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Nonlinear System State Estimation Mechanism Based on Kalman Filter for Wireless Sensor Networks Localization

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Abstract. The article deals with a class of state vector estimation problems of nonlinear systems, which is derived from single node localization in wireless sensor networks. And a new strong adaptive Kalman filter mechanism is implemented by combining the original nonlinear filtering algorithm such as Square-root Cubature Kalman Filter (SCKF) with Kalman filter. Firstly, the mechanism utilizes the state estimation algorithm based on SCKF to estimate and correct the state vector in state-space model. Kalman filter is then performed for further processing due to the linear changes of state equation. Furthermore, the strong adaptive filter mechanism with Extended Kalman Filter (EKF) is established for comparative purposes, and the Cramer-Rao Bound (CRB) based on the nonlinear model is also derived. Finally, to verify the effectiveness of the mechanism, numerical simulation is made. Results analysis illustrates that the proposed mechanism has high location accuracy and is better than that of the original filtering algorithm without strong adaptive recursion.

Keywords: node localization, state vector estimation, strong adaptive filter mechanism, wireless sensor networks

1 Introduction

Wireless sensor networks (WSNs) are self-organizing networks within which a large number of distributed autonomous nodes are arbitrarily deployed. Recent advancements in the wireless communications have facilitated the development of WSNs for a wide variety of real-world applications, including traffic control, environmental monitoring, object tracking, and so on [1]. Many applications of WSNs require location information of the randomly deployed nodes.

Existing localization systems basically consists of three distinct components [2-3]: The first component known as ranging, distance measurement may be completed by using received signal strength indication (RSSI), time of arrival/time difference of arrival (TOA/TDOA). Second, position estimation of the unknown node is carried out using the ranging information and positions of reference nodes, which is done by solving a set of simultaneous equations. Such methods include trilateration, multilateration and triangulation. The third is localization algorithm, which determines how the available information will be manipulated in order to allow most or all of the nodes of a WSN to estimate their positions.

For range-free localization algorithms omitting ranging, such as centroid localization and DV-Hop localization [4-5], have already caused much attention. However, these advantages make them only suitable for the coarse localization. Conversely, range-based localization has higher location accuracy. Sometimes these two kinds of techniques are applied by using an optimization algorithm that minimizes the localization error to locate most or all of the nodes [6].

The case of location estimation of a single node is considered here. In [7], it formulates a dynamical system that encodes both the target moving manners and coarse sensor locations in an augmented state by integrating augmented transition and observation models. Ho et al. [8] reviews simultaneous localization

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and mapping problem based on different filtering techniques used to do the state estimation of the mobile robot.

In this article, we will transform the single node localization problem in WSNs into the state vector estimation problem of nonlinear system, where nonlinear filtering estimation is performed by using the parameters of the node location as the state vector, and the real-time correction is made with the increase of the number of sampling points.

In recent years, nonlinear state estimation has played an important role, which has important applications in target tracking, information processing, parameter estimation and localization. Kalman filter [9] is computationally efficient due to its recursive nature and its optimality for linear systems. Here we will adopt the derived Kalman filter [10-11] to address the nonlinearity issue aroused in the considered problem. Extended Kalman Filter (EKF) and Cubature Kalman Filter (CKF), and other nonlinear filtering methods are developed. However, these derived Kalman filters are not the optimal filter for non-Gaussian model [12]. Particle filter is also a class of nonlinear filtering method [13]. When the nonlinearity of the system is stronger, the effect of particle filter is more obvious than that of Kalman filter and its improved method. In particular, the CKF algorithm proposed by Arasaratnam et al. [14], which described nonlinear transfer of system state by using the integral rule of the spherical radial volume criterion. It not only overcomes the application limitation of EKF in strongly nonlinear system, but also has higher filtering accuracy than the particle filter.

In order to improve the performance of CKF, the square root of the error covariance matrix is added in its filtering process, which is used to guarantee the symmetry of the error covariance matrix. Thus, the Square-root Cubature Kalman Filter (SCKF) algorithm is formed [15].

In [15], the RSSI state estimation algorithm based on SCKF converts the RSSI localization problem into the state vector estimation problem of nonlinear system, which utilizes the SCKF to estimate the target's position and the channel attenuation parameter, and uses dynamic channel parameter to correct the node's position in real time in order to improve accuracy.

The SCKF algorithm does not have the adaptability to the change of measurement condition and model uncertainty of system. Measurement malfunctions or system model changes will affect the filter performance and even lead to filter failure. The robustness of the measurement malfunctions can be improved by building adaptive filter with innovation covariance matching techniques, and the influence of the model uncertainty can be solved by viewing Strong Tracking Filter (STF) as the basic theory framework [16]. However, STF has theoretical limitations of low precision and calculation of Jacobian matrix etc. Considering the influence of noise, especially in the case of low signal to noise ratio, the SCKF filter will result in divergence, its performance will be restricted.

Motivated by these, we propose a strong adaptive Kalman filter mechanism which combines the ordinary nonlinear filtering algorithm with Kalman filter for improved estimation results. The mechanism performs Kalman filter for further processing on the basis of SCKF or EKF state vector estimation. The key technical problems that need to be solved are how to establish the correlation between common filtering location algorithm and Kalman filtering recursion, and how to apply the two specific algorithms such as SCKF and EKF to the solution process of the state estimation problems of nonlinear systems derived from single node localization.

The technical achievements of this work are as follows: (1) Compared with single Kalman filter only adapted to the linear system model, the hybrid filter method does not depend on the inappropriate initial conditions and can be well applicable to highly non-linear system; (2) The strong adaptive filtering algorithm can obtain lower root mean square error lower bound than that of the original filtering algorithm without strong adaptive recursion; (3) The stability of the strong adaptive Kalman filter mechanism is good and divergent failure will not easily happen.

The article is organized as follows. In the next section, related work has been presented. Section 3 presents the formulation of the proposed mechanism. Numerical simulation and results analysis is illustrated in section 4. In Section 5, conclusions are given.

2 Related Work

2.1 Wireless Channel Propagation Model

In range-based schemes, RSSI technique [17] has been widely used for distance measurement.

As depicted in Fig. 1, a sender node sends a signal with a determined strength that fades as the signal propagates, and a known wireless channel propagation model can be used to convert the signal strength received by various known nodes into distance.

