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Research on Fire Risk Assessment of High-rise Buildings Based on Fuzzy Mathematics and Set-value Statistics Theory

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Abstract. Once a high-rise building fire will cause serious casualties and huge loss of property, meanwhile, it will also have bad social impact. In order to avoid or reduce the occurrence of fire, effective prevention and control of high-rise building fire has become an important subject. Building fire risk assessment is the basis of building fire risk management, fire risk assessment of buildings can effectively prevent the occurrence of fire accidents and control the loss of the accident. Based on the study of the role and relationship of all factors in building fire safety, this paper establishes an index system for fire risk assessment of high-rise buildings , which can be evaluated from five aspects, it includes the active fireproofing ability of building, passive fireproofing ability of building, fire fighting and rescue ability, evacuation capability and fire management level. Considering the relationship between the measured value and the evaluation index range from the point of view of system security, the corresponding weight system is determined. Considering that the fire risk assessment of high-rise buildings is characterized by many evaluation parameters, fuzzy evaluation conclusions and complicated evaluation process, in this paper, based on the traditional fuzzy evaluation model, the set value statistics method is introduced in the fuzzy mathematics, and the corresponding evaluation interval is redefined. it adopts the Linear interpolation method to show the evaluation results which is expressed in membership degree in quantitative form, and a new linear weighted average fuzzy comprehensive evaluation model is established. The evaluation results of the new model are more objective, clear and universal.

Keywords: fuzzy mathematics, high building fire, membership function, risk assessment, set value statistics

1 Introduction

With the continuous growth of China's economy and the accelerating process of urbanization, the construction industry has developed rapidly, and the number of high-rise buildings is more and more. Due to the number of layers, the large volume and the concentration of personnel, high-rise building fire have the characteristics of rapid fire spread, difficult rescue and evacuation difficulties. Once a fire happens, it will cause serious casualties and huge property losses, and at the same time, there will also be bad social impacts. Therefore, we need to establish a fast and reliable assessment method to detect and evaluate the fire risk of high-rise buildings, and make reasonable transformation so as to prevent fires and reduce fire losses, it has become an urgent subject to study at present [1-2].

Sun Xiaoqian expounds the steps and methods of high-rise building fire risk assessment, in a high-rise building as an example, including five steps of hazard identification, scenario design fire, quantitative risk analysis, risk assessment comparing to acceptable criteria, and risk management, both the probability

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and the consequence of fires are quantified in this paper [3]. However, this method is not universal, and the evaluation is high and time-consuming.

Wang Li constructed a grey correlation evaluation model of fire system in high-rise buildings by using the grey correlation analysis method [4]. In the paper, the grey relational matrix is established by calculating the direct grey correlation degree between the evaluation index series and the real sample sequence, and the influencing factors and the overall risk level of the system are obtained. The direct evaluation method can not only protect the existing information but also reduce the error. It has the advantages of simple, intuitive and small calculation, but the main drawback lies in the requirement that the optimal value of each index should be determined at present, and the subjectivity is too strong.

Xin Jing, Xia Dayou, Pang Xilei puts forward the qualitative with the cloud theory to solve some evaluation index description to quantitative representation problems in high-rise building fire risk assessment, considering fuzzy factors appear in the course of evaluation and random factors [5], the method is an effective attempt in the risk assessment of high-rise building fire. Although this method is practical and maneuverable, its evaluation results are not quantitative, and the results are not scientific and intuitive.

In general, things in the objective world are mostly described by the concept of traditional mathematics, because they have the characteristics of accuracy and certainty. But the objective world is at the same time with diversity and Complexity, which makes it difficult for many things to be described with a precise, definite concept. In order to describe the randomness of the occurrence and change of events, statistical mathematics has been developed. In order to describe fuzziness of things, fuzzy mathematics has been developed.

The risk of building fire is a typical fuzzy problem, there is no definite boundary in quantity; on the other hand the evaluation of high-rise building fire risk involves multiple influencing factors, the attributes of each factor are classified into different categories and levels, and many factors are difficult to quantify, so they have characteristics of randomness and fuzzinessy. While fuzzy mathematics is a mathematical method to study and deal with fuzzy phenomena, and expresses some qualitative descriptions and human subjective judgements in a form of magnitude, through the way of fuzzy operation use membership to determine the risk level of system. Therefore, It is especially suitable for the objective judgment of multi-factor events which are unsuitable for quantitative analysis, such as the fire risk assessment of high-rise buildings, and it can be more accurate to evaluate the risk status of fire in high-rise buildings.

The set value statistics is an extension of classical statistics and fuzzy statistics [6]. The classical statistics get a definite point of phase space in each experiment, and the set value statistics get a subset of the phase space in each experiment. That is to say, when determining the index safety, we give an interval value instead of past statistics to give a fixed value method, which solves the quantification of some vague concepts by some experts. The expression of fuzzy comprehensive evaluation results is vector form, which can not fully quantify the fuzzy concept of final evaluation index. The introduction of set-valued statistics can solve this problem and quantify the evaluation results.

2 The Composition of the Risk Evaluation Index System and the Determination of the Level Interval

2.1 The Composition of the Risk Evaluation Index System for High-rise Buildings

According to the modern accident causation theory, the direct cause of the accident is the unsafe behavior of human beings and the unsafe state of the objects, so the basic reason is the mismanagement. Therefore, the first level of impact on the risk of building fire mainly focuses on the safety management system and the status of the building itself.

