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Stability Analysis of Hybrid Impulsive and Switching Stochastic Neural Networks with Time-Delay

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Abstract. This paper studies a class of hybrid impulsive and switching stochastic neural networks with time-delay. Using stochastic analysis theory and impulsive differential inequality theory, we establish the *p*th moment global asymptotical stability, *p*th moment global exponential stability and the mean square stability of the considered systems. Furthermore, we covert the stability analysis to solving a LMI (linear matrix inequality). Numerical examples are provided to illustrate the theoretical results.

Keywords: impulsive and switching, stability, stochastic neural networks, time-delay

1 Introduction

In recent years, differential impulsive systems are employed to model many real processes where parameters may abruptly change in system structure [1-2]. Researchers find that the performances of differential impulsive models are better than other models [1-5]. Therefore, these systems have widely been applied to control theory, biology and so on. A differential impulsive system is a dynamic system with the interaction between discrete dynamics and continuous dynamics [3-5]. As is known to all, stability is one of the most important dynamical behavious of a system. In the real world, the stability of a differential impulsive system is often influenced by many factors, such as impulses, time-delay and so forth [6]. In realty, a lot of practical problems can be described by differential impulsive systems naturally. For instance, many biology researchers point out that each cell contains a number of "regulatory" genes which act as switches by turning one another on and off. Similarly to the systems of "regulatory" genes, many practical systems are subject to known or unknown abrupt parameter variations or sudden change of system structures due to the failure of a component. It is worth mentioning that the impulsive control scheme can provide an effective approach for controlling some highly nonlinear complex dynamical systems. But, at same time, the mechanisms may bring many new challenges on system stabilization, which is beyond the conventional theory.

Furthermore, besides impulsive effects, stochastic effects exist widely in real systems [7]. In fact, the stochastic perturbation is unavoidable in the real word [8]. These stochastic effects may come from abrupt phenomena such as stochastic failures and repairs of the components, sudden environment changes, and changes of the interconnections of subsystems.

In the past decade, the stability of stochastic dynamic systems has also been intensively investigated [8-13]. For example, the convergence of Euler Maruyama (EM) method for SDEs has been studied by Buckwar et al. [9]. Furthermore, in [8] and [11], Mao and Gard have studied the existence and uniqueness of the analytic solution of stochastic differential equations (SDEs). The stability of EM method for SDEs has been studied by Cao et al [10] and Liu et al. [12]. Omar et al. [13] have investigated the stability and convergence of composite Milstein method for SDEs.

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The stochastic differential impulsive systems have been extensively intensified. The reason is that the corresponding problems are not only academically challenging but also of practical importance in many branches of science and engineering [14-15]. These systems are adequate mathematical models for many processes and phenomena, which have been studied in biology, physics technology, population dynamics, solar-powered systems, and so on [14, 16-17]. In [18], some sufficient conditions for the existence and global p-exponential stability of periodic solution for impulsive stochastic neural networks with time-delay have been established. In addition, references [19-24] have discussed the stability of some stochastic systems. In [24], Wu et al. have studied the p-moment stability of stochastic differential equations with impulsive jump and Markova switching.

Recently, researchers have transferred their attention to impulsive stochastic delay differential systems [25-29]. In [25-26], the authors have considered the global exponential stability of impulsive stochastic delay differential systems. By using auxiliary ordinary differential equations, the criteria of pth moment asymptotical stability has been obtained for impulsive stochastic delay differential systems [27] and for stochastic differential systems with Markova switching [28]. In [29], the authors have investigated pth moment exponential stability of impulsive delay differential systems.

However, although the stability of stochastic differential impulsive systems has stirred some initial research interest, there are few results about hybrid impulsive and switching stochastic neural networks with time-delay. Thus, in this paper, we aim to establish the stability criteria for a class of hybrid impulsive and switching stochastic neural networks, which can be written as follows

$$\begin{cases} dx(t)=\left[A_{\delta(t)}x(t)+B_{\delta(t)}f_{\delta(t)}\left(t,x(t)\right)\right]dt+g_{\delta(t)}\left(t,x(t),x(t-\tau)\right)d\omega(t), t\in[t_{k-1},t_k), \\ \Delta x(t_k)=J_k\left(t_k^-,x(t_k^-)\right), t=t_k, \\ x(t_0,\delta(k))=\xi(t_0+s), s\in[-\tau,0), \\ x(t_k)=Z_{\delta(k)}, \end{cases} \quad \textbf{(1)}$$

where $t\in R^+$, $x\in R^n$ is the variable; $t_0\geq 0$ is the initial time; $\delta(t):R^+\to I$, $I=\{1,2,...,m\}$, R^+ is the positive real number set, and the time sequence $\{t_k\}$ satisfies

$$0\leq t_0<t_1<\cdots<t_k<\cdots \quad \textbf{(2)}$$

and $\lim_{k\to\infty} t_k=\infty$.

Comparing with other existing results, the model is more generalization. In fact, many existing models are included in systems (1). For instance, if $g_{\delta(t)}\left(t,x(t),x(t-\tau)\right)=0$, then systems (1) becomes a hybrid impulsive and switching NN model in [30] without considering stochastic effects. If $\delta(k)$ is a constant function, systems (1) becomes an impulsive stochastic differential equation without switching in [31]. Since switching, time-delay, impulsive and stochastic effects are all considered in systems (1), it is very difficult to analyze the stability. And many existed stability criteria for differential dynamical systems [29, 31-37] may be ineffective for systems (1). Moreover, several sufficient conditions are established to ensure the global asymptotical stability and global exponential stability for systems (1) in pth moment and mean square, respectively. The time-delay upper bound and convergence rate of our work are better than existing results, such as [23] and [34].

In this paper, unless otherwise specified, we employ the notation as follows. Let $(\Omega,F,\{F_t\}_{t\geq 0},P)$ be a complete probability space with filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., F_0 contains all p-null sets and F_t is right continuous) and $\|\cdot\|$ denote the Euclidean norm. Let $E[\bullet]$ be the expectation operator with respect to the probability space. $\omega(t)=(\omega_1(t),\omega_2(t),\cdots,\omega_n(t))$ presents n-dimensional Brownian with $E(\mathrm{d}\omega(\mathrm{t}))=0$ and $E((\mathrm{d}\omega(\mathrm{t}))^2)=\mathrm{dt}$.

This paper is organized as follows. Some assumption of hybrid impulsive and switching stochastic neural networks with time-delay, definitions and lemmas are listed in Section 2. Section 3 has established several criteria for pth moment global asymptotical and exponential stability. In addition, 2th moment global asymptotical and exponential stability for a kind of stochastic hybrid impulsive and switching neural networks are presented. In Section 4, several numerical examples are given to illustrate the theoretical results. Finally, Section 5 contains some conclusions and further work.

2 Preliminaries

To begin with, some conditions on systems (1), basic definitions and useful lemmas are introduced. Suppose that systems (1) admits a unique solution and the conditions (C1-C3) are satisfied at any bounded interval $0 < t_k - t_{k-1} \le T$.

(C1) Continuous functions $J_k(t_k^-, x(t_k^-)) : R^+ \times R^n \to R^n (k \ge 1)$, and $J_1(t,0) \equiv 0, t \in R^+$.

(C2) There exist nonnegative constant sequences $\{C_{\delta(k)}\}, \{D_{\delta(k)}\}$ such that

(i) $\|f_{\delta(k)}(t, x(t))\| + \|g_{\delta(k)}(t, x(t))\| \le C_{\delta(k)}(1 + \|x\|)$.

