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Quantum Gray-scale Image Dilation/Erosion Algorithm Based on Quantum Loading Scheme

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Abstract. Gray-scale image morphological processing is extensively applied to the field of boundary detection, image segmentation and feature extraction, etc in traditional computer. Its time complexity, however, is very high. On account of the unique superiority of quantum computation, this paper proposes a novel dilation and erosion processing algorithm based on quantum loading scheme and quantum reversible adder/subtraction circuits. Compared to electronic computer, the algorithm can store images with less qubits, and the complexity is lower. Besides, the proposed meet the optimal circuit designs. Finally, the simulation experiments are done to prove the correctness of the program.

Keywords: gray-scale image morphological processing, quantum loading scheme, quantum reversible adder/subtraction

1 Introduction

Dilation and erosion as the most basic operations in the mathematical morphology, are the foundations of opening and closing operations. However, the algorithms of dilation and erosion are very time-consuming. When the sizes of image and structuring element are relatively large, the time complexity is $O(n^4)$. Therefore, how to optimize the two algorithms is always a hot research issue.

Generally, there are two traditional ways to improve the arithmetic speed. The first method is based on the chain rule that a large structuring element is decomposed into a series of small structuring elements [1-3]. For example, Zhuang [1] put forward a searching tree algorithm, which decomposes arbitrary structuring element into only contains two pixels structuring elements. And Xu [2] resolve convex structuring element into the structuring element of 3×3. The second method regards the structuring element as a whole without decomposing [4].

Meanwhile, with the development of quantum computer [5-7], the quantum versions have been proposed for classical algorithm. For example, geometric transformation [8-10, 40], color transformation [11], image scaling [12-14], image scrambling [15-16], image segmentation [17], feature extraction [18], quantum morphological algorithm [19-23]. The superiority of quantum morphological algorithms has been proved, compared to classical morphological algorithms. However, the current research is limited to quantum binary image, which is not enough to meet the actual demand. Consequently, in this paper, quantum gray-scale image morphological processing has been studied, including dilation and erosion algorithms. The proposed algorithms use QSL to load classical information into quantum computer, and select the optimal adder/subtraction comparator to realize circuits. Besides, GGI is used to design algorithms.

Gray-scale morphology is built on binary morphology. It is known that the transforms of set still play a key role in the binary morphology, such as translation, intersection, union. But there isn't a simple binary

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set for the gray-scale image, while needs to use a function to represent. For example, the definition of gray-scale dilation:

$$(f+b)(s,t)=\max\left\{f(s-x,t-y)+b(x,y)\,|\,(s-x,t-y)\in D_f;(x,y)\in D_b\right\}. \quad \textbf{(1)}$$

Where D_f and D_b are the domains of f and b respectively. f and b are functions here while are sets in binary morphology. The condition that $(s-x)$ and $(t-y)$ must be limited in the domain of f, and x and y must be limited in the domain of b, which is analogous to the condition in the binary definition of dilation, where the two sets have to overlap by at least one element.

The contributions of this paper are listed as follows: by using QSL, the time complexity to load all information of vector into quantum registers of quantum CPU from classical memory is $O(\log_2^N)$, while loading same information into classical registers of CPU from classical registers, the time complexity is $O(N)$. For a image with size of 8×8, electronic computer needs 2^{19} bits to store all information of this image, while this paper only needs 24 qubits. Besides, the time complexity of the proposed algorithm for dilation/erosion is $O(7m^2)$, while classical algorithm is $O(m^2n^2)$. Besides, the optimized circuits are designed with less qubits cost and less garbage outputs, by using a novel quantum BCD adder/subtraction and 4-bit reversible comparator.

This paper is organized as follows: In section 2, the loading scheme of this paper is designed based on quantum loading scheme. In section 3, the quantum reversible adder/subtraction and quantum reversible comparator are applied to this algorithm. In section 4, the specific algorithm procedure is illustrated. In section 5, we analyze, theoretically, the performance of the novel dilation and erosion algorithm. In section 6, the experiment simulation is given. Summary and outlook are proposed in Section 7.