2.2 Determination of Grade Interval for Fire Risk Assessment of High-rise Buildings

According to China’s civil building reliability evaluation standard GB50292-1999, the reliability level of existing buildings is divided into four levels, and similar grading standards are adopted here. By analogy analysis, according to the degree of the building meeting or not meeting the current standard and the technical countermeasures taken to the building at last, the fire risk level of the building is divided into four levels, and a grade that basically meets the current standards and specifications is set up. The grading standard is as follows:

Level one. meet the requirements of the current standard of the state, the level of fire risk is acceptable, no or only local areas need to be improved.

Level two. slightly lower than the current standard requirements of the state, The level of fire risk is basically acceptable, only the local area need to be improved.

Level three. not meet the requirements of the current standard of the state, The level of fire risk is unacceptable, and measures must be taken**.**

Level four. serious failure to meet the requirements of the current standard of the state, the level of fire risk is absolutely unacceptable, and measures must be taken immediately.

The evaluation indexes of fire risk are divided into four grades. If the percentage of the scores in most of the evaluation methods is evenly distributed, the range of each grade is shown in Table 1.

The score is a more specific description of the evaluation index, and to a certain extent, it reflects people's psychological measure of the change of things. In fact, the hazard class of things, people used to maintain a conservative and pessimistic attitude of things, People are sensitive to the change of things, and the changes in the psychological measure increase quickly. On the contrary, if things are in a favorable state and continue to develop in a favorable way, people's reaction to things will weaken and the change of psychological measurement will slow down. In this case, the psychological measure of the change of things is nonlinear. According to the relevant literature [7], we choose the power function as follows:

$$y = 100 - 0.24(100 - x)^{1.31}\,. \tag{1}$$

In formula 1: x —the value of the uniform distribution (linear uniform distribution)
y —the value of Considering the nonlinear variation of people's psychological measure (nonlinear distribution)

According to the power function of the formula1, considering the regularity of the data, the range of each grade of the fire risk evaluation index is shown in Table 1.

Table 1. The range of each level after optimization

Level	The score of a uniform partition	Consider the score of the nonlinear change of psychological measure
Level one	75-100points	85~100points
Level two	50-75points	60~85 points
Level three	25-50points	30~60 points
Level four	0-25points	0~30 points

The evaluation index system should contain sufficient information and reduce the correlation between factors. Based on the national standards and safety regulations of many kinds of buildings, The article draw lessons from a large number of researches on the past building fire evaluation index system, the high-rise building fire risk evaluation index system is established, as shown in Table 2, at the same time, according to the current fire code in our country, we have established the detailed scoring rules for each index, it will not be described in detail here.

Table 2. Fire risk evaluation index system for high-rise buildings

Evaluation content	Firs level evaluation index	Second level evaluation index	First-level	Second-level	Third-level	Fourth-level
Passive fireproofing ability of Building	Plane layout	Building durability endurance	[100,150]	[50,100)	[15,50)	[0,15)
		Fire separation distance	[13,16]	[9,13)	[4,9)	[0,4)
		Perimeter risk	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Building height	[0,24)	[24,50)	[50,80)	[80,110]
		Weather conditions	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]

Table 2. Fire risk evaluation index system for high-rise buildings (continu)

Evaluation content	Firs level evaluation index	Second level evaluation index	First-level	Second-level	Third-level	Fourth-level
Passive fireproofing ability of Building	Fire Resistance Rating	Fire resistance rating of building structure	(1.4,1.7]	(1,1.4]	(0.5,1]	(0,0.3]
	Electric ire prevention	Current status of electrical accessory	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Protection of electrical accessory	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Fire load	Fire load density	[0,170)	[170,500)	[500,1000)	[1000,2000]
		Distribution of fire load	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Fire partition	Fire separation	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		fireproof plugging of special parts	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		fire partition area	[1700,2000)	[2000,2500)	[2500,3100)	[3100,3700]
Active fireproofing ability of building	Smoke control system	Smoke exhausting mode	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Smoke emission volume	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Air supplement setting	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Pressurized air supply device	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Automatic fire alarm system	Type of detector	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Detector reliability	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Monitoring mode	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	automatic fire extinguishing system	Fire extinguishing device	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		System reliability	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Other fire extinguishing systems	Fire extinguisher configuration standard	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Fire water supply	Municipal Pipeline	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		fire-fighting pool	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Hydrant	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
Fire fighting and rescue ability	Fireman	The number of firemen	(0,1200]	(1200,3000]	(3000,5000]	(5000,7000]
		working life of firemen	(10,8]	(8,6]	(3,6]	(0,3]
		Business building	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Fire station	Protection area of fire station	(4000,5500]	(5500,7000]	(7000,8800]	(8800,10600]
		Building area of fire station	(3500,5000]	(2600,3500]	(1520,2600]	(500,1520]
		Fire fighting equipment	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Building fire fighting conditions	Fire escape	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Fire fighting area	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
Evacuation capability	Evacuation route	Safe evacuation distance	[0,15)	[15,40)	[40,70)	[70,100)
		Safe export quantity	[4,5)	[3,4)	[2,3)	[1,2)
		Hundred people's width index	(1,3]	(0.5,1]	(0.3,0.5]	(0,0.3]
	Evacuation facilities	Emergency broadcast lighting and evacuation indicator	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Evacuation staircase	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Refuge	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Auxiliary evacuation facility	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Personnel load	Personnel distribution status	[0.28,0.4)	[0.4,0.6)	[0.6,0.84)	[0.84,1.08]

Table 2. Fire risk evaluation index system for high-rise buildings (continu)