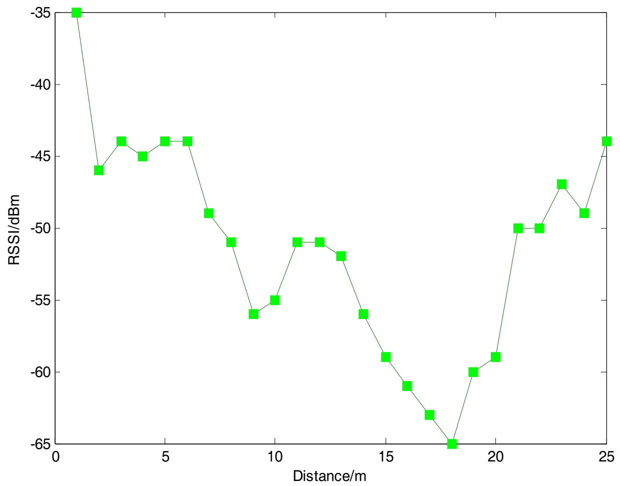

Fig. 1. Variation of RSSI for different distances

The propagation model at a given transmission distance can be described as:

$$P_r(d)[dBm] = P_r(d_0) - 10\eta \lg\left(\frac{d}{d_0}\right) + v_\alpha , \qquad \textbf{(1)}$$

where d is the transmission distance, $P_r(d_0)$ is the received signal power at reference distance d_0 ($d_0 = 1m$), $P_r(d)$ is the received signal power from the sender. η is the channel attenuation parameter, and v_α is a zero-mean Gaussian noise with standard deviation α.

The relationship between RSSI values and the received signal power from the receiving node is expressed by:

$$P_r(d)[dBm] = RSSI(d) + OFFSET , \qquad \textbf{(2)}$$

where $OFFSET$ is a constant value. $RSSI(d)$ is the received signal strength from the sender.

It follows from (1) and (2) that there exists

$$RSSI(d)[dBm] = RSSI(1) - 10\eta \lg(d) + v_\alpha . \qquad \textbf{(3)}$$

The RSSI can be estimated when the unknown nodes receive the signal from the beacon nodes, and the distance can be calculated via (3).

2.2 State-Space Model

It is assumed that wireless communication can be formed between two nodes if and only if they are within their communication range [18]. We assume that all nodes have the same communication range. The relationship about the distance d_i and RSSI value between the unknown node and the i^{th} beacon node can be written as

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}\ , \tag{4}$$

$$RSSI_i(\gamma)[dBm] = RSSI_{d_0} - 10\eta\lg(d_i) + v_i, 1 \le i \le n\,, \tag{5}$$

where $\gamma = \left[x, y, \eta, RSSI_{d_0}\right]^T$ is the state vector to be estimated, $RSSI_i(\gamma)$ received by the unknown node is the signal strength from the i^{th} beacon node, $RSSI_{d_0}$ is the received signal strength at reference distance d_0, v_i denotes the value obtained from the noisy range measurements corresponding to the i^{th} beacon node.

Noise-free received signal strength $H_i(\gamma)$ derived from Eq. (4) and (5) is given as

$$H_i(\gamma) = RSSI_{d_0} - 10\eta\lg\sqrt{(x_i - x)^2 + (y_i - y)^2}, 1 \le i \le n\,. \tag{6}$$

Eq. (5) is represented as a vector form: $RSSI = H(\gamma) + v$, where $RSSI = [RSSI_1, RSSI_2, \cdots RSSI_n]^T$, $H(\gamma) = [H_1(\gamma), H_2(\gamma), \cdots H_n(\gamma)]^T$, $v = [v_1, v_2, \cdots v_n]^T$.

Let the state vector of blind node at time k^{th} be $\gamma_k = \left[x_k, y_k, \eta_k, RSSI_{d_0,k}\right]^T$, we can obtain a nonlinear system with additive noise, whose state-space model is defined as:

$$\text{Process equation: } \gamma_k = A\gamma_{k-1} + w_{k-1}\,, \tag{7}$$

$$\text{Measurement equation: } \mathrm{RSSI}_k = H(\gamma)_k + v_k\,, \tag{8}$$

where $A_{4\times4}$ is the unit state transition matrix; $\{w_{k-1}\}$ and $\{v_k\}$ are respectively independent process and measurement gaussian noise sequences with zero means and covariance matrices $Q_{k-1} = \mathrm{cov}(w_{k-1}) = diag\left(\sigma_x^2, \sigma_y^2, \sigma_\eta^2, \sigma_{RSSI_{d_0}}^2\right)$ and $R_k = \mathrm{cov}(v_k) = diag\left(\sigma_{v,1}^2, \sigma_{v,2}^2, \cdots, \sigma_{v,n}^2\right)$, respectively.

2.3 RSSI State Estimation Based on SCKF

For the state-space model depicted in Eq. (8) and (9), it is ready to apply SCKF which consists of two consequent stages at step $k(k = 1, 2, \cdots N)$.

Time update

(1)Evaluate the cubature points $(i = 1, 2, \cdots m)$: $\Gamma_{i,k-1|k-1} = S_{k-1|k-1}\varepsilon_i + \hat{\gamma}_{k-1|k-1}$, where $m = 2n_\gamma$.

(2)Evaluate the propagated cubature points $(i = 1, 2, \cdots m)$: $\Gamma^*_{i,k|k-1} = A\Gamma_{i,k-1|k-1}$.

(3)Estimate the predicted state: $\hat{\gamma}_{k|k-1} = \sum_{i=1}^{m} \frac{1}{m}\Gamma^*_{i,k|k-1}$.

(4) Estimate the square-root factor of the predicted error covariance: $S_{k|k-1} = tria\left(\gamma^*_{k|k-1}\ S_{Q,k-1}\right)$, where $S_{Q,k-1}$ denotes a square-root factor of Q_{k-1} such that $Q_{k-1} = S_{Q,k-1}S_{Q,k-1}^T$ and the weighted, centered matrix $\gamma^*_{k|k-1} = \frac{1}{\sqrt{m}}\left[\Gamma^*_{1,k|k-1} - \hat{\gamma}_{k|k-1}, \cdots, \Gamma^*_{m,k|k-1} - \hat{\gamma}_{k|k-1}\right]$.