2 Quantum Loading Scheme (QSL)

QSL is to load all information of vector into quantum registers of quantum CPU from classical memory [24-29]. QSL is designed based on path interference with time complexity $O\left(\log_2^N\right)$, while loading same information into classical registers of CPU from classical registers, the time complexity is $O(N)$.

In computer science, binary is often used to represent data. Such as an N-dimensional vector $a=\{a_0,a_1,\cdots,a_{N-1}\}$, where the components $a_0,a_1,\cdots,a_{N-1}$ are integer numbers.

According to the Pang's interpretation [24, 38], the whole information of the vector consist of three parts, component a_i, subscript i, and the one-to-one mapping relationship between them, and any storing manner should include the three parts completely. This paper utilize Pang's theory to load all of the information of the image into the quantum state. The quantum representation of image is defined as the superposition states, by which the whole information of image is represented. Let

$$\left|qVector\right\rangle=\frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}\left|i\right\rangle_{q_1q_2\cdots q_n}\left|a_i\right\rangle_{p_1p_2\cdots p_m}\left|ancilla\right\rangle. \quad \textbf{(2)}$$

Where the n qubits $q_1q_2\cdots q_n$ and the m qubits $p_1p_2\cdots p_m$ are used to represent subscript i and the corresponding value a_i respectively, and the ancillary state $\left|ancilla\right\rangle$ is known. Register $\left|i\right\rangle_{q_1q_2\cdots q_n}$ is entangled with register $\left|a_i\right\rangle_{p_1p_2\cdots p_m}$.

Let the initial states ϕ_0 is

$$\left|\phi_0=\left|0\right\rangle_{q_1q_2\cdots q_n}\left|0\right\rangle_{p_1p_2\cdots p_m}\left|ancilla\right\rangle. \quad \textbf{(3)}$$

The unitary operation $U_{(0,1,\cdots N-1)}$ was designed by pang [5] to obtain $\left|qVector\right\rangle$.

$$\left|U_{(0,1,\cdots N-1)}(\phi_0)=\frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}\left|i\right\rangle_{q_1q_2\cdots q_n}\left|a_i\right\rangle_{p_1p_2\cdots p_m}\left|ancilla\right\rangle. \quad \textbf{(4)}$$

In this paper, a size of 4×4 of structuring element and a gray-scale image of 256×256 are selected. We need to use the quantum loading scheme (QLS) to load structuring element and gray-scale image into quantum registers from electronic memory. After performing QSL, the structuring element and gray-scale image are represented as (5) and (6) respectively.

$$|\phi_{SE}=\frac{1}{256}\sum_{i=0}^{255}|i\rangle_{q_1q_2\cdots q_8}|a_i\rangle_{p_1p_2\cdots p_8}|ancilla2\rangle. \quad \textbf{(5)}$$

$$|\phi_{GI}=\frac{1}{4}\sum_{i=0}^{15}|i\rangle_{q_1q_2}|a_i\rangle_{p_1p_2}|ancilla1\rangle. \quad \textbf{(6)}$$

3 Quantum Circuits for Dilation and Erosion

3.1 Introduction of GGL

It is known that binary morphology is the operations for intersection, union of a set. The gray-scale morphology is the operations for minimum value, maximum of a function [30]. In this paper, the algorithms for dilation and erosion of gray-scale image is to use addition and subtraction operations instead of convolution's product, and replacing the convolution's addition and subtraction with maximum and minimum value, respectively. So we need to use the measured result by (general Grover iteration) GGI to do the addition, subtraction and comparison. GGI is the improved Grover iteration by Pang [25, 39] and defined as

$$|G'=\left(2|\xi\rangle\langle\xi|\right)\left(U_L\right)^{+}\left(O_c\right)^{+}O_fO_cO_L. \quad \textbf{(7)}$$

where U_L denotes the unitary operation, O_f denotes the oracle that flips the phase of state in Grover iteration [30] and O_c denotes another computation oracle [27-29]. In GGI, U_L is included, which can load the content of the record into the register to be entangled with its index. That is, U_L entangles index with its corresponding record so that the index and its corresponding record both can be measured out as a last answer.

