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Bounded Confidence Opinion Dynamics in Virtual Networks and Real Networks

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Abstract. It is well known that opinion evolution is a kind of dynamics running on the substrate structure (i.e. topology). The opinions and substrate topology act on and influence each other. However, the substrate topology is regarded as a background of opinion dynamics, and there has been less previous research has focused on the transverse comparison of impacts of different topologies on opinion dynamics, to our knowledge. In particularly, as the technology of Web 2.0 develops, the real user relationship-based network can be available. For this end, we apply the Hegselmann–Krause (HK) model on the 4 virtual networks (i.e. square lattice, random network, small-world network, and scale-free network) and 2 real networks (i.e. twitter friendship network and Facebook friendship network). It is found that, opinions converge better in random network that in other virtual networks; the topology characteristics of real network is one of the reason of opinion stalemate in online social networks.

Keywords: complex network, Hegselmann–Krause (HK) model, opinion dynamics, social network

1 Introduction

In the last few years, the interplay between the topological structure and dynamics running on it has attracted extensive attention worldwide [1-2]. Without a doubt, the effects of interpersonal network on opinion dynamics are also an interesting problem in social physics [3-4]. Accompany with presentation of a new opinion dynamics model, a fair amount of works on the new model will be raised every time, including the universality verification of conclusions about the new model on different topologies [5]. Among them, the HK model reveals one possible mechanism of the opinion evolution and presents abundant phenomena. However, the network topology is studied as a substrate in the previous literature, the parallel comparison between the mechanics of HK model on different topologies, especially real networks, is rarely investigated. And this is exactly the starting point of this paper.

In statistical physics, the study of how human behavior forms macroscopic phenomena in a society, e.g. culture dynamics [6], language dynamics [7], information or rumor dissemination [8], opinion formation [9], crowd behavior [10], has already been a research hotspot. Among these interesting problems, the study of opinion formation has become a main stream [11]. Opinion dynamics attempts to describe how individuals exchange opinions, persuade each other, make decisions, and implement actions, employing diverse tools furnished by statistical physics, e.g. probability and graph theory [5]. It describes social phenomena and simulates the evolution process of opinions. Opinion dynamics focus on understanding the interplay between the microscopic local interactions of individuals and emergence of collective public opinion on the macroscopic scale [12]. The opinion dynamics models may reach different regimes, i.e.

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consensus, polarization or fragmentation [13]. The consensus regime corresponds to the emergence of an agreement, i.e. every agents share a same opinion; the polarization regime means the coexistence of two opinions, i.e. two clusters holding different opinions coexist; the fragmentation regime corresponds to a disordered state where the distribution of numerous opinions tends to be uniform and is rather scattered). And every regime could be the absorbing state based on different assessments of parameters. This means that once the system randomly enters such a configuration, it stays there forever.

In establishing a framework on opinion dynamics, the most important part includes specifying any possible opinion states of agents and defining the elementary processes or rules that determine opinion transitions of every agent between such states. According to the way opinion variables are defined, the main investigations can be divided into two groups: discrete opinion models and continuous opinion models. The former, e.g. the voter model [14], the majority rule model [15], and the Sznajd model [16], describe situations when people confronted with only two choices on a certain topic, e.g. yes or no, left of right, Samsung or iPhone. While the latter, e.g. the Deffuant model [17], and the Hegselmann–Krause (HK) model [18-19], adequate at explaining cases in which individual's opinion can vary smoothly from one extreme to the other, e.g. political orientation of an individual, worthiness of a choice. As a typical representative of continuous models, the HK mechanism of opinion evolution is based on bounded confidence, that is, each agent can only interact with the agents whose opinion values lie within its confidence range. Due to the reason that it can present the opinion phenomena found in social networks, including consensus, polarization, and fragmentation, by defining one possible mechanism of opinion evolution, the HK model has been widely studied. Wongkaew et al. [19] investigated different control strategies for HK model with leadership and demonstrated the ability of the proposed strategies to drive the system to attain consensus. Zhao et al. focuses on the evolution of opinion interaction (consumer behavior) as a direct relationship between opinion leaders and opinion followers (consumers), and established a new bounded confidence-based dynamic model for opinion leaders and followers [20]. Zhu et al. investigated the formation of continuous opinion dynamics based on virtual gambling mechanism where agents fight for a limited resource [21]. Zhang et al. [13] proposed an opinion dynamics model with time-varying bounded confidence, and the Opinion formation with time-varying bounded confidence.