Measurement update

(1) Evaluate the cubature points $(i = 1, 2, \cdots m)$: $\Gamma_{i,k|k-1} = S_{k|k-1}\varepsilon_i + \hat{\gamma}_{k|k-1}$.

(2) Evaluate the propagated cubature points $(i = 1, 2, \cdots m)$: $Z_{i,k|k-1} = H\left(\Gamma_{i,k|k-1}\right)$.

(3) Estimate the predicted measurement: $\hat{z}_{k|k-1} = \sum_{i=1}^{m} \frac{1}{m} Z_{i,k|k-1}$.

(4) Estimate the square-root of the innovation covariance matrix: $S_{zz,k|k-1} = tria\left(P^{*}_{k|k-1} \; S_{R,\mathrm{k}}\right)$, where $S_{R,\mathrm{k}}$ denotes a square-root factor of R_k such that $R_k = S_{R,\mathrm{k}} S^{T}_{R,\mathrm{k}}$ and the weighted, centered matrix $P^{*}_{k|k-1} = \frac{1}{\sqrt{m}}\left[Z_{1,k|k-1} - \hat{z}_{k|k-1}, \cdots, Z_{m,k|k-1} - \hat{z}_{k|k-1}\right]$.

(5) Estimate the cross-covariance matrix: $M_{xz,k|k-1} = A_{k|k-1} {P^{*}_{k|k-1}}^{T}$, where the weighted, centered matrix $A_{k|k-1} = \frac{1}{\sqrt{m}}\left[\Gamma_{1,k|k-1} - \hat{\gamma}_{k|k-1}, \cdots, \Gamma_{m,k|k-1} - \hat{\gamma}_{k|k-1}\right]$.

(6) Estimate the Kalman gain: $W_k = \left(M_{xz,k|k-1} / {S_{zz,k|k-1}}^{T}\right) / S_{zz,k|k-1}$.

(7) Estimate the updated state: $\hat{\gamma}_{k|k} = \hat{\gamma}_{k|k-1} + W_k\left(\mathrm{RSSI}_k - \hat{z}_{k|k-1}\right)$.

(8) Estimate the square-root factor of the corresponding error covariance $S_{k|k} = tria\left(A_{k|k-1} - W_k P^{*}_{k|k-1} \;\; W_k S_{R,\mathrm{k}}\right)$

3 Proposed Mechanism

When the system state model and observation model is linear and consistent with the Gaussian distribution, Kalman filter is optimal linear filter, assuming that noise is also a Gaussian distribution.

Considering the defects of low precision and divergence of the RSSI state estimation results, in this section, a strong adaptive Kalman filter mechanism which combines the original nonlinear filtering algorithm such as SCKF and EKF with Kalman filter is proposed. The mechanism firstly utilizes the RSSI state estimation algorithm based on SCKF to estimate and correct the state vector as depicted in state-space model, and then the Kalman filter is performed for further processing due to the linear changes of state equation. For application and comparison, the strong adaptive filter mechanism with EKF is established, and the Cramer-Rao Bound (CRB) is also derived.

3.1 Strong Adaptive SCKF Design

See from Eq. (7), the state vector of the unknown node at time k^{th} satisfies order estimation $\hat{\gamma}_k = A\hat{\gamma}_{k-1} + w_{k-1}$, then error between $\hat{\gamma}_k$ and the estimated value $\hat{\gamma}^{SCKF}_k$ determined by SCKF should be in accordance with the relationship of an additive noise such as $\hat{\gamma}^{SCKF}_k = \hat{\gamma}_k + e_k$, where measurement noise e_k submitting Gaussian distribution with zero means and covariance matrix $\Phi_k = \mathrm{cov}\left(e_k\right) = diag\left(5,5,5,5\right)$.

Therefore, we obtain the following linear system whose state-space model can be expressed by:

$$\text{Process equation: } \hat{\gamma}_k = A\hat{\gamma}_{k-1} + w_{k-1}, \tag{9}$$

$$\text{Measurement equation: } \hat{\gamma}^{SCKF}_k = \hat{\gamma}_k + e_k. \tag{10}$$

For the linear Gauss model, Kalman filter is considered to be the optimal filter, so the optimal estimation $\hat{\gamma}_k$ can be deduced by Kalman filter based on $\hat{\gamma}^{SCKF}_{k-1}$.

$$P_{k|k-1} = AP_{k-1|k-1}A^{T} + Q_{k-1}, \tag{11}$$

$$G_k = P_{k|k-1}\left(\Phi_k + P_{k|k-1}\right)^{-1}, \tag{12}$$

$$\hat{\gamma}_k = \hat{\gamma}_{k-1} + G_k\left(\hat{\gamma}^{SCKF}_{k-1} - \hat{\gamma}_{k-1}\right), \tag{13}$$

$$P_{k|k} = \left(I - G_k\right)P_{k|k-1}, \tag{14}$$

where $P_{k-1|k-1}$ is covariance matrix of the optimal estimation $\hat{\gamma}_{k-1}$ at time $k-1^{th}$, Q_{k-1} is process noise's covariance matrix which is the same as $Q_{k-1} = diag\left(\sigma_x^2, \sigma_y^2, \sigma_\eta^2, \sigma_{RSSI_{d_0}}^2\right)$ mentioned in section 2, $P_{k|k-1}$ is covariance matrix of the predicted state $\hat{\gamma}_{k-1}$ at time k^{th}, $P_{k|k}$ is covariance matrix of the optimal estimation $\hat{\gamma}_k$ at time k^{th}, G_k is the Kalman gain. About the initial value setting, let $\hat{\gamma}_0$ and $P_{0|0}$ be the initial value of state vector and its covariance matrix (not square root) before SCKF estimation. This strong adaptive Kalman filter design for SCKF algorithm is summarized in the following way:

Algorithm 1 [Strong Adaptive SCKF]:

(1) Initialize the filter: Denote $\hat{\gamma}_0$ and $P_{0|0}$, the square root $S_{0|0}$ of $P_{0|0}$ obtained by QR decomposition, simulate RSSI values received by the unknown node at n beacon nodes which is within communication range.

(2) *For* $k = 1, 2, \cdots N$, N is sampling times.

(3) Execute SCKF: Estimate the state estimation $\hat{\gamma}_{k-1}^{SCKF}$ and the related square root $S_{k|k}$ of the error covariance matrix determined by time update and measurement update in accordance with section 2.3.