Evaluation content	Firs level evaluation index	Second level evaluation index	First-level	Second-level	Third-level	Fourth-level
Fire management level	Implementation of the security system	Fire protection facilities maintenance and periodical maintenance	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Autocratic duty	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Fire emergency plan	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Safety responsibility system	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Managers' business level	Fire prevention education	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Organization skills	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Fire fighting knowledge and skills	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
	Other members of the building	Safety awareness level	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Fire prevention training	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Hidden trouble rectification	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
		Organization	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]

3 Fuzzy Comprehensive Evaluation Model for Fire Risk of High Rise Buildings

Traditional mathematics describes the objective world with precise and deterministic mathematical concepts. However, the diversity and complexity of the objective world make it difficult to describe many things with precise and definite concepts. In order to describe the randomness of event occurrence and change, mathematics is generated. In order to describe the ambiguity of the characteristics, fuzzy mathematics is generated and developed. Fuzzy sets in fuzzy mathematics can be used to describe fuzzy concepts without explicit epitaxy. Due to the degree of membership of each element to the fuzzy set on domain U, Because each element of the domain U of the membership degree of fuzzy sets, which can be expressed by a certain number of between 0 to 1, this means that a mapping is constructed, that is, the mapping of every element from domain U to the membership of a fuzzy set, which is called the membership function of the fuzzy set. The fuzzy set and the membership function can be uniquely determined each other [8-9].

3.1 Normalization of Data

The evaluation object is divided into *m* class (M grade), *n* evaluation index, the upper and lower limits of the *h* classification criteria for the *i* evaluation index are shown in Table 3 respectively.

Table 3. Index standard value and object value

Standard value of index classification			
Class 1	Class 2	…	Class M
$y_{11} \sim y_{12}$	$y_{12} \sim y_{13}$	$\cdots$	$y_{1m} \sim y_{1,m+1}$
$y_{21} \sim y_{22}$	$y_{22} \sim y_{23}$	$\cdots$	$y_{2m} \sim y_{2,m+1}$
…	…	…	…
$y_{n1} \sim y_{n2}$	$y_{n2} \sim y_{n3}$	$\cdots$	$y_{nm} \sim y_{n,m+1}$

$y_{i1} \le y_{i2} \le \cdots \le y_{i,m+1}$ or $y_{i1} \ge y_{i2} \ge \cdots \ge y_{i,m+1}$ $\quad i = 1,2,\cdots,n$

$h = 1,2,\cdots,m$, x_i is the *i*th index value of the object.

Regulation: y_{i1} is the upper limit value of the class 1 index standard of index i, the membership degree of y_{i1} to the fuzzy concept is equal to 1; The membership degree of the class M standard lower limit for the index *i* to the fuzzy concept is equal to 0, The membership of the standard value between y_{i1} and $y_{i,m+1}$ is in the [0, 1] interval, It can be determined by linear interpolation, which is the following normalization formula:

$$s_{ih}=\frac{y_{ih}-y_{i,m+1}}{y_{i1}-y_{i,m+1}}. \tag{2}$$

Among them: the S_{ih} is the normalized number, that is, the membership degree of the index classification standard value to the fuzzy concept, $h=1,2,.....,m$

From the perspective of engineering security of fuzzy concept, the upper limit of standard value interval at all levels can be regarded as the standard value of all levels, and there is a standard index membership matrix:

$$S=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1m} \\ S_{21} & S_{22} & \cdots & S_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nm} \end{bmatrix}=\left[S_{ih}\right]_{n\times m}. \tag{3}$$

The *n* index value of the object is:

$$x=\left[x_1,x_2,\cdots,x_n\right]^T. \tag{4}$$

The rule: the larger (smaller) the index value, the better (stability) index, the greater than (less) equal to the upper limit of the 1 class standard, and the degree of membership of the fuzzy concept is 1;less than (greater) or equal to the lower limit value $y_{i,m+1}$ of the *m* class standard, and the membership degree to the fuzzy concept is 0.

The membership of the standard value between y_{i1} and $y_{i,m+1}$ is in the [0, 1] interval, It can be determined according to the following normalization formula:

$$a_i=\frac{x_i-y_{i,m+1}}{y_{i1}-y_{i,m+1}}. \tag{5}$$

From Table 3 and formula 5, the index can be normalized to obtain the membership degree vector of the sample index:

$$a=\left[a_1,a_2,\cdots,a_n\right]^T. \tag{6}$$

Matrix *S* gives the standard index membership grade of *m* class to some fuzzy concept. Vector a represents the degree of membership of each object to a fuzzy concept. It can be seen that the normalized number not only eliminates the different effects of the physical dimension of the index, but also has a clear mathematical and physical meaning, that is the degree of membership of the standard values of each index and the value of the object index to a fuzzy concept.

3.2 Determination of Weight

Weight is an important information of comprehensive evaluation. There are many methods for determining weight, such as Delphy Fa, principal component method and so on. Therefore, it can be concluded that the importance of the index itself has been reflected in the classification standard values of varying amplitude and magnitude. However, from the perspective of system security, the greater the classification of the index *i* membership value, the more unfavorable the influence of this index on the system security, and the greater the weight should be given. However, from the perspective of system security, the larger the index *i* membership value a_i falls into, the more adverse the impact of this index will be on the system security, and the greater the weight should be given. According to this principle, it is determined that the S_{im} is equal to or less than the *m* class standard value of index *I,* and its unnormalized weight $\omega_i^{'}$ equals 1, for each category of decline, the unnormalized weight $\omega_i^{'}$ decreases by 0.1, for example, m= 4, the rules are shown in Table 4.