(ii) $\|f_{\delta(k)}(t, x) - f_{\delta(k)}(t, y)\| + \|g_{\delta(k)}(t, x) - g_{\delta(k)}(t, y)\| \le D_{\delta(k)}(\|x - y\|)$.

(C3) Let $x(t_k) = Z_{\delta(k)}$ be a random variable which is independent of the $\delta-$generated $F_\infty^{(m)}$ by $\omega_s(\cdot)$ ($\omega_s(\cdot)$ is a m-dimensional normal Brownian motion), and $E\left[\left\|Z_{\delta(k)}\right\|^2\right] \le M < \infty$.

Let $k_0 = \min\{k \in Z, t_k \ge k\}$, $\Lambda = \{1,2,...,k_0\} \cap \{k \in Z, \delta(k) = i\}$, $T_i(t_0, t) = \sum_{k\in\Lambda}\left[\min(t, t_k) - t_{k-1}\right]$ denotes the working time of the ith subsystem during the interval $[t_0, t]$, and $\mu(T_i(t_0, t))$ denote the Lebesgue measure of the set $T_i(t_0, t)$. Then systems (1) can be written as

$$\begin{cases} dx(s) = \left[A_i x(t) + B_i f_i(t, x(t))\right]dt + g_i(s, x(s), x(s-\tau))\mathrm{d}\omega(s), s \in T_i(t_0, t), \\ \Delta x(s_k) = x(s_k) - x(s_k^-) = J_k\left(s_k^-, x\left(s_k^-\right)\right), s = t_{k,} \\ x(t_0, \delta(k)) = \xi(t_0 + l), l \in [-\tau, 0), \\ x(s_k) = Z_i. \end{cases} \tag{3}$$

where $i \in I$ and $\bigcup_{i=1}^{m} T_i(t_0, t) = [t_0, t]$, $f_i(0, x(0)) = g(0, x(0)) = 0$ for all $i \in N$.

Let $\Gamma = \{t_i : i = 1,2,..., \}$, $\{\delta \le t_1 < t_2 < ... < t_i < ...\}$, $R_\delta = \{x \in R : x \ge \delta\}$, where $\delta \in R$ is a given constant. Moreover, $C^{1,2}\left(R^n \times R_\delta \times I; R^+\right)$ denotes the family of all nonnegative functions $V(x,t,i)$ on $R^n \times R_\delta \times I$ that are twice continuously differentiable in x and once in t. If $V(x,t,i) \in C^{2,1}$ $\left(R^n \times R_\delta \times I; R^+\right)$, define an operator L associated with systems (3) from $R^n \times R_\delta \times I$ to R by

$$LV(t, x(t), i) = V_t(t, x(t), i) + \mathrm{V}_x(t, x(t), i) f_i(t, x(t)) + \frac{1}{2} trace\left[g_i^T(t, x(t)) V_{xx} g_i(t, x(t))\right], \tag{4}$$

where

$$\begin{cases} V_t(t, x(t), i) = \partial V(t, x(t), i) / \partial t \\ V_{xx}(t, x(t), i) = \left(\dfrac{\partial^2 V(t, x(t), i)}{\partial x_i \partial x_j}\right)_{n\times n} \\ V_x(t, x(t), i) = \left(\dfrac{\partial V(t, x(t), i)}{\partial x_1}, \dfrac{\partial V(t, x(t), i)}{\partial x_2}, ..., \dfrac{\partial V(t, x(t), i)}{\partial x_n}\right) \end{cases} \tag{5}$$

Definition 2.1

(i)(pth moment asymptotically stable)The trivial solution of systems (1) and (3) is pth moment asymptotically stable if there exists $\delta = \delta(t_0) > 0$ such that

$$E\left(\left\|x(t)\right\|^p\right) < \delta \tag{6}$$

and

$$\lim_{t\to\infty} E\left\|x(t)\right\|^p = 0, t \ge t_0 \ge 0 . \tag{7}$$

(ii) (pth moment exponentially stable) The trivial solution of systems (1) and (3) is pth moment exponentially stable if there exist positive constants $\alpha > 0, K \ge 1$ such that

$$\lim_{t\to\infty} E\left\|x(t)\right\|^p \le Ke^{-\alpha(t-t_0)} E\left(\left\|x_0\right\|^p\right),\ \ t \ge t_0 \ge 0 . \tag{8}$$

(iii) (Asymptotically stable in mean square)The trivial solution of systems (1) and (3) is asymptotically stable in mean square if there exist $\delta = \delta(t_0) > 0$ such that

$$E\left(\left\|x(t)\right\|^2\right) < \delta \tag{9}$$

and

$$\lim_{t\to\infty} E\left\|x(t)\right\|^2 = 0, t \ge t_0 \ge 0 . \tag{10}$$

(iv) (Exponentially stable in mean square)Tthe trivial solution of systems (1) and (3) is pth moment exponentially stable if there exist positive constants $\alpha > 0, K \ge 1$ such that

$$\lim_{t\to\infty} E\left\|x(t)\right\|^2 \le Ke^{-\alpha(t-t_0)} E\left(\left\|x_0\right\|^2\right),\ \ t \ge t_0 \ge 0 . \tag{11}$$

Definition 2.2

$D^+ f(x) = \lim_{\Delta x \to 0^+} \left(\left(f(x+\Delta x) - f(x)\right)/\Delta x\right)$ if $f(x)$ is differentiable at its right side.

Furthermore, in order to finish our results, we will introduce the following lemmas.

Lemma 2.1. Let $\omega(t)$ be a nonnegative function defined on the interval $[t_0 - \tau, \infty)$ and be continuous on the interval $[t_0, \infty)$. Assume that

$$\dot{\omega}(t) \le -a\omega(t) + b\omega(t-\tau), t \ge t_0 , \tag{12}$$

where a and b are nonnegative constants satisfying $a>b$. Then

$$\omega(t) \le \bar{\omega}_0 \exp\left(-\int_{t_0}^{t} \lambda ds\right), \tag{13}$$

$\bar{\omega}_0 = \sup_{t_0-\tau\le t\le t_0} \omega(\theta)$ and $\lambda > 0$ satisfies

$$\lambda - a + be^{\lambda\tau} = 0 . \tag{14}$$

Lemma 2.2. If $V \in C^{2,1}\left(R^n \times R_\delta \times S; R^+\right)$, then

$$EV\left(t_2, x(t_2), \delta(t_2)\right) = EV\left(t_1, x(t_1), \delta(t_1)\right) + E\int_{t_1}^{t_2} LV\left(s, x(s), \delta(s)\right) ds , \tag{15}$$

for $\delta \le t_1 < t_2 < \cdots < t_n < \cdots$.

3 Main Results

For systems (1) and (3), assuming conditions (C1)-(C3) hold, one obtains the stability criteria as follows.