(4) Execute the Kalman filter update: covariance matrix $P_{k|k-1}$ of $\hat{\gamma}_{k-1}$, Kalman gain G_k, optimal estimation $\hat{\gamma}_k$ of the state vector and covariance matrix $P_{k|k}$ of $\hat{\gamma}_k$ are obtained as Eq. (11-14), respectively.

(5) $\hat{\gamma}_k$ derived from (4) and $S_{k|k}$ derived from (3) are needed as input to circulation. Steps 3 to 4 are repeated until the smooth completion of N sampling.

3.2 Strong Adaptive EKF Design

Similarly, we also perform strong adaptive filter design with EKF algorithm.

For the state vector $\gamma_k = \left[x_k, y_k, \eta_k, RSSI_{d_0,k}\right]^T$ at time k^{th} in the state-space model, there's no need to take the derivative of process equation $\gamma_k = A\gamma_{k-1} + w_{k-1}$ because A is the unit state transition matrix. So we just take into consideration how nonlinear function $h(\gamma_k) = \left[h_1(\gamma_k), h_2(\gamma_k), \cdots h_n(\gamma_k)\right]^T$ at measurement equation where $h_i(\gamma_k) = RSSI_{d_0,k} - 10\eta_k \lg\sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}, 1 \le i \le n$ works in EKF. The related Jacobian matrix H_k [19] describes the partial derivatives of $h(\gamma_k)$ with respect to γ_k as:

$$H_k = \frac{\partial h}{\partial \gamma_k} = \begin{bmatrix} \frac{10\eta(x_1 - x_k)}{ln10\left((x_k - x_1)^2 + (y_k - y_1)^2\right)} & \frac{10\eta(y_1 - y_k)}{ln10\left((x_k - x_1)^2 + (y_k - y_1)^2\right)} & -10\lg\sqrt{(x_k - x_1)^2 + (y_k - y_1)^2} & 1 \\ \frac{10\eta(x_2 - x_k)}{ln10\left((x_k - x_2)^2 + (y_k - y_2)^2\right)} & \frac{10\eta(y_2 - y_k)}{ln10\left((x_k - x_2)^2 + (y_k - y_2)^2\right)} & -10\lg\sqrt{(x_k - x_2)^2 + (y_k - y_2)^2} & 1 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{10\eta(x_n - x_k)}{ln10\left((x_k - x_n)^2 + (y_k - y_n)^2\right)} & \frac{10\eta(y_1 - y_k)}{ln10\left((x_k - x_n)^2 + (y_k - y_n)^2\right)} & -10\lg\sqrt{(x_k - x_n)^2 + (y_k - y_n)^2} & 1 \end{bmatrix}. \quad \textbf{(15)}$$

Therefore, this strong adaptive Kalman filter design for EKF algorithm is summarized in the following way:

Algorithm 2 [Strong Adaptive EKF]:

(1) Initialize the filter: Denote $\hat{\gamma}_0$ and $C_{0|0}$, the initial value $P_{0|0}$ of covariance matrix of state vector in Kalman filter, simulate RSSI values.

(2) $For\ k = 1,2,\cdots N$, N is sampling times.

(3) Execute EKF: Estimate the state estimation $\hat{\gamma}_{k-1}^{EKF}$ and the error covariance matrix $C_{k|k}$ according to the following series of formulas.

$$C_{k|k-1} = AC_{k-1|k-1}A^T + Q_{k-1}, \quad \textbf{(16)}$$

$$W_k = C_{k|k-1}H_k^T[H_k^T C_{k|k-1}H_k + R_k]^{-1}, \quad \textbf{(17)}$$

$$\hat{\gamma}_{k-1}^{EKF} = \hat{\gamma}_{k-1} + W_k\left(RSSI_k - h\left(\hat{\gamma}_{k-1}\right)\right), \quad \textbf{(18)}$$

$$C_{k|k} = \left(I - W_k H_k\right)C_{k|k-1}. \quad \textbf{(19)}$$

(4) Execute the Kalman filter update: $P_{k|k-1}$ of $\hat{\gamma}_{k-1}$, G_k, $\hat{\gamma}_k$ of $P_{k|k}$ of $\hat{\gamma}_k$ are obtained as Eq. (11-14) where $\hat{\gamma}_{k-1}^{SCKF}$ needs to be replaced with $\hat{\gamma}_{k-1}^{EKF}$, respectively.

(5) $\hat{\gamma}_k$ from (4) and $C_{k|k}$ from (3) are needed as input to circulation. Steps 3 to 4 are repeated.

3.3 CRB Analysis

For the strong adaptive Kalman filter localization algorithm, the estimate $\hat{\gamma}_k$ of the state vector γ_k converges to the optimal estimation. In order to analyze the performance of the algorithm, we compare the estimation variance of $\hat{\gamma}_k$ and the variance lower bound. For the fixed state estimation problem which is converted by the system model, the lower bound of variance can be obtained by CRB. CRB is derived as follows:

For $v = \left[v_1, v_2, \cdots v_n\right]^T \sim N\left(0, \sigma_v^2\right)$, v_i denotes the random measurement noise corresponding to $RSSI_i$ received by the unknown node from the i^{th} beacon node, and $\mathrm{cov}\left(v\right) = \sigma_v^2 = diag\left(\sigma_{v,1}^2, \sigma_{v,2}^2, \cdots, \sigma_{v,n}^2\right)$, so $RSSI_i \sim N\left(H_i\left(\gamma\right), \sigma_v^2\right)$ and its conditional probability density function exists [20] as follows:

$$f\left(RSSI_i|\gamma\right) = \frac{1}{\sqrt{2\pi\sigma_v^2}} exp\left\{-\frac{\left(RSSI_i - H_i\left(\gamma\right)\right)^2}{2\sigma_v^2}\right\}, 1 \le i \le n. \quad \textbf{(20)}$$

$RSSI_i$ ($1 \le i \le n$) are assumed to be independent and identically distribute, the joint PDF of $RSSI$ is

$$f\left(RSSI|\gamma\right) = \prod_{i=1}^{n} f\left(RSSI_i|\gamma\right). \quad \textbf{(21)}$$