Table 4. Index standard value and object value

Class	*h*	1	2	3	4
Class standard value	S_{ih}	S_{i1}	S_{i2}	S_{i3}	S_{i4}
Unnormalized weight	$\omega_i^{'}$	0.7	0.8	0.9	1

In Table 4, the $\omega_i^{'}$ line is the unnormalized weight scale interval. a_i corresponds to the h-class interval of the index *i*, whose unnormalized weight is determined by linear interpolation formula 7:

$$\omega_i^{'} = 0.5 + 0.1 \times (h + \frac{S_{i,ha} - a_i}{S_{i,ha} - S_{i,hb}}) . \tag{7}$$

$S_{i,hb} \le a_i \le S_{i,ha}$, *h*= 1,2,3,4; *i*=1,2,3,4 ($S_{i,ha}$, $S_{i,hb}$ are the upper and lower limits of grade *h*, respectively), and then normalized to get the weight:

$$\omega_i = \frac{\omega_i^{'}}{\sum_{i=1}^{5} \omega_i^{'}} . \tag{8}$$

3.3 Determination of Membership Function

When fuzzy methods are used to deal with fuzzy concepts, the elements of the theory domain are mapped to [0, 1] by the selected membership function to obtain the comparability of different indexes. The membership function should be objectively and reasonably determined based on the previous work data and factor analysis results. The commonly used membership functions are Ridge functions, Normal functions, Trapezoid functions (Trig functions) [10], etc, in the process of drawing, these commonly used membership functions are based on the equality of the length of each level. But if at the division level or interval when applied psychological measurement formula, caused the interval length is stretched or shortened, membership function should also change, regular membership function will no longer use at this time. It is necessary to modify the formula of membership function to obtain the new membership function formula to adapt to the interval length of the change.

after the confirmation of the fire control levels corresponding to fire performance range, in all kinds of membership function is used to establish the specific expression in the process, common to follow the principle of fuzzy and the most clear principles, namely interval endpoint in the fuzzy state, the membership degree is 0.5; The middle point of the interval is clear, membership is 1; The category of the endpoint of the boundary interval is the freshest and the membership degree is 1. The total membership of each point is 1. According to the previous work data and factor analysis, the membership function of building fire level should be triangulation membership function.

Its membership function curves are as follows (the range of measured values is [0, 1], the range of membership grade at all levels is [0, 1]):

The red curve is a four grade membership curve, the greater the vertical axis of the red curve, the greater the membership degree of the four grade. when the measured value is greater than 0.45, the membership degree of the four grade is 0. In the same way, the orange curve is the three grade membership curve, the purple curve is the two grade membership curve, and the green curve is the first order degree of membership curve. At any point within the range of measured values, the sum of the first to four membership degrees is 1.

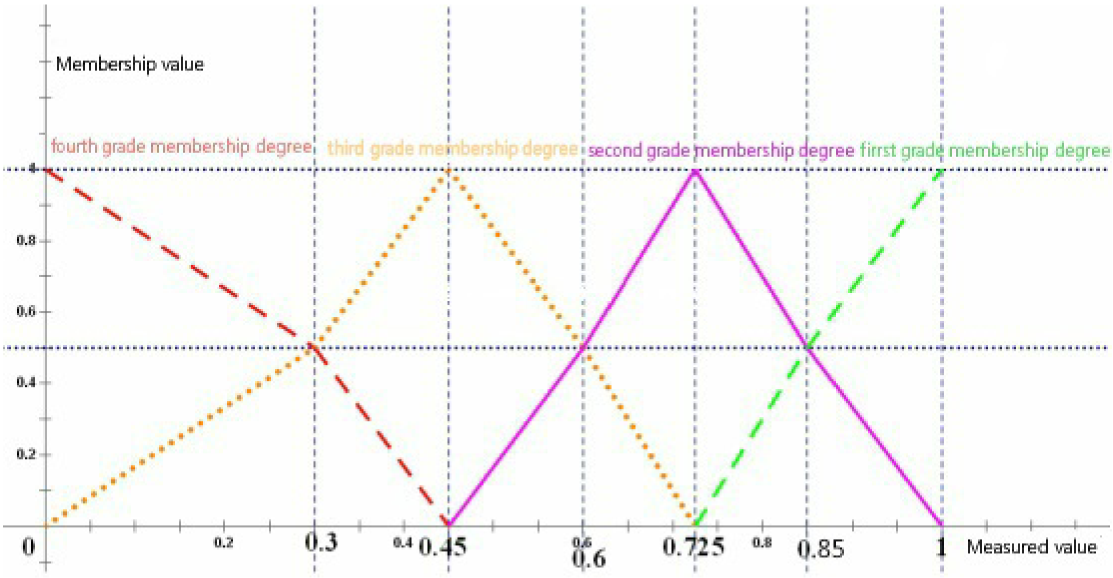

Fig. 1. The membership function curve of existing building fire hazard assessment

If y_{ij} monotonic ascending membership function is known as:

$$r_{i1}=\begin{cases}1, & x_i \le y_{i1}\\ 1-0.5\times\dfrac{x_i-y_{i1}}{y_{i2}-y_{i1}}, & y_{i1}<x_i\le y_{i2}\\ 0.5\times\left(1-\dfrac{x_i-y_{i2}}{\dfrac{y_{i2}+y_{i3}}{2}-y_{i2}}\right), & y_{i2}<x_i\le\dfrac{y_{i2}+y_{i3}}{2}\\ 0, & x_i>\dfrac{y_{i2}+y_{i3}}{2}\end{cases}. \tag{9}$$