Theorem 3.1

If there exist switching Lyapunov functions $V_i(t) \triangleq V_i(x(t),t,i) \in C^{2,1}(R^n \times R_\delta \times I; R^+)$ and positive numbers c_i and d_i $(i=1,2,\cdots,N)$ such that

(C4) $$E\left[LV_i(t)\right] \le -a_i E\left[V_i(t)\right] + b_i E\left[V_i(t-\tau)\right],\ i \in N, \quad \textbf{(16)}$$

$$\sum_{i=\delta(k)}\left[\ln\left(\frac{d_i}{c_i}\right) + p\ln\beta_k\right] + \sum_{k=1}^{n}\left[-\int_{t_{k-1}}^{t_k}\lambda_i ds\right] \le \varphi(t_0,t), \quad \textbf{(17)}$$

where $\varphi(t_0,t)$ is a continuous function on $R^+, a_i > b_i > 0,$ and λ_i satisfy the following equation:

$$\lambda_i + a_i + b_i e^{\lambda_i \tau} = 0, \quad \textbf{(18)}$$

(C5) $$c_i E\left[\left\|x(t)\right\|^p\right] \le EV_i(t) \le d_i E\left[\left\|x(t)\right\|^p\right], \quad \textbf{(19)}$$

(C6) $$\left\|x(t_k^-) + J_k\left(t_k^-, x(t_k^-)\right)\right\| \le \beta_k \left\|x(t_k^-)\right\|, k=1,2,\cdots, \quad \textbf{(20)}$$

then

$$\lim_{t\to+\infty}\varphi(t_0,t) = -\infty, \quad \textbf{(21)}$$

implies that the trivial solution of systems (1) and (3) is pth moment globally asymptotically stable, and

$$\varphi(t_0,t) \le -c(t-t_0),\quad t \ge t_0, \quad \textbf{(22)}$$

implies that the trivial solution of systems (1) and (3) is pth moment globally exponentially stable.

Proof. For any $x(t_0,i) \in PC_{F_0}^b\left(\left[-\tau,0\right]; R^n\right)$, we denote the solution $x(t,t_0,i)$ of systems (1) and (3) by $x(t)$. Let $V_i(t)$ be the switching Lyapunov function with $t \in \mu_i(t_0,t), i=\delta(k) \in N$. According to the Itô formula, for $t \in [t_k, t_{k+1})$, we have

$$dV_i(t) = LV_i(t)dt + g_i\left(t,x(t)\right)\frac{\partial_i V(t)}{\partial t}d\omega(t), \quad \textbf{(23)}$$

and $$V_i(t+\Delta t) - V_i(t) = \int_t^{t+\Delta t} LV_i(s)ds + g_i\left(t,x(t)\right)\frac{\partial V_i(t)}{\partial t}d\omega(t). \quad \textbf{(24)}$$

Then taking expectations in both side of (24), we have

$$V_i(t+\Delta t) - V_i(t) = \int_t^{t+\Delta t} LV_i(s)\mathrm{d}s + g_i(t,x(t))\frac{\partial V_i(t)}{\partial t}\mathrm{d}\omega(t). \quad \textbf{(25)}$$

By the continuity and the definition of the Dini derivation, we have

$$D^+V_i(t) = ELV_i(t), t \in [t_k, t_{k+1}), i=\delta(k) \in I. \quad \textbf{(26)}$$

It then follows from condition (C4) and Lemma 2.1 that exists λ_i such that

$$E\left[V_i(t)\right] \le E\left[V_i(t_k)\right] e^{-\int_{t_k}^{t_{k+1}} \lambda_i ds}, \quad \textbf{(27)}$$

where λ_i satisfies $\lambda_i - a_i + b_i e^{\lambda_i \tau} = 0$. Hence, using equation (19) and (20), we arrive at

$$E\left(V_i(t)\right) \le \left[EV_i(t_k)\right] e^{-\int_{t_k}^{t_{k+1}} \lambda_i ds} \le d_i E\left[\left\|x(t_k)\right\|^p\right] e^{-\int_{t_k}^{t_{k+1}} \lambda_i ds} \le d_i \left(\beta_k\right)^p E\left[\left\|x(t_k^-)\right\|^p\right] e^{-\int_{t_k}^{t_{k+1}} \lambda_i ds}, \quad \textbf{(28)}$$

for $t \in (t_k, t_{k+1}]$,

$$E\left(\left\|x(t)\right\|^p\right) \le \frac{d_i}{c_i}(\beta_k)^p \left[EV_{\delta(k)}(t_k^-)\right] e^{-\int_{t_k}^{t_{k+1}} \lambda_i ds} \le \cdots \le \prod_{i=\delta(k)} \frac{d_i}{c_i}(\beta_k)^p EV_{\delta(0)}(t_0^-) e^{\sum_{i=1}^{N} -\int_{t_k}^{t_{k+1}} \lambda_{\delta(k)} ds}, \quad \textbf{(29)}$$

where

$$\prod_{i=\delta(k)} \frac{d_i}{c_i}(\beta_k)^p EV_{\delta(0)}(t_0^-) e^{\sum_{i=\delta(k)} -\int_{t_k}^{t_{k+1}} \lambda_{\delta(k)} ds} = e^{\sum_{i=\delta(k)} \left[\ln(d_i/c_i) + p\ln(\beta_k)\right] + \sum_{k=1}^{n} -\int_{t_k}^{t_{k+1}} \lambda_{\delta(k)} ds} EV_{\delta(0)}(t_0) \le EV_{\delta(0)}(t_0) e^{\varphi(t_0,t)}. \quad \textbf{(30)}$$

It follows from the above discussion that

$$E\left[\left\|x(t)\right\|^p\right] \le E\left[V_{\delta(0)}(t_0)\right] e^{\varphi(t_0,t)}, t \in (t_k, t_{k+1}], \quad \textbf{(31)}$$

which implies the conclusions of the theorem. The proof is completed.

Using the proof techniques in Theorem 3.1, we can obtain the following Corollaries.

Corollary 3.1. Suppose that conditions in Theorem 3.1 hold. Then the trivial solution of systems (1) and (3) is *p*th globally exponentially stable if one of the following conditions is satisfied.

(D1) For all $i \in N$, $\alpha = \min\{\lambda_i\}$, if there exists a constant ε $(0 < \varepsilon < \alpha)$ such that

$$\ln\frac{d_i}{c_i} + p\ln\beta_k - \int_{t_k}^{t_{k+1}} \varepsilon ds \le 0, k = 1, 2, \ldots. \quad \textbf{(32)}$$

(D2) For any $t_{k+1} - t_k < T$, if there exist constant α and positive constant $\bar{\alpha}$ such that $\bar{\alpha} > \lambda_i \ge |\alpha|$ and

$$\ln\frac{d_i}{c_i} + p\ln\beta_k + \int_{t_k}^{t_{k+1}} \bar{\alpha} ds \le 0, k = 1, 2, \ldots. \quad \textbf{(33)}$$

Proof. When $\alpha = \min\{\lambda_i\}$, it follows from (32) that

$$\sum_{i=\delta(k)} \left[\ln\frac{d_i}{c_i} + p\ln\beta_k\right] + \sum_{k=1}^{n} \left[-\int_{t_k}^{t_{k+1}} \lambda_i ds\right] \le -(\alpha - \varepsilon)(t - t_0). \quad \textbf{(34)}$$

Let $\varphi(t_0, t) = -(\alpha - \varepsilon)(t - t_0)$ and note $(\alpha - \varepsilon) > 0$. Based on Theorem 3.1, we obtain that the trivial solution of system (1) and (3) is *p*th moment globally exponentially stable. When $\bar{\alpha} > \lambda_i \ge |\alpha|$, it follows from (33) that

$$\sum_{i=\delta(k)} \left[\ln\frac{d_i}{c_i} + p\ln\beta_k\right] + \sum_{k=1}^{n} \left[-\int_{t_k}^{t_{k+1}} \lambda_i ds\right] \le \bar{\alpha}T - (\bar{\alpha} - \alpha)(t - t_0). \quad \textbf{(35)}$$

Let $\varphi(t_0, t) = -(\bar{\alpha} - \alpha)(t - t_0)$. Noting that $\bar{\alpha} - \alpha > 0$, we can conclude the proof.