After logarithm fetch on joint PDF, which get log-likelihood function:

$$l\left(RSSI|\gamma\right) = ln\left[f\left(RSSI|\gamma\right)\right] = -\frac{1}{2\sigma_v^2}\sum_{i=1}^{n}\left(RSSI_i - H_i\left(\gamma\right)\right)^2. \quad \textbf{(22)}$$

By the CRB inequality, it meets $\mathrm{cov}\left(\hat{\gamma}\right) \ge F^{-1}$ where $F = -E\left[\frac{\partial^2 l\left(RSSI|\gamma\right)}{\partial\gamma^2}\right]$, F can be calculated as:

$$F_{kj}=\left[F\right]_{kj}=-E\left[\frac{\partial^2 l\left(RSSI|\gamma\right)}{\partial\gamma^2}\right]=\begin{cases}\dfrac{1}{\sigma_v^2}\sum_{i=1}^{n}\left(\dfrac{\partial H_i\left(\gamma\right)}{\partial\gamma_k}\right)^2,k=j\\ \dfrac{1}{\sigma_v^2}\sum_{i=1}^{n}\left(\dfrac{\partial H_i\left(\gamma\right)}{\partial\gamma_k}\dfrac{\partial H_i\left(\gamma\right)}{\partial\gamma_j}\right),k\neq j\end{cases}. \quad \textbf{(23)}$$

Above all, minimum mean square error lower bound of γ can be expressed by:

$$\sigma_k^2=Var\left(\gamma_k-\hat{\gamma}_k\right)\geq F_{kk}^{-1},k=1,2,3,4. \quad \textbf{(24)}$$

4 Numerical Simulation and Results Analysis

The algorithms described above are implemented in MATLAB to evaluate their performance by measuring the accumulative root mean square error over the state vector. All experiments were performed at a PC with CPU of 2.40 GHz and RAM of 8.00 GB.

4.1 Description of the Simulation Environment

To ease our illustration, each sensor node has a transmission radius of r = 10m. Assuming that the node's position is changeless, let the related data of the node be sampled 100 times according to the time interval of 0.01s.

For comparison, the root mean square of localization error of the unknown node at time k^{th} is defined as

$$RMSE=\sqrt{\left(\hat{x}_k-x_k\right)^2+\left(\hat{y}_k-y_k\right)^2}, \quad \textbf{(25)}$$

where $\left(x_k,y_k\right)$ is the actual position of the unknown node, $\left(\hat{x}_k,\hat{y}_k\right)$ is the estimated value.

Similarly, the accumulative root mean square of localization error is as follows:

$$ARMSE=\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[\left(\hat{x}_k-x_k\right)^2+\left(\hat{y}_k-y_k\right)^2\right]}. \quad \textbf{(26)}$$

As for the state vector $\gamma_k=\left[x_k,y_k,\eta_k,RSSI_{d_0,k}\right]^T$, the accumulative root mean square error of each component of which is expressed by

$$ARMSE\_each=\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[\left(\hat{\gamma}_{k,i}-\gamma_{k,i}\right)^2\right]},i=1,2,3,4. \quad \textbf{(27)}$$

The accumulative root mean square error of all the components of which is also calculated as :

$$ARMSE\_all=\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[\sum_{i=1}^{4}\left[\left(\hat{\gamma}_{k,i}-\gamma_{k,i}\right)^2\right]\right]}. \quad \textbf{(28)}$$

4.2 Performance Evaluation of SCKF

In the case of single-node's localization without strong adaptive recursion, we set initial value $\hat{\gamma}_{0|0}=\left[0.2,2,3,-20\right]^T$, $S_{0|0}=diag\left(0,0,0,0\right)$, the actual position of the unknown node is $\left(1,4\right)$. RSSI values collected at four known nodes are simulated as Eq. (5) where $d_i=\sqrt{\left(x_i-1\right)^2+\left(y_i-4\right)^2}$, v_i is 0.8 times of random noise, $i=1,2,3,4$ [21].

The experimental results shown in Fig. 2 to Fig. 5 demonstrate that the RSSI state estimation algorithm based on SCKF is efficient. From the Fig. 5, we can find that estimated parameter tends to be the true value with the increase in the number of samples.

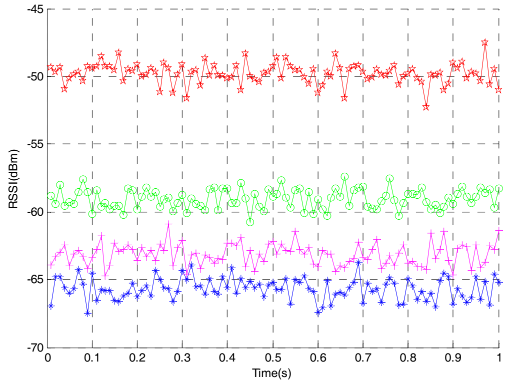

Fig. 2. RSSI distribution generated by simulation

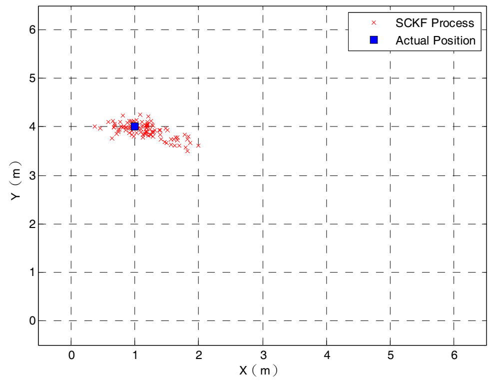

Fig. 3. Location estimation of a single node

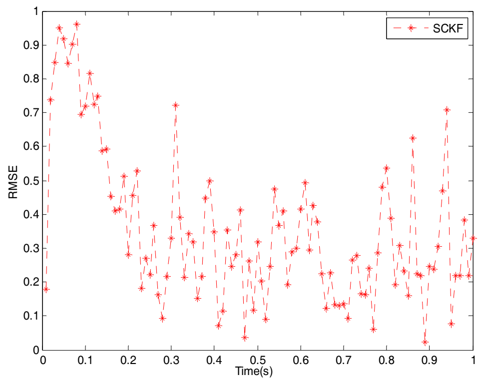

Fig. 4. Variation of RMSE with time

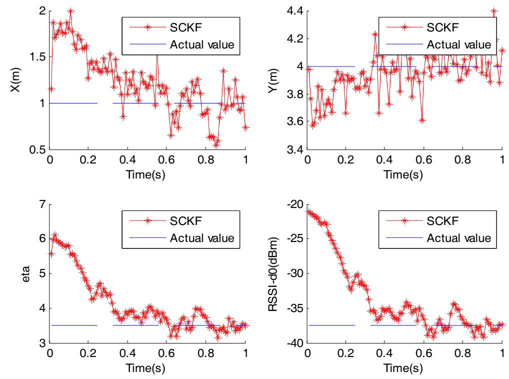

Fig. 5. Comparison of estimated state and actual value

4.3 Analysis of Strong Adaptive Mechanism

Let the geographical region be marked by a 6m×6m area. Now consider the case where the unknown node and 5 beacon nodes are deployed randomly in the region.

