angry, disgust, afraid, resentment}, {disgust, happy, hope, delight, love, arrogance, relax, reproach}, {angry, disgust, hope, reproach}, {disgust, disappoint, pain, fear, sympathy, remorse, resentment, shame}}.

According to Definition 5.2, the emotional granularity is given as follows:

$$G(TR(x,y/0.3,\omega))=\frac{\sum_{i=1}^{m}|C_i|^2}{|U|^2}=0.7639\,.$$

Because of $G(TR(x,y/0.15,\omega))<G(TR(x,y/0.3,\omega))$, there is $TR(x,y/0.3,\omega)\prec TR(x,y/0.15,\omega)$ according to Deduction 5.1, which means that the emotional granule of $d_1=0.15$ is thinner than the

emotional granule of $d_1 = 0.3$, and $[u]_{0.3}$ is the parental granule of $[u]_{0.15}$. In other words, $[u]_{0.3}$ describes the emotional state more macroscopically than $[u]_{0.15}$, which demonstrates the person's ability to cognize and understand the emotion with multi-granularity.

As the table 1 shows, in the space of PAD, the 24 basic emotional states are quantized with 24 points. However, except the 24 points, others have no emotional implication. Meanwhile, those points are given no exact semantics to describe and define them under current emotional model of PAD, while it is indispensable to endow these points with emotional implications realistically. For example, in the speech emotion recognition system, it demands that the mapping model between speech emotion and emotional space of PAD should be constructed to quantize the speech emotion in emotional space of PAD by which it can be recognized. Besides building the adequate mapping model, there are more critical problems how to express the speech emotion perfectly. Generally, the speech emotion is mapped into the space of PAD on the 24 points so that the quantized speech emotion is discredited artificially, which cannot afford to express the emotion abundantly and exquisitely. Fortunately, the model of consistent fuzzy set is adept in describing the details of emotion effectively.

Definition 5.3 Assuming $f : E \to U : P \times A \times D$ as a mapping model between emotion and space of PAD, when the $\Delta E(E_1, n)$ is mapped into space of PAD by f to be ΔU, then a fuzzy set $\Delta\tilde{U}$ is obtained as follows:

$$\Delta U(x) = \sup\{1 - d(x, y/\omega) / y \in \Delta U\}.$$

Where $\Delta E(E_1, n) = \{E_1 + \Delta E_1 \quad E_1 + \Delta E_2 \quad \cdots \quad E_1 + \Delta E_n\}$ denotes the drifting phenomena when person wants to show the emotion E_1. $d(x, y/\omega)$ is a normalized distance function defined in $U : P \times A \times D$. The $\Delta\tilde{U}$ derived from $\Delta U = \{u_1 \quad u_2 \quad \cdots \quad u_n\}$ can denote emotional implication. And the $\Delta U(x) = \sup\{1 - d(x, y/\omega) / y \in \Delta U\}$ is regarded as an emotional fuzzy set where E_1 is mapped into emotional space of PAD.

Definition 5.3 Specifying a rule that any imported outer emotion has a corresponding fuzzy set to show when mapped into emotional space of PAD. Furthermore, the emotional implications of fuzzy set need to be explained in advance. The points except the 24 basic points have no definite implication and computable formalized model in emotional space of PAD. While consistent fuzzy set can describe any fuzzy subset for U in emotional space of PAD from viewpoint of consistent granule which includes the specific emotional fuzzy set defined in Definition 5.3.

Definition 5.4 Assuming $TR(x, y/r, w)$ as a consistency relation for $U : P \times A \times D$, $C = \{[x]_{TR} / x \in U\}$ $= \{C_1, C_2, C_i \cdots C_m\}$ as the corresponding maximum consistent covering and $\Delta\tilde{U}$ as a fuzzy set defined in Definition 5.3, then the upper and lower approximations membership function for $\Delta\tilde{U}$ relating to the covering approximation space (U, C) are as follows:

$$\Delta\bar{U}(x) = \max\{\Delta U(x), \Delta U'(x)\}, \ \Delta\underline{U}(x) = \min\{\Delta U(x), \Delta U'(x)\}.$$

Then $\Delta\bar{U}(x)$ and $\Delta\underline{U}(x)$ is regarded as emotional covering fuzzy set for $\Delta\tilde{U}$ relating to $TR(x, y/r, w)$, where $\Delta U'(x) = \dfrac{\sum_{y \in \bigcup Md(x)} \Delta U(y)}{\left|\bigcup Md(x)\right|}$.