Corollary 3.2. In Theorem 3.1, if there exist switching Lyapunov functions $V_i(t) = x^T(t) P_i x(t)$, P_i is a positive definite matrix $(i = 1, 2, \cdots, n)$ such that

(D3) $$\left\| x\left(t_k^-\right) + J_k\left(t_k^-, x\left(t_k^-\right)\right)\right\| \le \beta_k \left\| x\left(t_k^-\right)\right\|, k = 1, 2, \cdots \quad \textbf{(36)}$$

(D4) $$E\left[LV_i\left(t\right)\right] \le -a_i E\left[V_i\left(t\right)\right] + b_i\left[V_i\left(t-\tau\right)\right], i \in N. \quad \textbf{(37)}$$

$$\sum_{k=1}\left[\ln\frac{d_i}{c_i} + p\ln\beta_k\right] + \sum_{i=1}^{N}\left[-\int_{t_{k-1}}^{t_k}\lambda_i ds\right] \le \varphi\left(t_0, t\right), \quad \textbf{(38)}$$

where $c_i = \lambda_{min}(P_i)$, $d_i = \lambda_{max}(P_i)$, $a_i > b_i > 0$ satisfying $\lambda_i - a_i + b_i e^{\lambda_i \tau} = 0$ and $\varphi(t_0, t)$ is a continuous function on R^+. Then $\lim_{t\to\infty}\varphi\left(t_0, t\right) = -\infty$, which implies that the trivial solution of systems (1) and (3) is globally asymptotically stable in mean square, and $\varphi\left(t_0, t\right) \le -c\left(t - t_0\right), t \ge t_0, c > 0$, which implies that the trivial solution of systems (1) and (3) is globally exponentially stable in mean square.

Proof. When P_i is a positive-definite matrix, there exists an orthogonal matrix U_i ($U_i^T U_i = I$, I is an identity matrix) such that

$$U_i^T P_i U_i = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}, \quad \textbf{(39)}$$

$\lambda_j > 0$ $(j = 1, 2, \cdots, n)$. Thus, We have

$$x^T\left(t\right)P_i x\left(t\right) = z^T\left(t\right)U_i^T P_i U_i z\left(t\right) = z^T\left(t\right)\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix} z\left(t\right) = \sum_{j=1}^{n}\left(\lambda_j z_j^2\right), \quad \textbf{(40)}$$

and

$$\left\|z\left(t\right)\right\|^2 = z^T\left(t\right)z\left(t\right) = \left[U_i x\left(t\right)\right]^T U_i x\left(t\right) = x^T\left(t\right)U_i^T U_i x\left(t\right) = x^T\left(t\right)x\left(t\right) = \left\|x\left(t\right)\right\|^2. \quad \textbf{(41)}$$

Obviously,

$$\lambda_{\min}\left(P_i\right)\sum_{j=1}^{n} z_j^2 \le \sum_{i=1}^{n}\left(\lambda_j z_j^2\right) \le \lambda_{\max}\left(P_i\right)\sum_{j=1}^{n} z_j^2 .$$

Thus, we obtain that

$$\lambda_{\min}\left(P_i\right)\left\|x\left(t\right)\right\|^2 \le V_i\left(t\right) = x^T\left(t\right)P_i x\left(t\right) \le \lambda_{\max}\left(P_i\right)\left\|x\left(t\right)\right\|^2 . \quad \textbf{(42)}$$

Let $c_i = \lambda_{\min}\left(p_i\right)$ and $d_i = \lambda_{\max}\left(p_i\right)$. In this part, it is similar to (C5) of Theorem 3.1 and p=2. In addition, equation (37) is similar to equation (17) of Theorem 3.1 ($\rho_i = d_i / c_i = \lambda_{\max}\left(P_i\right)/\lambda_{\min}\left(P_i\right)$, $\rho = \max_{i\in I}\left(\rho_i\right)$).

Similar to the proof of Theorem 3.1, we can easily complete the proof of Corollary 3.3.

Corollary 3.3. In Theorem 3.1, if $p = 2$ in (17) and $c_i E\left[\left\|x\left(t\right)\right\|^2\right] \le EV_i\left(t\right) \le d_i E\left[\left\|x\left(t\right)\right\|^2\right]$ in (19), then

$$\lim_{t\to\infty}\varphi\left(t_0, t\right) = -\infty, \quad \textbf{(43)}$$

which implies that the trivial solution of systems (1) and (3) is asymptotically stable in mean square, and $\varphi\left(t_0, t\right) \le -c\left(t - t_0\right), t \ge t_0, c > 0$, which implies that the trivial solution of systems (1) and (3) is exponentially stable in mean square.

Remark 3.1. From corollary 3.3, the results can be applied to the practical case of stochastic hybrid impulsive and switching neural networks. If we construct switching Lyapunov function by $V_i(t)=x^T(t)P_ix(t)$, a mean square stability condition for systems (1) and (3) can be obtained. Furthermore, the problem can be turn into solving a linear matrix inequality (LMI) via Schur complement. The LMI can be regarded as an optimization problem.

Next, we consider the global asymptotical exponential stability and global asymptotical stability for systems (1) and (3) in mean square. For simplify, suppose the solution of the model always exists, and several Lemmas are introduced as follows.

Lemma 3.1 [23]. Let Y be the real matrix of $n\times m$ dimensions and let P be a positive matrix. Then

$$trace\left[Y^TPY\right]\leq\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}trace\left[Y^TPY\right]. \tag{44}$$

Lemma 3.2. Let $P\in R^{n\times n}$ be a symmetric and positive definite matrix and $Q\in R^{n\times n}$ be a symmetric matrix. Then $\lambda_{\min}\left(P^{-1}Q\right)X^TPX\leq X^TQX\leq\lambda_{\max}\left(P^{-1}Q\right)X^TPX$.

Lemma 3.3. Let $A\in R^{n\times n}$ be a symmetric and positive definite matrix and ε be a positive constant. Then $2X^TY\leq\varepsilon X^TAX+\frac{1}{\varepsilon}Y^TA^{-1}Y$.

Theorem 3.2

If systems (1) and (3) satisfy the following conditions:

(E1) for $i\in N, j=1,2,\cdots,n$, there exist positive constants $M_j^{(i)}$ such that $\left|f_j^{(i)}(\alpha)\right|\leq M_j^{(i)},\forall\alpha\in R^n$.

(E2) for $i\in N, j=1,2,\cdots,n$, there exists positive constants $L_j^{(i)}$ such that $\left|f_j^{(i)}(\alpha)-f_j^{(i)}(\beta)\right|\leq L_j^{(i)}\left|\alpha-\beta\right|$.

(E3) for $k=1,2,\cdots$, there exist positive constants β_k such that $\left\|x\left(t_k^-\right)+J\left(t_k^-,x\left(t_k^-\right)\right)\right\|\leq\beta_k\left\|x\left(t_k^-\right)\right\|$.

(E4) for systems (1) and (3), there exists a constant $\sigma\geq1$ such that $t_k-t_{k-1}\geq\sigma\tau$ for all k.