Let the actual value of $\gamma = [x, y, \eta, RSSI]^T$ be $[1, 4, 3.5, -37.5]^T$, initialize $\hat{\gamma}_0$, its covariance matrix $P_{0|0} = diag(0,0,0,0)$ (this is the same as what in Kalman filter), covariance matrix $\Phi_k = diag(5,5,5,5)$ of measurement noise in Kalman filter update.

We execute the SCKF, Strong Adaptive SCKF, EKF and Strong Adaptive EKF algorithms in 50 independent runs and compare them according to $ARMSE$, $ARMSE\_each$ and $ARMSE\_all$ as following Table 1.

Table 1. Comparison of four algorithms for ARMSE

Algorithms	$ARMSE$	$ARMSE\_each$	$ARMSE\_all$
EKF	0.3279	(0.2716, 0.1838, 0.3014, 2.0909)	2.1378
Strong Adaptive EKF	0.1886	(0.1509, 0.1131, 0.2261, 1.6196)	1.6461
SCKF	0.1572	(0.1109, 0.1114, 0.1162, 0.6843)	0.7116
Strong Adaptive SCKF	0.0901	(0.0620, 0.0654, 0.0704, 0.4047)	0.4206

It can be seen from Table 1 that the lower root mean square error and higher estimation accuracy are obtained for the state vector estimation of nonlinear systems, when the strong adaptive filter mechanism is applied to original nonlinear filtering algorithm such as SCKF and EKF. This shows that the mechanism is feasible and efficient.

When $\hat{\gamma}_0$ is initialized to $\hat{\gamma}_0 = [1.4777, 4.6001, 3.6596, -37.6673]^T$, the actual coordinate of the unknown node and the location estimation process determined by four different approaches in a trial run is shown in Fig. 6. We aim to find how the accuracy of the estimation relies on the strong adaptive filter mechanism. The location estimation results of SCKF and strongly adaptive SCKF, EKF and strongly adaptive EKF are respectively compared. It can be seen that the convergence of the estimated coordinates of the strong adaptive algorithm is better than that of the ordinary algorithm without strong adaptive recursion.

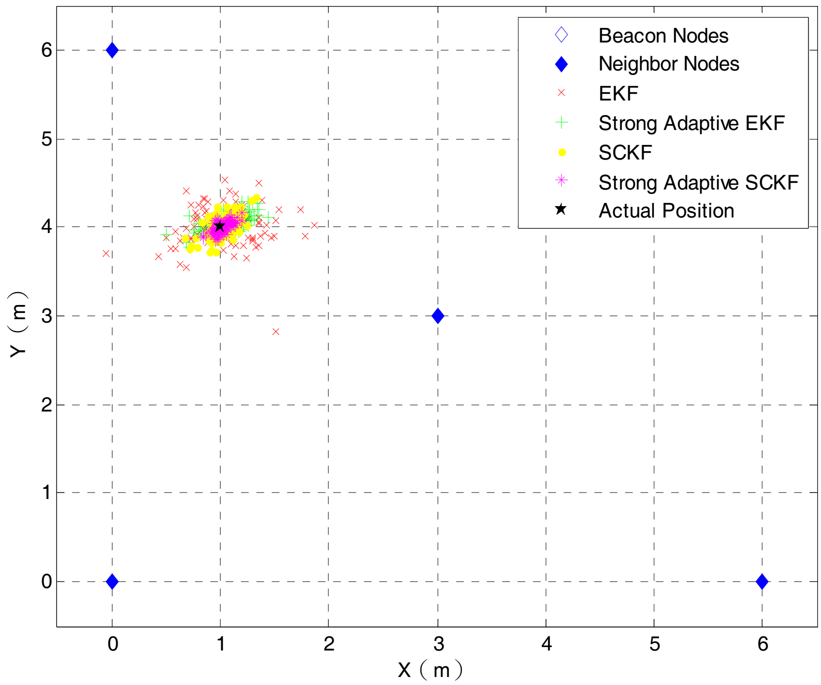

Fig. 6. Filtering location estimation for the unknown node

The reason that led to the conclusion is that the effect of additive noise is considered. Firstly, the SCKF based on RSSI state estimation algorithm is used to estimate the node location information and the channel attenuation parameter in WSNs, and the continuous correction of the node position estimation is obtained in successive iterations. Then, then error between the state vector of the unknown node at time k and the estimated value determined by SCKF should be in accordance with the relationship of an additive noise such as $\hat{\gamma}_k^{SCKF} = \hat{\gamma}_k + e_k$, according to the linear change of the state equation such as $\hat{\gamma}_k = A\hat{\gamma}_{k-1} + w_{k-1}$, a new state-space model such as Eq. (9) and (10) is established. Next, the recursive estimation is carried out on the basis of the Kalman filtering algorithm, so that the optimal estimation can be obtained.

In the process of N data samplings of Kalman filter (that is 1s time length), Fig. 7 shows the root mean square error curve of four different filtering algorithms. For the estimation of the position coordinates of the unknown node, the strong adaptive filtering design has achieved a lower error and has better stability.