Definition 5.5 Assuming $TR(x, y/r, w)$ as a consistency relation for $U : P \times A \times D$, $C = \{[x]_{TR} / x \in U\}$ $= \{C_1, C_2, C_i \cdots C_m\}$ as the corresponding maximum consistent covering and $\Delta\tilde{U}$ as a fuzzy set defined in Definition 5.3, then the certainty describing to what extent C_i certainly belong to $\Delta\tilde{U}$ in the (U, C) and likelihood describing to what extent the C_i likely belong to $\Delta\tilde{U}$ in the (U, C) are as follows:

$$Bel_{\Delta\tilde{U}}(C_i) = \min_{x \in C_i}\{\Delta\underline{U}(x)\}, \ PI_{\Delta\tilde{U}}(C_i) = \max_{x \in C_i}\{\Delta\bar{U}(x)\}.$$

Example 5.3 $\Delta U = \{u_1 \quad u_2 \quad u_3 \quad u_4\}$ with $u_1 = (0.32, 0.24, 0.25)$, $u_2 = (0.36, 0.25, 0.22)$, $u_3 = (0.34, 0.12, 0.28)$, $u_4 = (0.27, 0.14, 0.23)$ is the result that the emotional E_1 is mapped into the emotional space of PAD by f . C is a maximum consistent covering derived from $U: P \times A \times D$ using consistency relation $TR(x, y / r, w)$, where $d = 0.15$. Please solve the emotional state under the (U, C) .

(1) Solving the corresponding emotional fuzzy set $\Delta\tilde{U}$ of E_1 .

$$\Delta U(x) = \sup\{1 - d(x, y / \omega) / y \in \Delta U\} .$$

(2) Solving the maximum consistent covering C using $TR(x, y / 0.15, w)$.

The result of C can refer to the Example 5.1.

(3) Solving the upper and lower approximations membership function for $\Delta\tilde{U}$ relating to the covering approximation space (U, C) .

$$\Delta\overline{U}(x) = \sup\{\Delta U(y), y \in Md(x)\} ,\ \Delta\underline{U}(x) = \inf\{\Delta U(y), y \in Md(x)\} .$$

(4) Solving certainty and likelihood of C_i .

Let's take the C_1 for example.

C_1 : {admiration, grateful, happy, hope, delight, like, love}.

$$Bel_{\Delta\tilde{U}}(C_1) = \min_{x \in C_1}\{\Delta\underline{U}(x)\} = 0.84 ,\ Pl_{\Delta\tilde{U}}(C_1) = \max_{x \in C_1}\{\Delta\overline{U}(x)\} = 0.9832 .$$

The results show that the certainty is 0.84 and the likelihood is 0.9832. The distribution of $\Delta\tilde{U}$ and C in emotional space of PAD is shown in Fig. 1. In the figure, points marked with the same symbol belong to one of covering granules. The region included in the black circle indicates fuzzy set $\Delta\tilde{U}$ .The results demonstrate that $\Delta\tilde{U}$ is close to C_1 (points marked with '*', numbered 1, 10, 11, 13, 14, 15, 16 in the figure), which implies that certainty and likelihood can afford to describe the emotion in quantization.