(E5) $trace\left[g_i^T\left(t,x(t),x(t-\tau)\right)g_i\left(t,x(t),x(t-\tau)\right)\right]\leq\lambda_i^{(1)}\left\|x(t)\right\|^2+\lambda_i^{(2)}\left\|x(t-\tau)\right\|^2$.

(E6) there exist symmetric and positive-definite matrices P_i, positive-definite diagonal matrices Q_i,R_i and s_i positive constant scalars $\alpha_i\left(\alpha_i>\eta_i\right),i\in I$, such that

(e1) $\Omega_i+\alpha_iP_i\leq0$,

(e2) $\sum_{k=1}\left[2\ln\beta_j+\ln\rho_i\right]+\sum_{i=1}^{N}\left[-\lambda_i\mu\left(T_i\left(t_0,t\right)\right)\right]\leq\varphi\left(t_0,t\right)$, **(45)**

where $\varphi\left(t_0,t\right)$ and ρ_i are defined in Theorem 3.1, and

$$\Omega_i=P_iA_i+A_iP_i+P_iB_iQ_i^{-1}B_i^TP_i^T+\rho_i\lambda_i^{(1)}P_i+L_iQL_i, \tag{46}$$

and λ_i is the unique positive root of the equation $-\alpha_i+\eta_ie^{\lambda_i\tau}+\lambda_i=0$ with $\eta_i=\lambda_i^{(2)}$. Then $\lim_{t\to+\infty}\varphi\left(t_0,t\right)=-\infty$ implies that the trivial solution of (1) and (3) are globally square asymptotically stable, and $\varphi\left(t_0,t\right)\leq-c\left(t-t_0\right),c>0,t\geq t_0$ implies that the trivial solution of (1) and (3) are globally asymptotically mean square exponentially stable.

Proof. Consider the switching Lyapunov function

$$V_i(t)=x^T(t)P_ix(t),t\in[t_k,t_{k+1}),i=\delta(k). \tag{47}$$

Applying the Itô formula to (47), one has

$$dV_i(t)=LV_i(t)dt+g_i\left(t,x(t)\right)\frac{\partial V_i(t)}{\partial t}d\omega(t), \tag{48}$$

where

$$LV_i(t)=2x^T(t)P_i\times\left[A_ix(s)+B_if(s,x(s))\right]+trace\left[g_i^T(t,x(t),x(t-\tau))P_ig_i(t,x(t),x(t-\tau))\right].\quad\textbf{(49)}$$

Based on (E1-E5), Lemmas 2.1-2.2, and 3.1-3.3, we can prove that

$$\begin{aligned}LV_i(t)&\le x^T(t)(P_iA_i+A_iP_i)x(t)+x^T(t)P_iB_iQ_i^{-1}B_i^TP_i^Tx(t)+f_i^T(t)Q_if_ix(t)+trace\left[g_i^T(t,x(t),x(t-\tau))P_ig_i(t,x(t),x(t-\tau))\right]\\&\le x^T(t)\left[P_iA_i+A_iP_i+P_iB_iQ_i^{-1}B_i^TP_i^T\right]x(t)+f_i^T(t)Q_if_ix(t)+\lambda_i^{(1)}x^T(t)P_ix(t)+\lambda_i^{(2)}x^T(t-\tau)P_ix(t-\tau)\\&\le x^T(t)\left[P_iA_i+A_iP_i+P_iB_iQ_i^{-1}B_i^TP_i^T+\lambda_i^{(1)}P_i+L_iQ_iL_i\right]x(t)+\lambda_i^{(2)}x^T(t-\tau)P_ix(t-\tau)\\&\le-\alpha_iV_i(t)+\eta_iV(t-\tau),\end{aligned}\quad\textbf{(50)}$$

where $\eta_i=\lambda_i^{(1)}$ and λ_i satisfies $-\alpha_i+\eta_ie^{\lambda_i\tau}+\lambda_i=0$. Similar to the proof of Theorem 3.1, for any $t\in[t_k,t_{k+1})$, one can show

$$\lambda_{\min}(P_i)E\|x(t)\|^2\le E\left[V_i(t)\right]\le E\left[V_i(t_k)\right]e^{\int_{t_k}^{t_{k+1}}\lambda_ids}\le\lambda_{\max}(P_i)E\|x(t_k)\|^2e^{-\int_{t_k}^{t_{k+1}}\lambda_ids}\le\lambda_{\max}(P_i)\beta_k^2E\left\|x(t_k^-)\right\|^2e^{-\int_{t_k}^{t_{k+1}}\lambda_ids}.\quad\textbf{(51)}$$

Thus

$$\begin{aligned}E\|x(t)\|^2&\le\frac{1}{\lambda_{\min}(P_i)}E\left[V_i(t)\right]\\&\le\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_i)}E\left[\|x(t_k)\|^2\right]e^{-\int_{t_k}^{t_{k+1}}\lambda_ids}=\rho_iE\left[\|x(t_k)\|^2\right]e^{-\int_{t_k}^{t_{k+1}}\lambda_ids}\\&\;\;\vdots\\&\le\prod_{j=0,i=\delta(j)}^{k}[\rho_i]e^{\sum_{j=0,i=\delta(j)}^{k}\left[2\ln\beta_j\right]+\sum_{j=0,i=\delta(j)}^{k+1}\left[-\int_{t_j}^{t_{j+1}}\lambda_ids\right]}E\left[V_{\delta(t_0)}(t_0)\right]\\&=e^{\sum_{j=0,i=\delta(j)}^{k}\left[2\ln\beta_j+\ln\rho_i\right]+\sum_{j=0,i=\delta(j)}^{k}\left[-\lambda_i\mu(T_i(t_0,t))\right]}E\left[V_{\delta(t_0)}(t_0)\right]\le E\left[V_{\delta(t_0)}(t_0)\right]e^{\varphi(t_0,t)}.\end{aligned}\quad\textbf{(52)}$$

Then $\varphi(t_0,t)=-c(t-t_0),t\ge t_0$, which implies $E\|x(t)\|^2\le E\left[V_{\delta(t_0)}(t_0)\right]e^{\varphi(t_0,t)}$, and the trivial solution of systems (1) and (3) is globally asymptotically mean square exponentially stable. Then $\lim_{t\to+\infty}\varphi(t_0,t)=-\infty$, $E\left[\|x(t)\|^2\right]\to0$, which implies that the trivial solution of systems (1) and (3) are mean square globally asymptotically stable.

Remark 3.2. Via Schur complement, it is easy to show that (e1) of (45) is equivalent to the LMI

$$\begin{bmatrix}\Omega_i+\alpha_iP_i & -P_iB_i\\ -B_i^TP_i^T & Q_i\end{bmatrix}\le0.\quad\textbf{(53)}$$

where $\Omega_i=P_iA_i+A_iP_i+\rho_i\lambda_i^{(1)}P_i+L_iQ_iL_i$.

Remark 3.3. For computational consideration, the smaller α_i satisfying (e1) of (45) will be better. Because smaller α_i will lead to the smaller left side of (e2) of (45). Therefore, the smaller α_i is better to reduce the conservatism of (e2) of (45). In addition, based on the following optimization problem:

$$OP\begin{cases}\min\alpha_i\\ s.t.,\ LMI(53)\ \text{holds}.\end{cases}$$

We can obtain the smaller α_i in (53).