3.2 Quantum Reversible BCD Adder/Subtraction

Besides, in order to implement the operations, a novel reversible BCD adder and parallel adder/subtraction are used [31]. To begin with, the basic quantum reversible logic gates are used in the adder/subtraction, such as TR gate [32], Modified Toffoli gate [33] and quantum reversible ZRQ2 gate. The quantum reversible BCD adder/subtraction based on control line is indicated as following.

Here, $|A_3\rangle,|A_2\rangle,|A_1\rangle,|A_0\rangle$ are the binary numbers of addition/subtraction, $|B_3\rangle,|B_2\rangle,|B_1\rangle,|B_0\rangle$ are the another numbers of addition/subtraction. $|C_0\rangle$ for low input and $|C_3\rangle$ for high input. $|S_3\rangle,|S_2\rangle,|S_1\rangle,|S_0\rangle$ are the results of addition/subtraction. Ctrl is control signal: Ctrl=0, performing BCD adder, Ctrl=1, performing BCD subtraction.

Through quantum measurement, the superposition state is as the input of BCD adder/subtraction circuit.

3.3 A Novel 4-bit Reversible Comparator

In order to get the maximum values or minimum values, a kind of comparison circuit will be introduced. Fig. 2 is the circuit of quantum comparator. The quantum gates, such as the FG gate, PG gate, ZRQC1 gate, among the references [34-36] can be obtained respectively.