In opinion dynamics models, the local interactions happen only between neighbors (corresponding to friends in reality). The neighborhood may appear between each agents-pair in a small community such as an office team, while in a large community such as an online social platform, neighborhood may appear only between friends, most agents-pairs are not connected. In computer simulations, the friendship in a community is defined by a connected network with the same size of the community. This underlying topology plays a significant role in opinion dynamics [22]; many literatures investigate the effects of statistical quantities of topology on opinion evolution. However, most researchers use the graph-based mathematical models to describe the substrate structure, observe the outputs of the system by adjusting the statistical quantities. The topology is regarded as a background and the comparison of effects of different topologies on opinion dynamics, especially on bounded confidence opinion dynamics, is lacked. Only Felijakowski and Kosinski [23] studied the opinion formation process of bounded confidence model on the Barabási-Albert network and corruption spreading in a hierarchical network. But the scale of Barabási-Albert network is 1000, and the scale of hierarchical network is only 37. The virtual network is simple, and the results are not typical enough. Moreover, as a typical representative of Web 2.0 applications, social media have demonstrated their strength in attracting users and propagating information. The quantitative data become increasingly available, particularly from online social networks, there is the possibility the real user relationship-based network can be obtained. Comparing the opinion evolution of HK model on different topologies is useful for exploring the effects of different topologies on opinion consensus and polarization, verifying the universality of results of research on HK model, giving an insight into the effects of real networks on opinion consensus, polarization and fragmentation.

In this paper, we present two real networks from Twitter [24] and Facebook [25] respect, and carry out simulations systematically by employing the HK model on virtual network and real network respectively. Here, the virtual network means the network is constructed by mathematical models, including square lattices, random network, small-world network, and scale-free network. The real network is the topology, describing the friendships between users, gotten from the real data of Twitter or Facebook. We will concentrate on the detailed process of system evolution into the dynamic equilibrium, the transition between different steady states with different numbers of communities, and evolution of number of opinions.

The paper is organized as follows. In the next section, we introduce the Hegselmann-Krause (HK) model and .our method of measuring the opinion formation process. Then the opinion evolution process of HK model is investigated for 4 virtual networks and 2 real networks, and the results are compared and presented in Section 3. Finally, we sum up and draw our conclusions in Section 4.

2 Model

An individual opinion is a summary evaluation of a psychological object. In reality, people's opinions are usually not yes/no, balk/white, left/right, etc. Instead, opinions often vary smoothly from one extreme to the other, for example, the political orientation of individual agents. The human interaction happens among people whose opinions are sufficiently close to each other, and opinion clusters will always emerge along with interaction and evolution. However, the discrete opinion models are not good at describing these phenomena, and based on the realistic aspects abovementioned, several confidence-based models have been established. One of the widely known confidence-based models is the Hegselmann–Krause (HK) model in which each agent has bounded confidence (that is, each agent can only interact with one another when the distance of their opinions is close enough to a given confidence level). The HK model can reveal this common evolution mechanism, gain insight into the emergence of opinion clusters, and present rich phenomena of opinion dynamics found in social networks, including agreement/consensus, polarization, and fragmentation. Due to the reason that it can present complex phenomena by defining one possible mechanism of opinion evolution and it can be realized easily, the HK model has been widely studied. So the HK model is also used in this paper to investigate opinion evolution.