Corollary 3.4. Assume that (e1) of E6 in Theorem 3.2 holds. Then, the trivial solution of systems (1) and

(3) are mean square globally exponentially stable if one of the following conditions is satisfied.

(f1) Let $\zeta = min\{\alpha_i\}$, for all $i \in I$, there exists a constant $0 < \varepsilon < \varsigma$ such that

$$\ln\left(\rho\beta_k{}^2\right) - \int_{t_k}^{t_{k+1}} \varepsilon ds \le 0, k = 1,2,\cdots . \quad \mathbf{(54)}$$

(f2) For any $t_{k+1} - t_k \le T < \infty$, there exist constants γ, ς satisfying $|\varsigma| \le \alpha_i < \gamma$ such that

$$\ln\left(\rho\beta_k{}^2\right) + \gamma\int_{t_k}^{t_{k+1}} ds \le 0, k = 1,2,\cdots . \quad \mathbf{(55)}$$

Proof. When $\zeta = min\{\alpha_i\}$, it follows from (55) that

$$\sum_{i=1}^{k-1}\ln\left(\rho\beta_i^2\right) + \sum_{i=1}^{N}\alpha_i\mu\left(T_i\left(t_0,t\right)\right) \le -\left(\varsigma - \varepsilon\right)\left(t - t_0\right). \quad \mathbf{(56)}$$

Let $\varphi\left(t,t_0\right) = -\left(\varsigma - \varepsilon\right)\left(t - t_0\right)$ and note that $\varsigma - \varepsilon > 0$. Then the trivial solution of systems (1) and (3) are all mean square globally exponentially stable. When $|\varsigma| \le \alpha_i < \gamma$, it follows from (55) that

$$\sum_{k=1}\ln\left(\rho\beta_k^2\right) + \sum_{i=1}^{N}\alpha_i\mu\left(T_i\left(t_0,t\right)\right) \le -\left(\gamma - \varsigma\right)\left(t - t_0\right). \quad \mathbf{(57)}$$

Let $\varphi\left(t,t_0\right) = -\left(\gamma - \varsigma\right)\left(t - t_0\right)$. Then the trivial solution of systems (1) and (3) are mean square globally exponentially stable. The proof is completed.

Remark 3.4. According to Corollary 3.4, one can derive an estimation of exponential convergence rate of $x(t)$. Additionally, in the case of condition (f2) of Corollary 3.3, the parameter α_i resolves the systems are stable or not. For example, if α_i is positive, the stochastic switching subsystems might be stable, but the stochastic switching subsystems might be unstable provided that α_i is a negative or sign varying with respect to *i*.

Furthermore, if $V_i(t) = x^T(t)P_i x(t)$ is chosen in the proof of Theorem 3.1, the following result holds.

Corollary 3.5. Assume that $J_k\left(t_k^-, x\left(t_k^-\right)\right) = b_k x\left(t_k^-\right)$ and there exist constants T^+ and T^- such that $T^+ > t_k - t_{k-1} \ge T^- > 0$ for $k = 1,2,\cdots$. Moreover, if there exist symmetric and positive definite matrices P_i, positive definite diagonal matrices Q_i , and a constant δ such that one of the following conditions holds:

(g1) $P_iA_i + A_iP_i + P_iB_iQ_i^{-1}B_i^TP_i^T + \lambda_i^{(1)}P_i + L_iQL_i + \delta P_i < 0$ and $2\ln\left(1 + b_k\right) - \delta T^- \le 0$.

(g2) $P_iA_i + A_iP_i + P_iB_iQ_i^{-1}B_i^TP_i^T + \lambda_i^{(1)}P_i + L_iQL_i - \delta P_i < 0$ and $2\ln\left(1 + b_k\right) + \delta T^+ \le 0$.

Then the trivial solution of systems (1) and (3) are all globally exponentially stable.

Remark 3.5. Similarly, Via Schur complement, the former inequation of condition (g1) and (g2) are equivalent to the LMI with respect to Qi. Additionally, the LMI is easy to be written as follows:

$$\begin{bmatrix} \Omega_i \pm \delta P_i & -P_iB_i \\ -B_i^TP_i^T & Q_i \end{bmatrix} \le 0 .$$

4 Numerical Simulation

In this section, three examples are presented to illustrate the main theoretical results proposed in this paper.

Example 4.1

In this example, we consider a stochastic hybrid system form as follows:

$$\begin{cases} dx(t)=\left[Ax(t)+Bf\left(x(t)\right)\right]dt+g\left(t,x(t),x(t-\tau)\right)d\omega(t), t\in[iT,(i+1)T) \\ \Delta x(t)=bx\left(t^{-}\right), \qquad t=(i+1)T, i=1,2,\cdots \end{cases} \quad \textbf{(58)}$$

where $T=1,\tau=0.4,b=-1.92$,

$$A=\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, B=\begin{pmatrix} -0.6 & -0.3 \\ -0.21 & 0.15 \end{pmatrix},$$

$$g\left(t,x(t),x(t-\tau)\right)=\frac{1}{3}A\left(x(t)+x(t-\tau)\right)+B\left(\sin\left(x(t)\right)+\sin\left(x(t-\tau)\right)\right), \quad \textbf{(59)}$$

and the activation function are described as: $f_i\left(x\right)=\dfrac{0.2\cos(i\pi)x(t)}{1+x(t)^{10}}$.

By solving the LMI of the (e1) of condition (E6) of Theorem 3.2, we obtain a feasible solution which is shown as following:

$$\mathrm{P}=\begin{pmatrix} 2.04 & 0.32 \\ 0.32 & 2.84 \end{pmatrix}, \mathrm{Q}=\begin{pmatrix} 0.56 & 0 \\ 0 & 0.62 \end{pmatrix},$$

and α=0.12 . Noting that $t_i-t_{i-1}=T=1$, $\beta\approx0.92$, and $\eta=0.2421$, and letting γ=0.20 , we can obtain that $2\ln\left(1+b\right)-\delta T^{-}=2\ln0.92-0.12\times1\approx$ -0.244 <0 . Thus, as shown in Fig. 1, the system of (58) is globally exponentially stable.

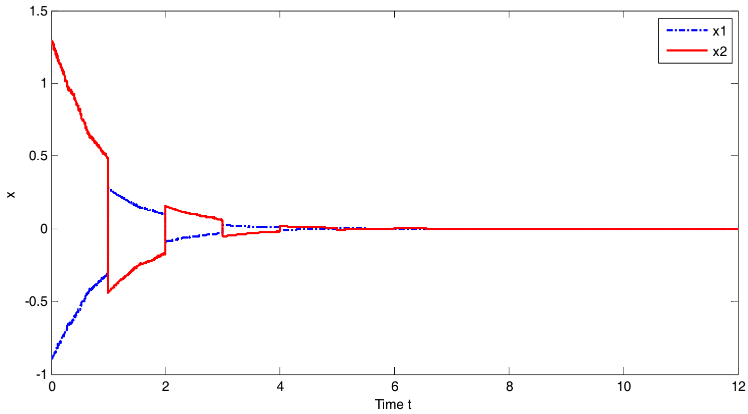

Fig. 1. Trajectories of the hybrid stochastic systems (58) with the initial value *x(0)=[-0.90 1.30]^T*