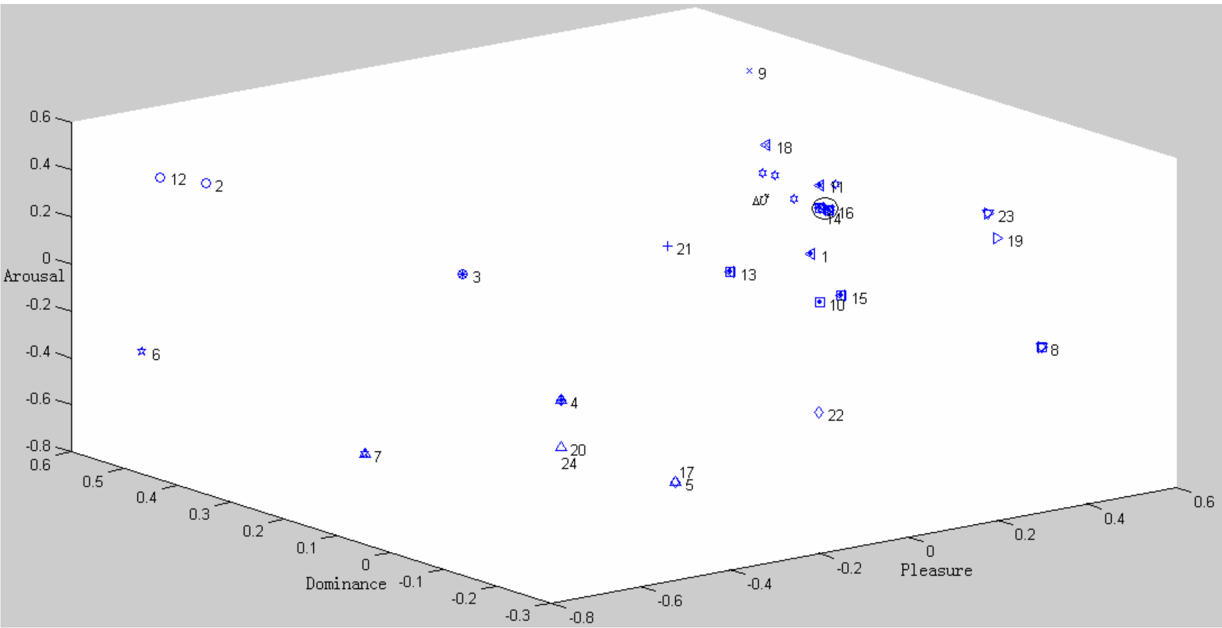

Fig. 1 The distribution of $\Delta\tilde{U}$ and C in PAD

6 Conclusions

The affective computable problem is a researched focus in the field of artificial intelligent. It is critical for affective computation how to construct mathematic models in effect to quantize the emotion that is mapped into the specific space. The result of affective computation can provide subsequent emotional reasoning model with the proof. This paper makes use of consistent covering granule model and granular computing to describe the emotion in the emotional space of PAD. From the granular viewpoint, the non-typical emotion is described in qualification and quantification. The examples demonstrate that the proposed emotion method is effective and practicable. But many shortages still exist due to just start researching the granular computing model. The research will continue to settle the emotional transition, synthesis and decomposition on multi-level. The next step is to build a multidimensional nonlinear discriminant function based on Support Vector Machines, in which the training samples and the testing samples require that the inputs are signals including the text and audio speech while the outputs are the points in PAD space, and the set of samples is annotated by subjects. Upon completion of the nonlinear discriminant function, an episode of text or audio speech that is imported to nonlinear discriminant function system is transformed into 10 points in the PAD space in 10 times test repetitively, where the 10 points can construct the corresponding emotional fuzzy set $\Delta\tilde{U}$ and then a given corresponding maximum consistent covering $C = \{[x]_{TR} / x \in U\} = \{C_1, C_2, C_i \cdots C_m\}$ is used to describe $\Delta\tilde{U}$ in terms of certainty and likelihood, i.e., $Bel_{\Delta\tilde{U}}(C_i) = \min_{x \in C_i}\{\Delta\underline{U}(x)\}$ and $PI_{\Delta\tilde{U}}(C_1) = \max_{x \in C_1}\{\Delta\overline{U}(x)\}$. The subjects are invited to verify how accurately the $Bel_{\Delta\tilde{U}}(C_i) = \min_{x \in C_i}\{\Delta\underline{U}(x)\}$ and $PI_{\Delta\tilde{U}}(C_1) = \max_{x \in C_1}\{\Delta\overline{U}(x)\}$ describe the emotions extracted from the episode of text or audio speech in the testing.

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