$$r_{i2}=\begin{cases}0, & x_i \le y_{i1}\\ 0.5\times\dfrac{x_i-y_{i1}}{y_{i2}-y_{i1}}, & y_{i1}<x_i\le y_{i2}\\ 0.5+0.5\times\dfrac{x_i-y_{i2}}{\dfrac{y_{i2}+y_{i3}}{2}-y_{i2}}, & y_{i2}<x_i\le\dfrac{y_{i2}+y_{i3}}{2}\\ 1-0.5\times\dfrac{x_i-\dfrac{y_{i2}+y_{i3}}{2}}{y_{i3}-\dfrac{y_{i2}+y_{i3}}{2}}, & \dfrac{y_{i2}+y_{i3}}{2}<x_i\le y_{i3}\\ 0.5\times\left(1-\dfrac{x_i-y_{i3}}{\dfrac{y_{i3}+y_{i4}}{2}-y_{i3}}\right), & y_{i3}<x_i\le\dfrac{y_{i3}+y_{i4}}{2}\\ 0, & x_i>\dfrac{y_{i3}+y_{i4}}{2}\end{cases}. \tag{10}$$

$$r_{ij}=\begin{cases}0, & x_i \le \dfrac{y_{i,j-1}+y_{ij}}{2}\\[2ex] 0.5\times\dfrac{x_i-\dfrac{y_{i,j-1}+y_{ij}}{2}}{y_{ij}-\dfrac{y_{i,j-1}+y_{ij}}{2}}, & \dfrac{y_{i,j-1}+y_{ij}}{2}<x_i\le y_{ij}\\[2ex] 0.5+0.5\times\dfrac{x_i-y_{ij}}{\dfrac{y_{ij}+y_{i,j+1}}{2}-y_{ij}}, & y_{ij}<x_i\le\dfrac{y_{ij}+y_{i,j+1}}{2}\\[2ex] 1-0.5\times\dfrac{x_i-\dfrac{y_{ij}+y_{i,j+1}}{2}}{y_{i,j+1}-\dfrac{y_{ij}+y_{i,j+1}}{2}}, & \dfrac{y_{ij}+y_{i,j+1}}{2}<x_i\le y_{i,j+1}\\[2ex] 0.5\times(1-\dfrac{x_i-y_{i,j+1}}{\dfrac{y_{i,j+1}+y_{i,j+2}}{2}-y_{i,j+1}}), & y_{i,j+1}<x_i\le\dfrac{y_{i,j+1}+y_{i,j+2}}{2}\\[2ex] 0, & x_i>\dfrac{y_{i,j+1}+y_{i,j+2}}{2}\end{cases}. \tag{11}$$

$$r_{i,m-1}=\begin{cases}0, & x_i \le \dfrac{y_{i,m-2}+y_{i,m-1}}{2}\\[2ex] 0.5\times\dfrac{x_i-\dfrac{y_{i,m-2}+y_{i,m-1}}{2}}{y_{i,m-1}-\dfrac{y_{i,m-2}+y_{i,m-1}}{2}}, & \dfrac{y_{i,m-2}+y_{i,m-1}}{2}<x_i\le y_{i,m-1}\\[2ex] 0.5+0.5\times\dfrac{x_i-y_{i,m-1}}{\dfrac{y_{i,m-1}+y_{im}}{2}-y_{i,m-1}}, & y_{i,m-1}<x_i\le\dfrac{y_{i,m-1}+y_{im}}{2}\\[2ex] 1-0.5\times\dfrac{x_i-\dfrac{y_{i,m-1}+y_{im}}{2}}{y_{im}-\dfrac{y_{i,m-1}+y_{im}}{2}}, & \dfrac{y_{i,m-1}+y_{im}}{2}<x_i\le y_{im}\\[2ex] 0.5\times(1-\dfrac{x_i-y_{im}}{y_{i,m+1}-y_{im}}), & y_{im}<x_i\le y_{i,m+1}\\[2ex] 0, & x_i>y_{i,m+1}\end{cases}. \tag{12}$$

$$r_{i,m-1}=\begin{cases}0, & x_i \le \dfrac{y_{i,m-1}+y_{im}}{2}\\[2ex] 0.5\times\dfrac{x_i-\dfrac{y_{i,m-1}y_{im}}{2}}{y_{im}-\dfrac{y_{i,m-1}y_{im}}{2}}, & \dfrac{y_{i,m-1}y_{im}}{2}<x_i\le y_{im}\\[2ex] 0.5+0.5\times\dfrac{x_i-y_{im}}{y_{i,m+1}-y_{im}}, & y_{im}<x_i\le y_{i,m+1}\\[2ex] 1, & x_i>y_{i,m+1}\end{cases}. \tag{13}$$

In the same way, we can get the membership function when y_{ij} is monotonically decreasing.

3.4 First-level Comprehensive Evaluation Model

According to the first level comprehensive evaluation model:

$$B=\omega\cdot R=\left[b_1,b_2,\cdots b_m\right]. \tag{14}$$

Among it: $\omega=\left[\omega_1,\omega_2,\cdots\omega_n\right]$ $\sum_{i=1}^{n}\omega_i=1$ $\omega_i\geq 0$; $\left[r_{ij}\right]_{n\times m}\in\left[0,1\right]$ $b_j=\sum_{i=1}^{n}\omega_i r_{ij}$

Here b_j is a function of $r_{1j},r_{2j},\cdots,r_{nj}$, which is the evaluation function. R is called a judgment matrix, which is composed of all the single factor evaluation set of F, $\omega=\left[\omega_1,\omega_2,\cdots\omega_n\right]$ said the weight distribution of various factors, it has to do with synthetic evaluation matrix R, is the comprehensive evaluation of various factors.