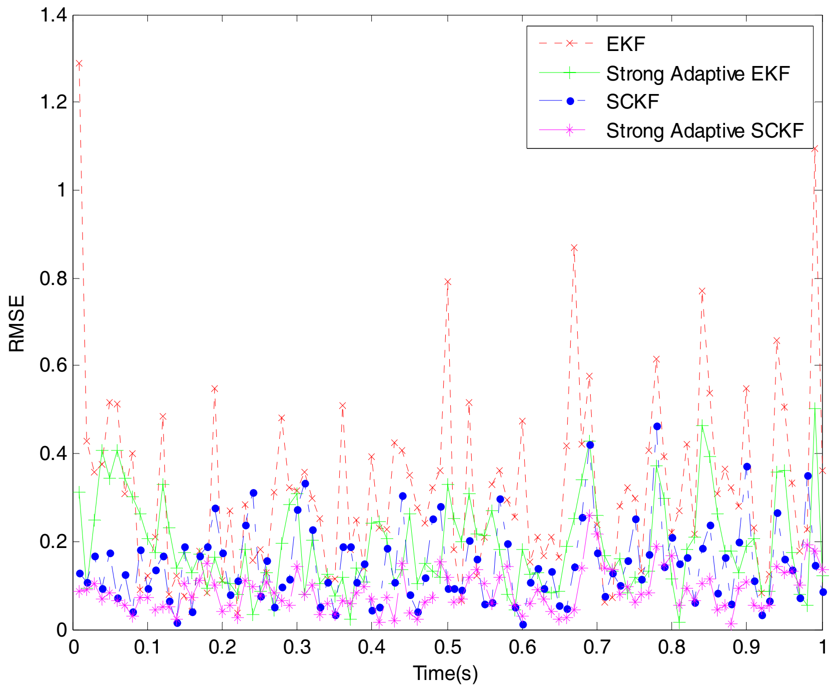

Fig. 7. Comparison of four algorithms for RMSE

As shown in Fig. 8, these four algorithms can asymptotically converge to the actual value of node state vector components. We can see that estimation result of the SCKF and EKF is not stable enough, will fluctuate in actual value, and the strong adaptation of the SCKF and EKF are more accurate and reliable because of the further filtering of the original algorithm. In addition, the strong adaptive filtering algorithm is more accurate for the estimation of the horizontal and vertical coordinates of the nodes, and the channel attenuation coefficient and the RSSI value at reference distance 1m is not ideal.

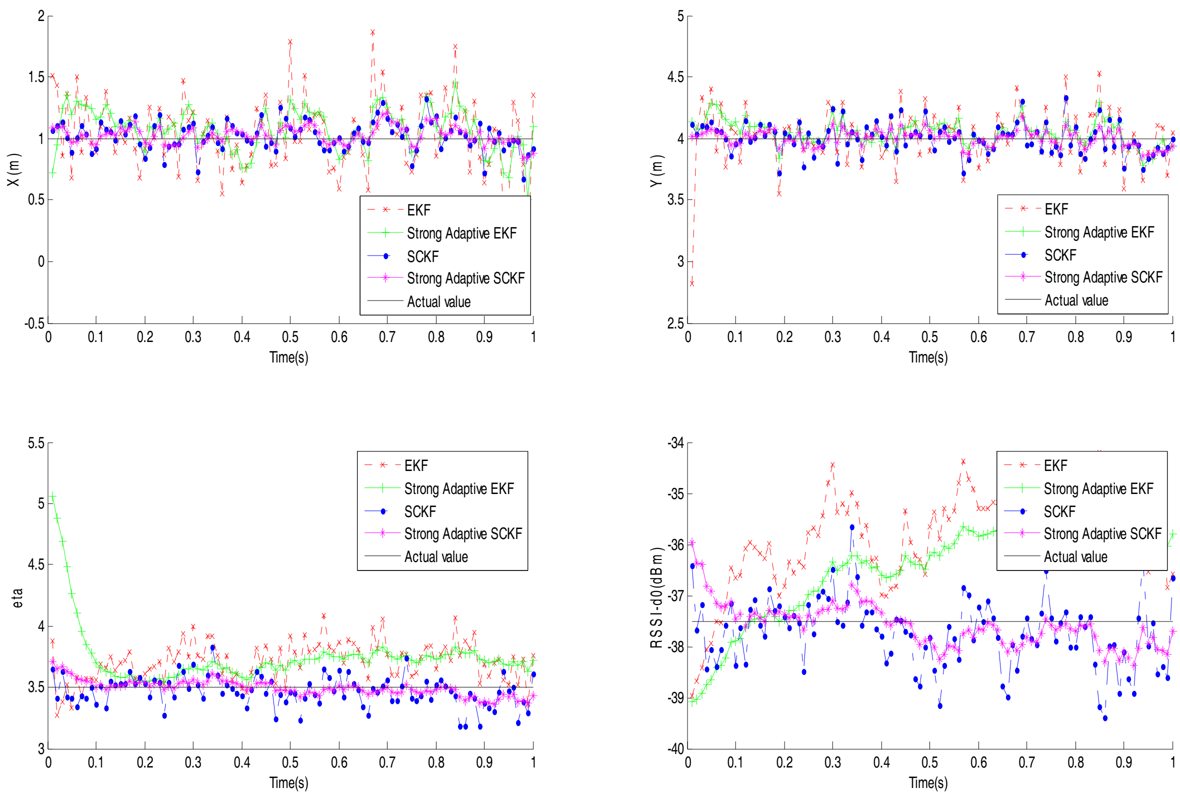

Fig. 8. Comparison of node state estimation

In the simulation results shown in Fig. 9, we describe an implementation of the root mean square error curve corresponding to node state estimation in Fig. 8. And the lower bound on the variance theory called CRB is compared, additionally.