As a kind of continuous opinions model, each agent i has an opinion represented by the variable $s_i(t)$, a real number from -1 to 1, in the HK model. Based on the phenomenon that people usually interact with peers whose opinion values lie within their confidence ranges, the model stipulate that the agent i updates its opinion $s_i(t)$ according to the formula (1)

$$s_i(t+1) = \delta_{M(i,t)} s_i(t) + (1-\delta_{M(i,t)}) \frac{1}{M(i,t)} \sum_{j \in F(i,t)} s_j(t), \tag{1}$$

where δ_x is the Kronecker delta function, δ_x=1 for x=0, and δ_x=0 for x>0; $M(i, t)$ means the number of elements of the set $F(i,t)$; the set $F(i,t)$ denotes the effective neighbors of agent i, defined by

$$F(i,t) = \{1 \le j \le N, j \ne i, A_{ij} = 1, \left| s_i(t) - s_j(t) \right| \le \varepsilon\}, \tag{2}$$

in which N is the scale of the model, i.e. the number of agents in the model; A is the adjacency matrix, and A_{ij} is the element of A with A_{ij} =1(0) if there is an edge between agent i and j (otherwise); | · | denotes the absolute value of a real number; $\varepsilon \in (0,1]$ represents the confidence threshold or interaction radius, which is taken as a constant in time and across the whole population.

At a given step, the following microscopic rules control the opinion dynamics:

(1) One random agent i is selected at random;

(2) Got all the neighbors of agent i according the relation topology;

(3) For every neighbor j of the focal agent i:

$$if \left| s_i(t) - s_j(t) \right| \le \varepsilon, \; sum = \sum_{j \in F(i,t)} s_j(t) \tag{3}$$

(4) The agent i updates opinion $s_i(t) = \left| M(i,t) \right|^{-1} \bullet sum$.

This process continues until N times updating completes and it constitutes one Monte Carlo Step t.

For comparing the effects of different topologies on opinion dynamics, 6 different kinds of topologies are used, including square lattices, random network, small-world network, scale-free network, Twitter friendship network, and Facebook friendship network. Considering that the impact of system size on opinion formation and evolution of the whole system can be ignored when the system size is larger than 2000 nodes, which has been demonstrated in [26], the size of the considered topology is assumed to be

$N \approx 2000$. Unless otherwise specified, the following assumptions are applied to all of the experiments:

(1) The initial state of the system is assumed to be fully disordered, that is, at the beginning of the dynamics, each agent has an opinion obeying the uniform distribution in the range [-1, 1].

(2) The square lattice is bi-dimensional, with periodic boundary conditions. And its size is N = 32*32. Each agent is regulated to only interact with its four nearest neighbors (von Neumann neighborhood). The boundaries of the lattice are connected to each other: the diagrams represent in fact the unfolding of a torus.

(3) The random network is constructed by ER random graph model proposed by Erdösand Rényi. The system size is N = 2000, and the connection probability of every nodes pair is p_{random}=0.25.

(4) The small world network is founded by NW model presented by Newman and Watts. We set the number of initial nodes is m_0=4, and the probability of adding a link between two randomly selected nodes is p_{WS} =0.1.

(5) The scale-free network is constructed by BA model proposed by Barabási and Albert. It starts with 3 initiating agents, and the number of new nodes added into the network is m=3.

(6) The Twitter friendship network [24] includes 2157 nodes and 5535 edges. The average degree is 4.9448, the mean shortest path length is 2.59 edges, and the clustering coefficient is 2.15%.

(7) The Facebook friendship network [25] is composed by 2888 nodes and 2981 edges. The average degree is 2.0644, the mean shortest path length is 3.98 edges, and the clustering coefficient is 0.0359%.

3 Results and Discussion

In order to compare the impacts of substrate topology on opinion formation and evolution, we perform a series of standard Monte Carlo simulations using random sequential updating. Firstly, for taking an intuitional look, we show the opinions evolution of a typical run of HK model on 6 substrate structures respectively, and compare the convergence of opinions and opinion clusters formation. Later, the relation between the order parameter and confidence threshold, as well as the relation between the number of opinions and confidence threshold, are investigated. Finally, as a further study, we inspect the change of the number of opinions over time in different topologies, compare and analyze the results.