3.5 The Quantitative Treatment of the Comprehensive Evaluation Result of First Level

The introduction of set-valued statistics. Set-valued statistics is an extension of classical statistics and fuzzy statistics. The classical statistics get a certain point in the phase space in each experiment, and the set value statistic every experiment obtains a subset of the phase space, when determining the index of safety performance, change the past statistics give a fixed value, and give an interval value, it solves the experts on some indicators "about what is" in the quantification of the fuzzy concept [11]. Assuming that m evaluation indexes are determined, n experts participate in the evaluation, Each expert gives the interval estimation value of each evaluation index's safety degree by line segment method, $X_{ij}=\left[a_{ij},b_{ij}\right]$ (i=1, 2, $\cdots$, q; j=1, 2, $\cdots$, m), as shown in Table 5. For the index r_j, the evaluation interval of the experts is y, which can form a collection statistic sequence.

Table 5. Statistical sequence of evaluation index set

Evaluation expert	Evaluation index					
	r_1	r_2	…	r_i	…	r_m
X_1	$\left[a_{11},b_{11}\right]$	$\left[a_{21},b_{21}\right]$	…	$\left[a_{i1},b_{i1}\right]$	…	$\left[a_{m1},b_{m2}\right]$
X_2	$\left[a_{12},b_{12}\right]$	$\left[a_{22},b_{22}\right]$	…	$\left[a_{i2},b_{i2}\right]$	…	$\left[a_{m2},b_{m2}\right]$
$\vdots$			…		…	
X_j	$\left[a_{1j},b_{1j}\right]$	$\left[a_{2j},b_{2j}\right]$	…	$\left[a_{ij},b_{ij}\right]$	…	$\left[a_{mj},b_{mj}\right]$
$\vdots$			…		…	
X_q	$\left[a_{1q},b_{1q}\right]$	$\left[a_{2q},b_{2q}\right]$	…	$\left[a_{iq},b_{iq}\right]$	…	$\left[a_{mq},b_{mq}\right]$

The safety performance value of the evaluation indicator x can be calculated according to the formula:

$$x_i=\frac{1}{2}\sum_{j=1}^{q}\left[b_{ij}^2-a_{ij}^2\right]\Big/\sum_{j=1}^{q}\left[b_{ij}-a_{ij}\right]. \tag{15}$$

In the formula 15: i = 1, 2, $\cdots$, m; j = 1, 2, $\cdots$, q. The following conclusions can be obtained by entering the membership degree model.

Table 6. Statistical sequence of evaluation index set

h	1	2	3	4
r_{ij}	r_{i1}	r_{i2}	r_{i3}	r_{i4}
X	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]

Among them, the range of X can be regarded as the set value statistics sequence; The r_{ij} can be regarded as the membership value of the number of experts assigned to the corresponding interval of X_i .

$$X_i^{'} = \frac{1}{2} \times \frac{r_{i1}[1^2 - 0.85^2] + r_{i2}[0.85^2 - 0.6^2] + r_{i3}[0.6^2 - 0.3^2] + r_{i4}[0.3^2 - 0^2]}{r_{i1}[1 - 0.85] + r_{i2}[0.85 - 0.6] + r_{i3}[0.6 - 0.3] + r_{i4}[0.3 - 0]} . \tag{16}$$

$X_i^{'} \in [0.15, 0.925]$

Then the results of the class level comprehensive evaluation can be obtained by the normalization of $X_i^{'}$

The normalizing processing of set value statistic score. The standard values of the degree of membership belong to the first and four class, as well as the boundary points of the adjacent level. The grading interval of the set value statistical score can be calculated..

The degree of membership is (1, 0, 0, 0), which completely belongs to the first level, substituting the formula 16, get $X_i^{'}$=0.925;

The degree of membership is (0.5, 0.5, 0, 0), which is at the juncture of the first and second level, substituting the formula 16, get $X_i^{'}$=0.8

The membership degree is (0, 0.5, 0.5, 0), which is in the demarcation point of the second and third level, substituting the formula 16, get $X_i^{'}$=0.575;

The membership degree is (0, 0, 0.5, 0.5), which is in the demarcation point of the third and fourth level, substituting the formula 16, get $X_i^{'}$=0.3;

The degree of membership is (0, 0, 0, 1), which completely belongs to the fourth level, substituting the formula 16, get $X_i^{'}$=0.15;

The corresponding classifications' standard values are obtained, thus Table 7 can be obtained.

Table 7. Normalized interval of set value statistical score

Class	h	1	2	3	4
Scoring interval	X_i	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
Normalized classification interval	Q	(0.8,0.925]	(0.575,0.8]	(0.3,0.575]	(0.15,0.3]

In Table 7, the X_i row is the score interval of the mental curve, and the Q row is the classification interval that the degree of membership is scored by the set value statistic model. $X_i^{'}$ corresponds to the *h* interval of the index *i*, the normalized results are calculated according to the linear interpolation formula 17:

$$X_i = S_{i,ha} + \frac{S_{i,ha} - S_{i,hb}}{Q_{i,ha} - Q_{i,hb}} \times (X_{\mathrm{i}}^{'} - Q_{i,ha}) . \tag{17}$$

$X_{i,hb} \le X_i^{'} \le X_{i,ha}$, *h*= 1, 2, 3, 4; *i*=1, 2, 3, 4 ($S_{i,ha}$, $S_{i,hb}$ is the upper and lower limit of scoring interval at the level *h* respectively. And $Q_{i,ha}$, $Q_{i,hb}$ is the upper and lower limits of the normalized hierarchical interval at the level *h*).

Take $X_i^{'}$ into formula 17, and obtain the normalized first-level comprehensive evaluation result, which is the score of the secondary evaluation index.