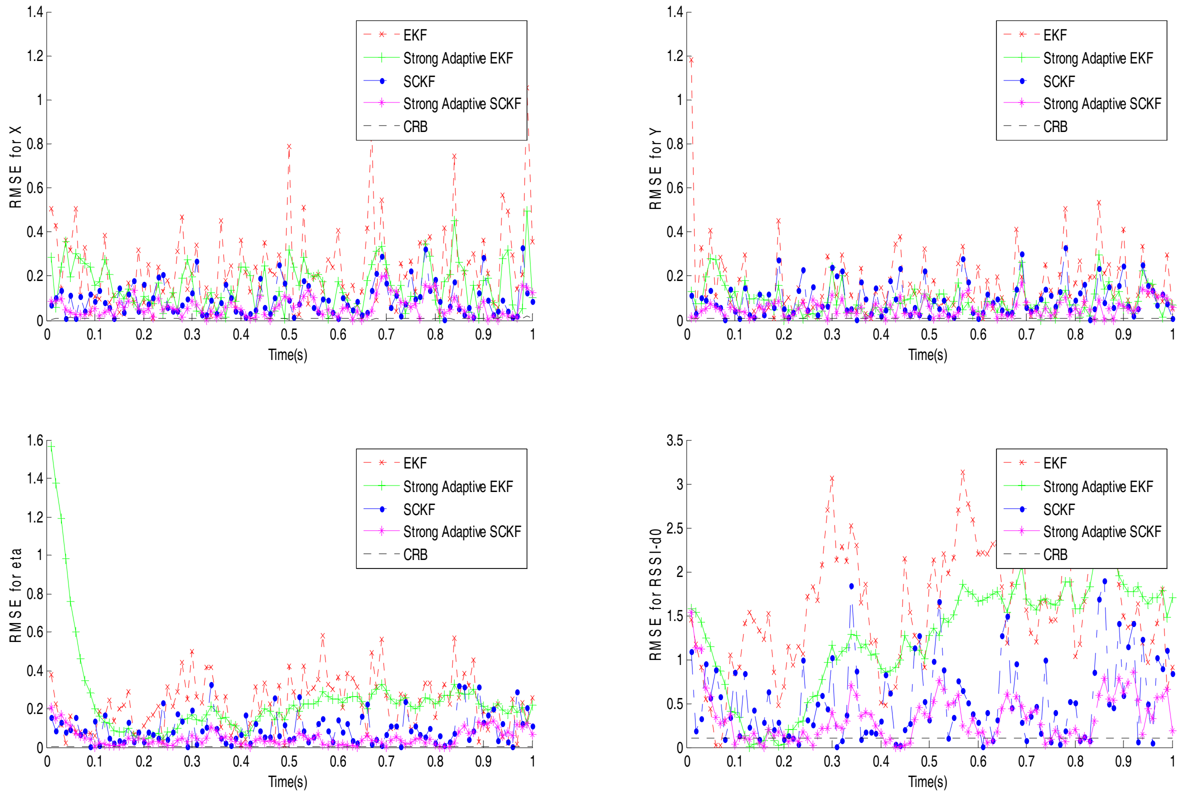

Fig. 9. Comparison of root mean square error of node state estimation

As can be seen from the results, four algorithms are almost all close to the lower bound of the error with the increase of the number of samples, but the strong adaptive filtering algorithm can obtain lower error lower bound. This is due to the CRB, whose derivation is based on the nonlinear model, can only be the lower bound of error for SCKF and EKF. The strong adaptive filtering algorithm is the linear recursive estimation based on SCKF and EKF, which can be used to eliminate the influence of noise, and the vector estimation error can be compared with CRB.

4.4 Comparison with Other Related Work

Now we provide a comparative table to indicate the contribution of this paper. We compare them with the Particle Swarm Optimization (PSO) [22], Bacterial Foraging Algorithm (BFA) [22], Maximum Likelihood Estimation (MLE) [23], Unscented Kalman Filter (UKF) and Centroid Localization in the numerical experiments. It can be seen from Table 2 that the Strong Adaptive SCKF outperforms all other methods.

Table 2. Comparison of all related algorithms for ARMSE

Algorithms	*ARMSE*
Particle Swarm Optimization (PSO) [22]	0.7218
Bacterial Foraging Algorithm (BFA) [22]	0.5167
Maximum Likelihood Estimation (MLE) [23]	0.4581
EKF	0.3279
Unscented Kalman Filter (UKF)	0.1265
Centroid Localization	0.2046
SCKF	0.1572
Strong Adaptive EKF	0.1886
Strong Adaptive SCKF	0.0901

Let the geographical region be marked by a 100m×100m area. Now consider the case where 100 sensor nodes are randomly placed while the number of unknown nodes is fixed as 50. We present an implementation of the iterative SCKF localization, and compare it with PSO, BFA, MLE and derivative iterative SCKF (That is Taylor Feedback Kalman Filter (TFKF) [24]) in the numerical experiments. The values in Table 3 denote average value over 50 runs.

As shown in Table 3, we can observe a slight reduction in the entire criterions so as to distinguish ours and others in more detail. It's also worth mentioning that TFKF is a new distributed iterative kalman filter localization technique, which is implemented by controlling the error with taylor expansion, to improve the deficiency of the Iterative SCKF and MLE methods. The article about TFKF [24] has been accepted and waiting to be published.

Table 3. A summary of results of 50 trial runs five algorithms for 50 unknown nodes

Algorithms	Average results of 50 trials			
	number of nonlocalized nodes	number of iterations	localization error	Elapsed time (s)
Particle Swarm Optimization (PSO)	0.3210	49	0.3511	288.4117
Bacterial Foraging Algorithm (BFA)	0.1956	49.4	0.2137	902.1132
Maximum Likelihood Estimation (MLE)	/	/	0.2624	0.8534
Iterative SCKF	0	3	0.1727	3.6271
Taylor Feedback Kalman Filter (TFKF) [24]	1	8	0.0857	9.1518

5 Conclusion

In this article we have developed a strong adaptive Kalman filter mechanism, which is implemented by combining the original nonlinear filtering algorithm with Kalman filter state estimation. A comparison of the performances in terms of the convergence of the estimated coordinates, node state vector components and the accumulative root mean square error is presented.

Although the study of Kalman filtering method in this paper can solve the locating problem under some specific circumstances, there are still many deficiencies that need further study. (1) When the range of the location area remains unchanged, the positioning accuracy achieved by SCKF can not be greatly improved by increasing the number of beacon nodes, this is because the beacon node can only bring more measurement information for more nodes remains to be positioned, which improves the measurement accuracy to some extent, but the effect on the positioning accuracy is limited, so the maximum number of reference nodes is limited to 6, and the nearest 6 from the target; (2) The strong adaptation mechanism proposed by this paper is based on the assumption that the system state model, the observation model and the noise are consistent with the Gauss distribution hypothesis, so the future work can be extended to the nonlinear state estimation research under the non Gauss background; (3) When a strong adaptation mechanism is established, then error between the state vector of the unknown node at time k and the estimated value determined by SCKF is assumed to be in accordance with the relationship of an additive noise such as $\hat{\gamma}_k^{SCKF} = \hat{\gamma}_k + e_k$, and this correlation can also be represented by a mathematical model that affects its change, in which parameters can be used for channel attenuation parameters. Therefore, it is appropriate to carry out the following work with modeling and parameter adjustment.

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