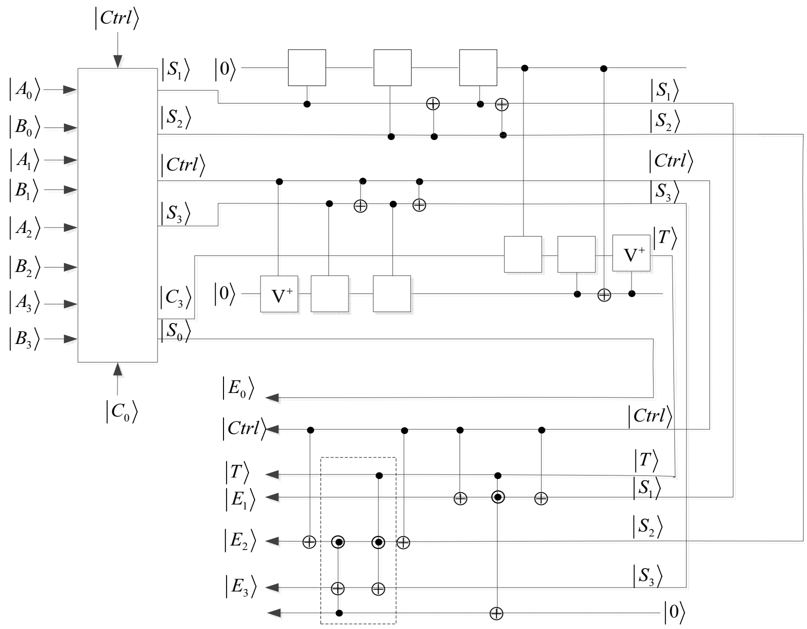

Fig. 1. A novel quantum BCD adder/subtraction

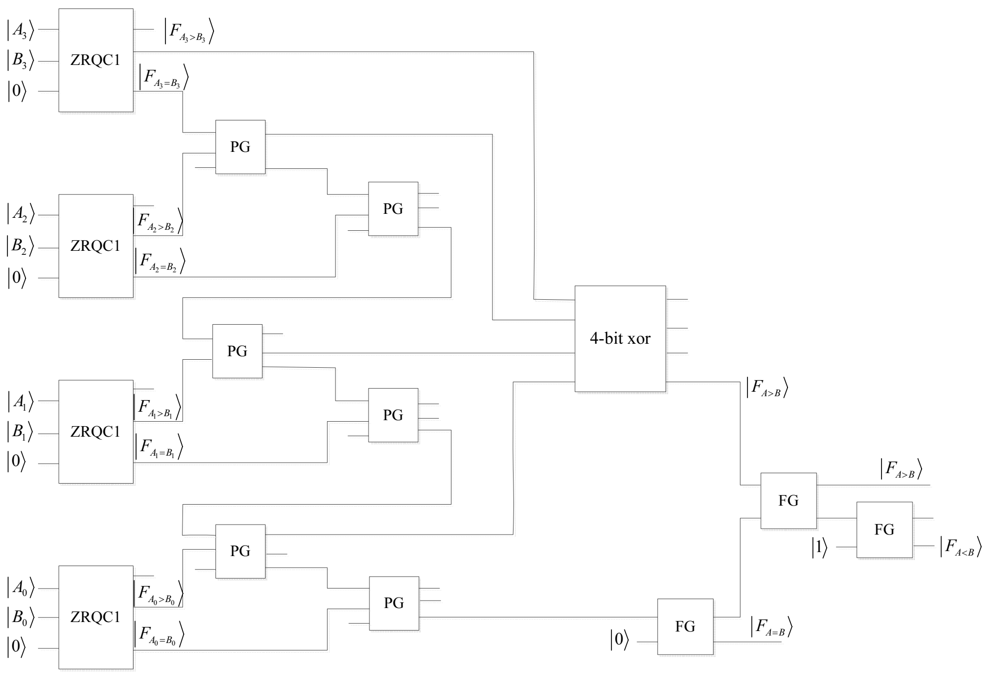

Fig. 2. The realization of 4-bit quantum comparator

The comparator greatly reduce waste output and quantum cost, and improved the performance of quantum reversible circuit and calculating speed. The circuit can implement the comparison of two 4 digits, thus it is very important for the algorithm to do the dilation and erosion of quantum image.

4 Quantum Algorithm of Dilation and Erosion

Algorithm thinking: The image A is regarded as a background image, and the structuring element B is regarded as the foreground image. The first point of structuring element as the origin and the method of point-by-point calculation to realize the gray-scale operation is used. The results of erosion and dilation come from calculating the difference and sum value between the local range of each point and the corresponding point of structuring element, respectively. and select the minimum value and maximum value as the final result of the correspond point.

Algorithm 1:

Step 1: Initialize $p=1$, $\lambda=6/5$.
Step 2: Choose a value q randomly, which is a non-negative integer and smaller than p.
Step 3: Perform GGI's iteration algorithm on the superposed state of image A and the structuring element B.
Step 4: Observe these registers: let $A_{00}\cdots A_{03}$ and $B_{00}\cdots B_{03}$ be the output.
Step 5: Judging the results. If the output is the desired results, exit the program. Otherwise, set $p=\min\left\{\lambda p,\sqrt{2}\right\}$ and go back to step 2.

Algorithm procedure:

Step 1: Initialize a set $C=\varnothing$, which size is equivalent to A. Then initialize an empty set D, which is used to store the calculation result and initialize a variable n=0 and i=0.

```
Step 2: While(n<256)
  {For(i=0,i<3,i++)
    {Call the algorithm 1 to load the data into the circuit. When the
     data via the adder/subtraction can get four sum/difference value.
     Then the comparator is used to obtain the maximum/minimum value.
        Finally, put the maximum/minimum value into the set D as the
        first retrieved data and i++.}
  Put the four data of set D into the comparator and the result as the
  first value of set C, n++.}
Step 3: Program ends and the data set C is result of dilation/erosion.
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In the procedure of dilation/erosion, as the particularity of structuring element, it's no need to move the structuring element. The only thing is to record the subscript and to do the corresponding calculation and the assignment of corresponding location, which can improve the speed of calculation as well.

5 Algorithm Performance Analysis

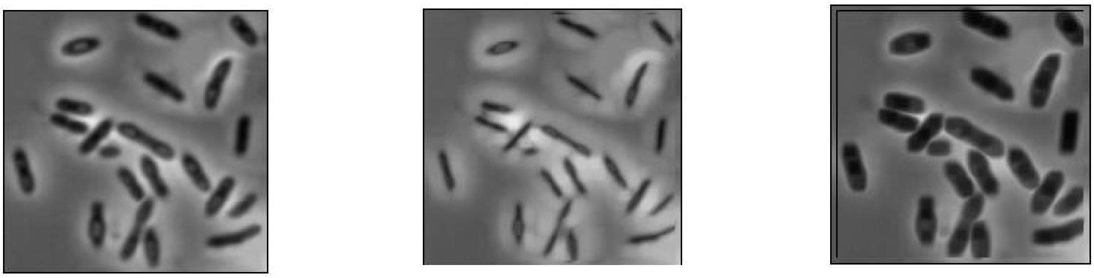
Firstly, the complexity of QLS is $O\left(\log_2^N\right)$, which is faster than classical loading scheme complexity $O(N)$. And, using the proposal representation, fewer qubits can store a whole image without losing information. For gray-scale image with the size $2^8\times2^8$, electronic computer needs $2^8\times2^8\times8$ bits to store all information of this image, while quantum computer only needs $8+8+8$ qubits using the entangled state.

Secondly, the reversible BCD adder/subtraction is complemented by reversible 4-bit adder/subtraction using ZRQ2 gate, which was unique and low of quantum cost. The total number of garbage outputs required to construct the proposed reversible BCD adder/subtraction is 8, the total number of constant input is 13 and the quantum cost is 32. Besides, the quantum reversible comparator is more optimized in terms of the number of reversible gates, garbage gates and quantum costs than the existing designs. Meanwhile, they improved the speed of quantum reversible logic circuit.

Finally, the performance of this algorithm is analyzed as the following. This paper used the structuring element is 4×4 and the image is 256×256. For convenience, the size of structuring element and image are denoted as n^2 and m^2 respectively. The erosion operation is used to explain the algorithm. According to the design of circuit, the algorithm needs to be done four times subtraction and three times comparison, so the whole time complexity is $O(7m^2)$. Compared with the classical algorithm $O(m^2n^2)$, the propose algorithm actually has the faster complexity. Yuan [22] has done the quantum version of dilation and erosion algorithms with complexity of $O(n \cdot q^2 \cdot m^2)$. There are also some improved algorithms, such as [1-4, 19-20, 37], but the propose algorithm has the better performance.

6 Simulation results

In this part, the simulation experiments are carried out by using the dilation and erosion algorithm, which are presented in Fig. 3. The structuring element is a 4×4 1-matric. The results show that the effect is superior to the classical algorithm.



(a) is the original image and the (b) present results by simulating the erosion and dilation algorithm in a classical computer (c) present results by simulating the erosion and dilation algorithm in a classical computer

Fig. 3. The simulation results of two algorithms

7 Conclusions

Erosion and dilation are fundamental operations in classical image processing. Its complexity is $O(m^2 n^2)$ (suppose the size of structuring element and image are $n \times n$ and $m \times m$). Many researchers have tried to lower complexity of erosion and dilation algorithms, but the results are not very satisfactory. As the unique advantages of parallel computing, entanglement, superposition state, quantum computer brings bright prospect. Giving this, the quantum versions of erosion and dilation algorithms have been studied, and the complexity is greatly reduced. However, the current research is limited to quantum binary image. Thus this paper proposes quantum dilation and erosion algorithms based on gray-scale image.

The proposed algorithms use QSL to load classical information into quantum computer, and select the optimal adder/subtraction, comparator to realize circuits. Besides, GGI is used to design algorithms. The time complexity of the proposed algorithm is less than the classical algorithm, and the time complexity of loading data is also faster than the classical loading scheme. Also, qubits to store structuring element and gray-scale image are far less than classical procedure. Beside, the design of circuit achieved the high efficiency and energy saving. However, quantum dilation and erosion algorithms based on color image have not been studied in this paper, so the issues should be solved in the future. And the target of next research will focus on the quantum watershed algorithm. It will bring us more broad application prospects.

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