3.6 Second-level Comprehensive Evaluation Model

Calculation of membership degree of the secondary-level comprehensive evaluation. The second-level evaluation is to judge the outcome of the first-level evaluation as a single factor in the second-level evaluation.

$$P = W \cdot B = W \cdot \begin{bmatrix} \omega_1 \cdot R_1 \\ \omega_2 \cdot R_2 \\ \cdot \\ \cdot \\ \cdot \\ \omega_c \cdot R_c \end{bmatrix} = W \cdot \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ \cdot \\ B_c \end{bmatrix}. \quad \textbf{(18)}$$

c is the number of the first class factor, and w_i is the weight vector of the second class; *W* is the first class factor weight, B_i is the evaluation result of class *i*, and *P* is the comprehensive evaluation result between the classes.

The determination of the weight in the secondary-level comprehensive evaluation. Take the X into the weight formula 19 to get the first class factor weight

In Table 8, the *Y* line is a scoring range that joins the heart curve, and the $W_i^{'}$ line is the unnormalized weight scale interval. a_i corresponds to the *h* interval of the index *i,* its unnormalized weight can be obtained by linear interpolation formula 19:

$$W_i^{'} = 0.5 + 0.1 \times (h + \frac{S_{i,ha} - a_i}{S_{i,ha} - S_{i,hb}}). \quad \textbf{(19)}$$

$S_{i,hb} \le a_i \le S_{i,ha}$, h= 1, 2, 3, 4;i=1, 2, 3, 4..($S_{i,ha}$, $S_{i,hb}$ is the upper and lower limit of scoring interval at the level h respectively)

Table 8. Unnormalized weight scale of the corresponding class standard values

Class	H	1	2	3	4
Class standard value	S_{ih}	S_{i1}	S_{i2}	S_{i3}	S_{i4}
Level division scope	Y	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]
Unnormalized weight	$W_i^{'}$	(0,0.25]	(0.25,0.5]	(0.5,0.75]	(0.75,1]

Then the weight is obtained by the normalization formula 20：

$$W_i = \frac{W_i^{'}}{\sum_{i=1}^{5} W_i^{'}}. \quad \textbf{(20)}$$

Quantitative treatment of secondary comprehensive evaluation results.

$$P = W \cdot B = \left[P_1, P_{2,} \cdots, P_m\right]. \quad \textbf{(21)}$$

The degree of membership is converted into a numerical value by the method of set-valued statistics.

Table 9. The estimation interval of the membership degree's eigenvalues

P_{ij}	P_{i1}	P_{i2}	P_{i3}	P_{i4}
R	(0.85,1]	(0.6,0.85]	(0.3,0.6]	(0,0.3]

Among them, the range of *R* can be regarded as the set value statistics sequence; P_{ij} can be regarded as the membership value of the number of experts who score the corresponding interval of the *R* . So there is :

$$R^{'} = \frac{1}{2} \times \frac{P_{i1}[1^2 - 0.85^2] + P_{i2}[0.85^2 - 0.6^2] + P_{i3}[0.6^2 - 0.3^2] + P_{i4}[0.3^2 - 0^2]}{P_{i1}[1 - 0.85] + P_{i2}[0.85 - 0.6] + P_{i3}[0.6 - 0.3] + P_{i4}[0.3 - 0]}. \quad \textbf{(22)}$$

Among it: put $R_i^{'}$ into formula 17 and normalize it, and get the secondary-level evaluation result *R*, which is the score value of the first level evaluation index in the evaluation index system of this paper, and the comprehensive evaluation of the next step can be carried out to obtain the total fire risk score of the high-rise building.

3.7 Comprehensive Evaluation

Shown in Table 2, in high-rise building fire risk evaluation index system, the different functions of high-rise building, its the weight of the first level index is different, therefore, it is necessary to take into account the occupancy nature of the object, the actual situation of the building and the expert opinion. The calculation formula of the overall fire risk level of high-rise buildings is as follows:

$$G = W \cdot R \, . \tag{23}$$

Among it: R—the weight vector of first-level index; W—the score of first-level index.

Table 10 lists the value scope of the fire risk rank interval, from this table, the high-rise building's risk rank of the fire risk score can be obtained.

Table 10. Fire risk classification

First-Level (Safe)	Second-Level (Relatively safe)	Third-level (relatively dangerous)	Fourth-level (dangerous)
$(0.85, 1]$	$(0.6, 0.85]$	$(0.3, 0.6]$	$(0, 0.3]$

4 Application Case Analysis

This chapter takes an office building in Beijing as an assessment object, and evaluates the fire risk based on the evaluation model established above. The office building has 20 floors, 73.5 meters high, the total floor area is 381600 square meters, the rental area is nearly 20000 square meters. The relevant data and evaluation index data of the office building are supported by the Fire Protection Institute of the Chinese Academy of Building Research.

4.1 Establishment of Evaluation Index System

In the process of evaluation, we investigate the actual situation of fire in buildings by consulting files, on-site in-quiries, building on-site verification, facilities and equipment function testing, and get the measured values of indicators combined with experts' opinions. The measured values are shown in the Table 11.