Example 4.2

In this example, consider a stochastic hybrid system (60). There are two subsystems in each stochastic hybrid system. The switching sequence is chosen as follows: subsystem 1→subsystem 2→subsystem 1→subsystem 2→⋯ Moreover, we suppose the activation function are piecewise liner functions, such as: $f_{ij}\left(x\right)=\alpha\left(\left|x+1\right|-\left|x-1\right|\right), 0<\alpha\leq1$.

$$\begin{cases} dx(t)=\left[A_1x(t)+B_1f_1\left(x(t)\right)\right]dt+g_1\left(t,x(t),x(t-\tau)\right)d\omega(t), & t\in[kT,kT+\sigma T), \\ \Delta x(t)=b_1x\left(t^{-}\right), & t=kT+\sigma T, \\ dx(t)=\left[A_2x(t)+B_2f_2\left(x(t)\right)\right]dt+g_2\left(t,x(t),x(t-\tau)\right)d\omega(t), & t\in[kT+\sigma T,(k+1)T), \\ \Delta x(t)=b_2x\left(t^{-}\right), & t=(k+1)T, k=0,1,2,\cdots \end{cases} \quad \textbf{(60)}$$

with $T=4,\sigma=0.5,b_1=b_2=-2.04,\tau=0.2$, and

$$A_1 = A_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.480 & -0.415 \\ -0.105 & 0.265 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.485 & 0.360 \\ 0.280 & 0.210 \end{pmatrix}, \tag{61}$$
$$g_i\left(t, x(t), x(t-\tau)\right) = A_1 B_2 \left(x(t) + x(t-\tau)\right) + \sin\left(A_2 B_1 \left(x(t) + x(t-\tau)\right)\right)$$

By solving the linear matrix inequality (LMI) of (e1) of condition (E6) of Theorem 3.2 for positive definite matrices P, and positive-definite diagonal Q_1, Q_2, we get a feasible solution as follows:

$$\mathrm{P} = \begin{pmatrix} 4.2161 & -0.0166 \\ -0.0166 & 4.6103 \end{pmatrix}, \mathrm{Q}_1 = \begin{pmatrix} 3.7937 & 0 \\ 0 & 4.7357 \end{pmatrix}, \mathrm{Q}_2 = \begin{pmatrix} 5.1489 & 0 \\ 0 & 5.4660 \end{pmatrix},$$

and $\delta = 0.04$. Note that $\beta = \beta_1 = \beta_2 = 1.04$, and $T^- = 2$. We can obtain that $2\ln(\beta) - \delta T^- = 2\ln 1.04 - 0.04 \times 2 \approx -0.0016 < 0$. Based on corollary 3.5, it can be verified that the system (60) is globally exponentially stable as shown in Fig. 2.

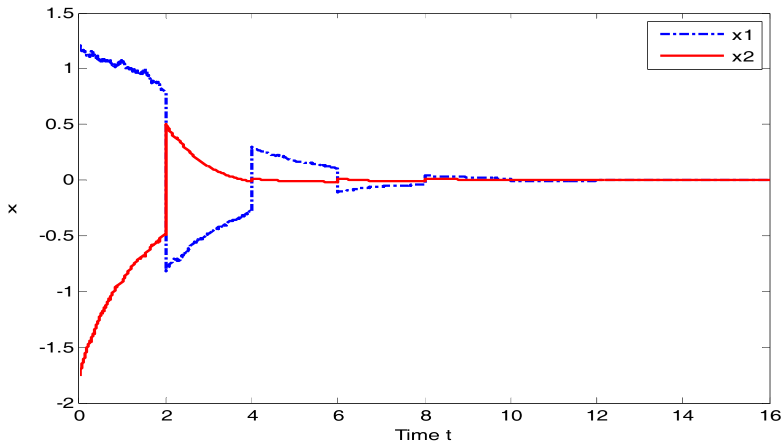

Fig. 2. Trajectories of the hybrid stochastic systems (60) with the initial value $x(0)=[1.20\ -1.76]^T$

Remark 4.1. To verify the performances of the approach presented in this paper, the comparison with results reported in [23] and [34] are listed in Table 1.

Table 1. Comparison of upper bound τ and exponential convergence rate c

Methods	Upper bound τ	Exponential convergence rate c
Pu et al. [23]	0.1900	0.3493
Li et al. [34]	0.2100	0.3229
Theorem 3.2	0.2000	0.4130

According to Table 1, it is easy to calculate that the stability criteria in this paper are less conservative in the sense of the computed time-delay upper bound. The convergence rate of hybrid stochastic systems also is faster.

Example 4.3

In this example, reconsider the stochastic hybrid system of the form of (58) with $T = 4, \tau = 2, b = -1.8084$,

$$A = \begin{pmatrix} -0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}, B = \begin{pmatrix} 0.2 & 0.1 & 0.1 \\ -0.2 & 0.3 & 0.1 \\ 0.1 & -0.2 & 0.3 \end{pmatrix},$$

and

$$g\left(t, x(t), x(t-\tau)\right) = x(t) + x(t-\tau) + \cos(t)\,. \tag{62}$$

Solving the maximal value of α satisfying the (e1) of condition (E6) of Theorem 3.2, we obtain that

$$\mathrm{P} = \begin{pmatrix} 1.458 & -0.0728 & 0.0710 \\ -0.073 & 1.8487 & -0.0003 \\ 0.071 & -0.0003 & 2.0748 \end{pmatrix}, \mathrm{Q} = \begin{pmatrix} 0.3338 & 0 & 0 \\ 0 & 0.4544 & 0 \\ 0 & 0 & 0.5122 \end{pmatrix},$$

and α=0.1. Noting that $t_i - t_{i-1} = T = 4$, $\beta = 0.8084$, and $\eta = 0.2553$ and letting δ=0.20, we can obtain that $2\ln\beta - \delta(t_i - t_{i-1}) = 2\ln(0.8084) - 0.2\times 4 \approx -1.2253 < 0$. Based on corollary 3.4, it can be verified that the system (58) is globally exponentially stable as shown in Fig. 3.

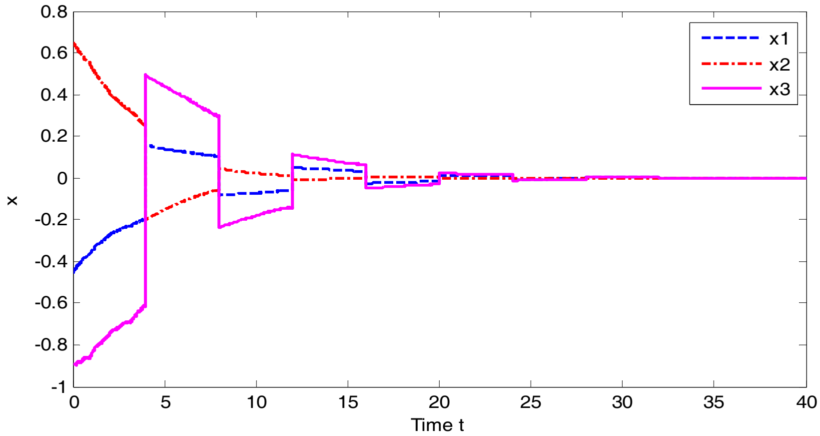

Fig. 3. Trajectories of the hybrid stochastic systems (58) with the initial value *x(0)=[-0.45 0.65 -0.9]*^{T}

5 Conclusion

In this paper, we have discussed the stability of hybrid impulsive and switching stochastic neural networks with time-delay. Switching Lyapunov functions and stochastic analysis techniques have been used to establish general criteria for the asymptotic and exponential stability of the new model. The new results can be used to analyze mean square global asymptotical and exponential stability for a kind of hybrid stochastic impulsive and switching neural networks. Furthermore, the results have illustrated by a number of numerical examples, which verified the effectiveness of the theoretical results. Although the exponential stability of systems in this work is derived by using Halanay inequality, the stability results are conservativeness. The reason is that it is difficult to construct a more effective Lyapunov-Krasovskii function to satisfy Halanay inequality. How to reduce the conservativeness and improve the convergence rate of hybrid stochastic systems is further work.