To have a direct-viewing understanding of effects of different substrate topologies on opinion evolution and clusters formation, the application of HK model on virtual network (including square lattice, random network, small world network, scale-free network) and real network (including Twitter friendship network and Facebook friendship network) are preliminarily compared. The detail parameters setting and give references of used topologies are introduced in Section 2, and so for the rest part of this paper. Fig. 1 shows configurations of a typical run of opinion evolution with different confidence threshold on square lattice (top left sub-figure), random network (top right sub-figure), small-world network (middle left), scale-free network (middle right), Twitter network (bottom left), Facebook network (bottom right), respectively. From top left to bottom right, the 6 sub-figures correspond to opinion evolutions on square lattice, random network, small world network, scale-free network, Twitter friendship network and Facebook friendship network. In every sub-figure, the 4 subplots correspond to opinion evolution with different confidence thresholds: d =0.2, 0.5, 0.7 and 1 form top left to bottom right. It is observed that, the system will always reach the steady state, no matter what the substrate topology is and what the value of confidence threshold is. In every system (i.e. sub-figure), with the increasing confidence threshold, the final state of system undergoes a process from chaos to order, it is much easier to distinguish the winning opinion in the final state, the final number of opinions decreases, and large opinion clusters emerge. All the above phenomena are irrelevant to the topology, so these are the characteristic of the HK model actually, due to the compromise rule.

However, there are some differences in 6 sub-figures considering the introduction of different topologies, on which opinions exchange and evolve. In Fig. 1 we can see that, when confidence threshold d=1, the agents can get consensus in virtual networks (i.e. square lattice, random network, small world network, scale-free network), while agents in real networks (i.e. Twitter network and Facebook network) can not. When opinions evolve on square lattice, small world network and scale-free network, as the confidence threshold enlarges, the system goes through 3 stages of fragmentation, polarization (here amounts to that opinions with two inclinations coexist with the central opinion), consensus (here amounts to that all agents get consensus on the central opinion). In contrast to the first stage, when opinions evolve on random network, opinions interact better and can converge into several large opinion clusters