Table 11. The measured value of second-rank evaluation index

Second-level evaluation index	measured value	Second-level evaluation index	measured value	Second-level evaluation index	measured value
Building durability endurance	100	Fire separation distance	40	Perimeter risk	0.875
Building height	73.5	Weather conditions	0.725	Fire resistance rating of building structure	1.55
Current status of electrical accessory	0.725	Protection of electrical accessory	0.725	Fire load density	580
Distribution of fire load	0.725	Fire separation	0.725	fireproof plugging of special parts	0.925
fire partition area	1700	Smoke exhausting mode	0.95	Smoke emission volume	0.925
Air supplement setting	0.925	Pressurized air supply device	0.925	Type of detector	0.95
Detector reliability	0.95	Monitoring mode	0.95	Fire extinguishing device	0.925

Table 11. The measured value of second-rank evaluation index (continu)

Second-level evaluation index	measured value	Second-level evaluation index	measured value	Second-level evaluation index	measured value
System reliability	0.925	Fire extinguisher configuration standard	0.925	Municipal Pipeline	0.925
fire-fighting pool	0.925	Hydrant	0.925	The number of firemen	600
working life of firemen	10	Business building	0.925	Protection area of fire station	4750
Building area of fire station	4250	Fire fighting equipment	0.925	Fire escape	0.925
Fire fighting are	0.925	Safe evacuation distance	25	Safe export quantity	5
Hundred people’s width inde	1.17	Emergency broadcast lighting and evacuation indicator	0.925	Evacuation staircase	0.925
Refuge	0.925	Auxiliary evacuation facility	0.925	Personnel distribution status	0.012
Fire protection facilities maintenance and periodical maintenance	0.925	Autocratic duty	0.95	Fire emergency plan	0.925
Safety responsibility system	0.95	Fire prevention education	0.95	Organization skill	0.95
Firefighting knowledge and skills	0.95	Safety awareness level	0.725	Fire prevention training	0.725
Hidden trouble rectification	0.925	Organization	0.925		

4.2 Modular Fuzzy Comprehensive Evaluation

After obtaining the data of various indicators, we evaluate the risk situation of the building from five aspects of index system, including the passive fireproofing ability of building, active fireproofing ability of building, fire fighting and rescue ability, evacuation capability and fire management level. This section will give a brief analysis of the calculation process in the specific evaluation.

4.2.1 The Calculation Process of the Passive Fireproofing Ability of the Building

1. The calculation process of first-level comprehensive evaluation

(1)According to formula 2-8, the standard index value and the measured value of the index are normalized to get the membership degree matrix of the standard index and the membership degree vector of the index value, then get the weight of the index. Based on he previously determined membership function and formula 14, get the result of first-level comprehensive evaluation:

$$B_1 = \begin{bmatrix} 0.3892 & 0.3676 & 0.1743 & 0.0689 \\ 0.75 & 0.25 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.6555 & 0.3445 & 0 \\ 0.5438 & 0.4563 & 0 & 0 \end{bmatrix}$$

(2)According to the formula 16 and 17, the results of the above evaluation are quantified, the results are shown in Table 12.

Table 12. The score of the first level evaluation indexe after normalization

i	1	2	3	4	5
Score before normalization X'_i	0.658	0.8536	0.725	0.6187	0.8084
Score after normalization X_i	0.6922	0.9143	0.7667	0.6485	0.8601

2. The calculation process of second-level comprehensive evaluation

(1) The weight of the first level evaluation index W_1 is calculated according to the formula 19 and 20:

$$W_1=[0.21,0.181,0.202,0.215,0.191]$$

The result of second-level comprehensive evaluation is obtained by the formula 18:

$$P = W_1 \cdot B_1 = [0.21,0.181,0.202,0.215,0.191] \cdot \begin{bmatrix} 0.3892 & 0.3676 & 0.1743 & 0.0689 \\ 0.75 & 0.25 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.6555 & 0.3445 & 0 \\ 0.5438 & 0.4563 & 0 & 0 \end{bmatrix}$$

$$= [0.3183,0.5564,0.1108,0.0145]$$

According to formula 17 and 22, the result of second-level comprehensive evaluation is quantified:

$$\mathrm{R} = 0.7563$$

According to the fire risk classification which is shown in the tabla10, therefore, the evaluation of the passive fireproofing ability of the object building belongs to the the second- level.

The evaluation process of the object in active fireproofing ability of building, fire fighting and rescue ability, evacuation capability and fire management level is the same as that of the above process.

4.3 Evaluation Results

The results of the fire hazard assessment of the office are shown in Table 13.

Table 13. The calculation results of risk level

Evaluation content	Fire performance score	Rank
the passive fireproofing ability	0.7563	Second- level (relatively safe)
active fireproofing ability of building	0.9204	First-level (Safe)
fire fighting and rescue ability	0.922	First-level (Safe)
evacuation capability	0.9124	First-level (Safe)
fire management	0.8875	First-level (Safe)
The whole building fire risk.	0.8847	First-level (Safe)

5 Summary

Based on the analysis of the present situation of high-rise building fire hazard, the author applies the theory and method of system safety and fuzzy mathematics to quantitative fuzzy evaluation for high-rise building fire hazard. The merits of this method are as follows:

This paper starts from the perspective of system engineering and combines with the fire prevention design standard, establish a modular and scientific high-rise building fire evaluation index system , which provides practical reference for building fire prevention design, safety management and fire hazard assessment. When dividing the standard intervals of evaluation indicators, we consider the nonlinear nature of people’s psychological measurement changes, so that the whole model is more objective and scientific, and at the same time, it is conducive to the flexible operation of evaluation.

Because of the complexity of the building fire itself and the fuzziness of the fire risk assessment,, it is very difficult to describe its risk level accurately and objectively. Fuzzy analysis is applied to deal with the uncertainty problem in building fire. Some qualitative descriptions and human subjective judgements are expressed in a magnitude manner, so that the risk assessment results of building fires are more consistent with objective reality.

The classical statistics and fuzzy statistics are extended to set-valued statistics, which solves the problem that traditional fuzzy comprehensive evaluation results can not be quantitatively expressed, making the evaluation result more intuitive and scientific.

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