Acknowledgements

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Reference

[1] I.A. Hiskens, Power system modeling for inverse problems, IEEE Transactions on Circuits and Systems I: Regular Papers 51(3)(2004) 539-551.

[2] J. Krupar, W. Schwarz, EMI tuning of hybrid systems by periodic patterns, IEEE Transactions on Circuits and Systems I:

Regular Papers 53(9)(2006) 2060-2067.

[3] G. Feng, J. Cao, Stability analysis of impulsive switched singular systems, IET Control Theory & Applications 9(6)(2015) 863-870.

[4] I. Stamova, Global stability of impulsive fractional differential equations, Applied Mathematics and Computation 237(3)(2014) 605-612.

[5] P. Cheng, F. Deng, F. Yao, Exponential stability analysis of impulsive stochastic functional differential systems with delayed impulses, Communications in Nonlinear Science and Numerical Simulation 19(6)(2014) 2104-2114.

[6] F. Yao, P. Cheng, H. Shen, L. Qiu, On input-to-state stability of impulsive stochastic systems with time delays, Abstract and Applied Analysis 2014(9)(2013) 299-311.

[7] B. Ksendal, Stochastic Differential Equations: An Introduction with Applications, vols. 1-3, Spring-Verlag, Berlin, 1995.

[8] X. Mao, Stochastic Differential Equations and Applications, Chichester, Horwood, 1997.

[9] E. Buckwar, Introduction to the numerical analysis of stochastic delay differential equations, Journal of Computational and Applied Mathematics 125(1-2)(2000) 297-307.

[10] W.R. Cao, M.Z. Liu, Z.C. Fan, MS-stability of the Euler-Maruyama method for stochastic differential delay equtions, Appl. Math. Comput 159(1)(2004) 127-135.

[11] T.C. Gard, Introduction to Stochastic Differential Equations, Marcel Dekker, New York, 1988.

[12] W. Liu, X.R. Mao, Strong convergence of the stopped Euler-Maruyama method for nonlinear stochastic differential equations, Appl. Math. Comput 223(4)(2013) 389-400.

[13] M.A. Omar, A.A. Hassan, S.I. Rabia, The composite Milstein methods for the numerical solution of stratonovich stochastic differential equations, Appl. Math. Comput 215(2)(2009) 727-745.

[14] H. Liu, L. Zhao, Z. Zhang, Y. Ou, Stochastic stability of Markovian jumping Hopfield neural networks with constant and distributed delays, Neurocomputing 72(16)(2009) 3669-3674.

[15] C. Huang, J. Cao, On pth moment exponential stability of stochastic Cohen-Grossberg neural networks with time-varying delays, Neurocomputing 73(4)(2013) 986-990.

[16] Y. Sun, J. Cao, Stabilization of stochastic delayed neural networks with Markovian switching, Asian Journal of Control 10(3)(2008) 327-340.

[17] L. Huang, X. Mao, On almost sure stability of hybrid stochastic systems with mode-dependent interval delays, IEEE Transactions on Automatic Control 55(8)(2010) 1946-1952.

[18] F. Yao, J. Cao, P. Cheng, L. Qiu, Generalized average dwell time approach to stability and input-to-state stability of hybrid impulsive stochastic differential systems, Nonlinear Analysis: Hybrid Systems 22(2016) 147-160.

[19] L. Pan, J. Cao, Robust stability for uncertain stochastic neural network with delay and impulses, Neurocomputing 94(3)(2012) 102-110.

[20] L. Xu, D. He, Q. Ma, Impulsive stabilization of stochastic differential equations with time delays, Mathematical and Computer Modelling 57(3-4)(2013) 997-1004.

[21] Q. Zhu, S. Song, T. Tang, Mean square exponential stability of stochastic nonlinear delay systems, International Journal of Control 90(11)(2017) 1-10.

[22] X. Wu, W. Zhang, Y. Tang, pth moment stability of impulsive stochastic delay differential systems with Markovian switching, Communications in Nonlinear Science and Numerical Simulation 18(7)(2013) 1870-1879.

[23] X. C. Pu, W. Yuan, Stability of hybrid stochastic systems with time-delay, ISRN Mathematical Anlysis 2014(41)(2014) 1-8.

[24] H. Wu, J. Sun, p-Moment stability of stochastic differential equations with impulsive jump and Markovian switching, Automatica 42(10)(2006) 1753-1759.

[25] M.S. Alwan, X.Z. Liu, W.C. Xie, Existence, continuation, and uniqueness problems of stochastic impulsive systems with time delay, J Franklin Inst 347(7)(2010) 1317-1733.

[26] P. Cheng, F. Deng, Global exponential stability of impulsive stochastic functional differential systems, Stat Probab Lett, 80(23-24)(2010) 1854-1862.

[27] L. Pan, J. Cao, Exponential stability of impulsive stochastic functional differential equations, J Math Anal Appl 382(2)(2011) 672-685.

[28] J. Liu, X. Liu, W. Xie, Impulsive stabilization of stochastic functional differential equations, Applied Mathematics Letters 24(3)(2011) 264-269.

[29] L. Shen, J. Sun, pth moment exponential stability of stochastic differential equations with impulse effect, Sci China Inf Sci 54(8)(2011) 1702-1711.

[30] X. Li, New results on global exponential stabilization of impulsive functional differential equations with infinite delays or finite delays, Nonlinear Analysis: Real World Applications 11(5)(2010) 4194-4201.

[31] F. Yao, J. Cao, L. Qiu, P. Cheng, Exponential stability analysis for stochastic delayed differential systems with impulsive effects: average impulsive interval approach. <https://onlinelibrary.wiley.com/doi/abs/10.1002/asjc.1320>, 2016.

[32] Q, Zhu, B. Song, Exponential stability for impulsive nonlinear stochastic differential equations with mixed delays, Nonlinear Anal RWA 12(5)(2011) 2851-2860.

[33] J. Tan, C. Li, T. Huang, The stability of impulsive stochastic Cohen-Grossberg neural networks with mixed delays and reaction-diffusion terms, Cognitive Neurodynamics 9(2)(2015) 213-220.

[34] C.D. Li, F. Gang, On Hybrid Impulsive and Switching Neural Networks, IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 38(6)(2008) 1549-1560.

[35] S. Peng, B. Jia, Some criteria on pth moment stability of impulsive stochastic functional differential equations, Stat Probab Lett 80(80)(2010) 1085-1092.

[36] W. Zhang, Y. Tang, W.K. Wong, Stochastic stability of delayed neural networks with local impulsive effects, IEEE Transactions on Neural Networks and Learning Systems 26(10)(2015) 2336-2345.

[37] M.H. Song, H.Z. Yang, M.Z. Liu, Convergence and stability of impulsive stochastic differential equations, International Journal of Computer Mathematics 94(9)(2016) 1-9.