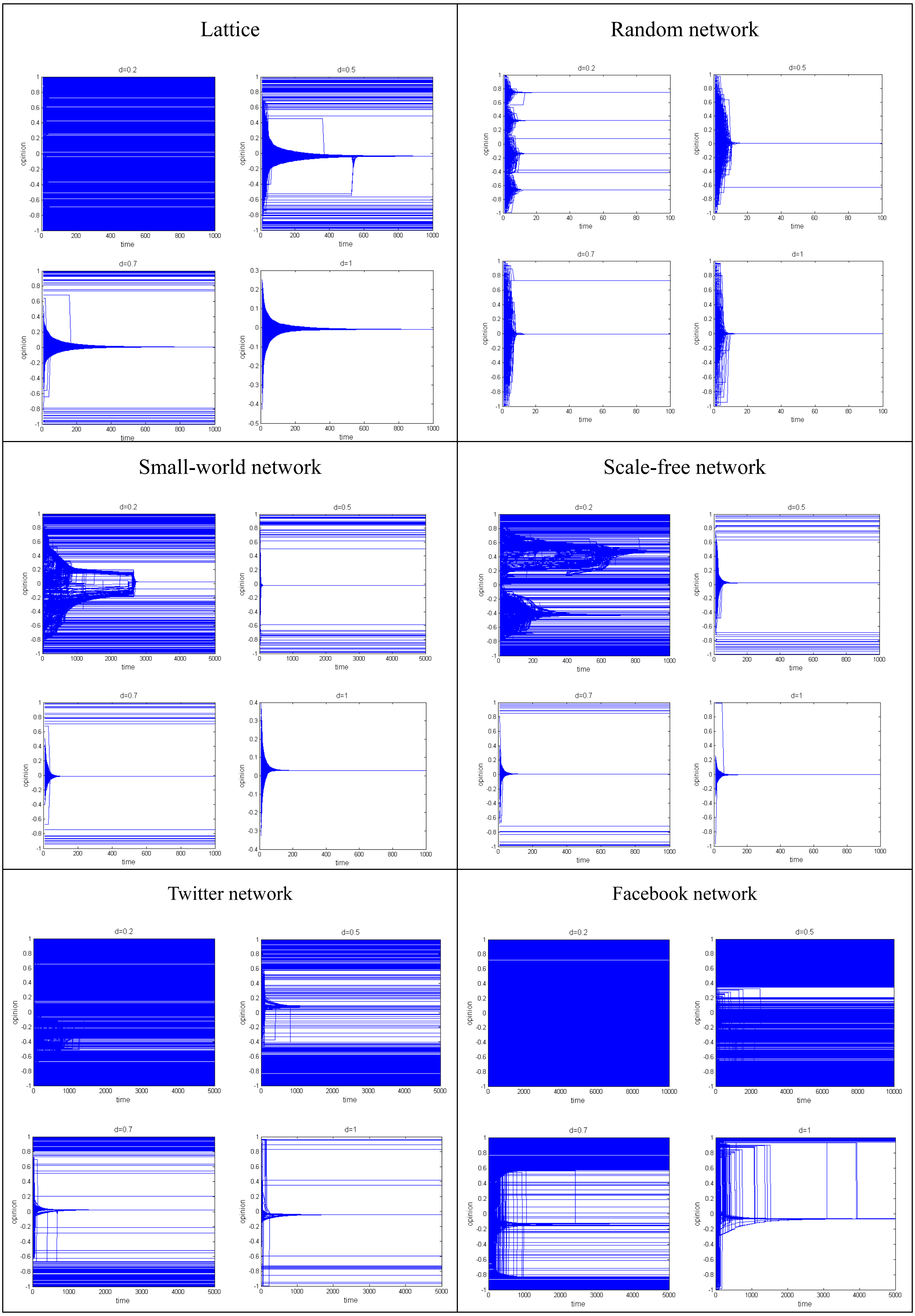

Fig. 1. A respectively typical run of opinion evolution with different confidence threshold on different topologies

as the confidence threshold d=0.2. In contrast to the final stage, when opinions evolve on Twitter network, it seems that the system gets a stable fragment situation, and several large clusters with scattered opinions confront with each other; when opinions evolve on Facebook network, a quasi-consensus situation is attained, i.e. a central opinion coexist with a dominant extreme opinion. The non-consensus final state of HK model on Twitter network and Facebook network is because of the high clustering of actual network. Actually, the final stalemate is much closer to the reality, because the people belonging to different communities always take different opinions and rarely interact with people in other communities.

Next, we make simulations to explore the detailed impact of substrate structures on opinion evolution and convergence. The number of opinions when the system gets stable as a function of confidence threshold , for HK model with about 2000 agents on different topologies: square lattice (●), random network (○), small world network (×), scale-free network (+), Twitter network (✶), Facebook network (□) is shown in Fig. 2. Each point is average results of 20 simulations. The results presented below correspond to observables measured over statistically-averaged ensembles in the stationary regime, which were obtained by averaging over 20 different initial independent configurations and simulations. The curve will be much smoother if the simulations are taken more times. It is observed that, the final number of opinions declines with enlarging confidence threshold, which has nothing to do with the substrate structure. Meanwhile, we also find that, with the same confidence threshold, the final number of opinions decreases as opinions evolve on Facebook network, Twitter network, square lattice, scale-free network, small world network and random network. This indicates that, the random link contributes to the interaction and convergence, and facilitates the consensus. While the real networks (i.e. Facebook network, Twitter network) and the square lattice goes against the convergence of opinions, due to the clustering and locality of the topologies. This is meaningful because the previous empirical study stated that the stalemate of opinions in reality is because of the quit of users from discussion. Our results demonstrate that the topology is also a reason for the stalemate.

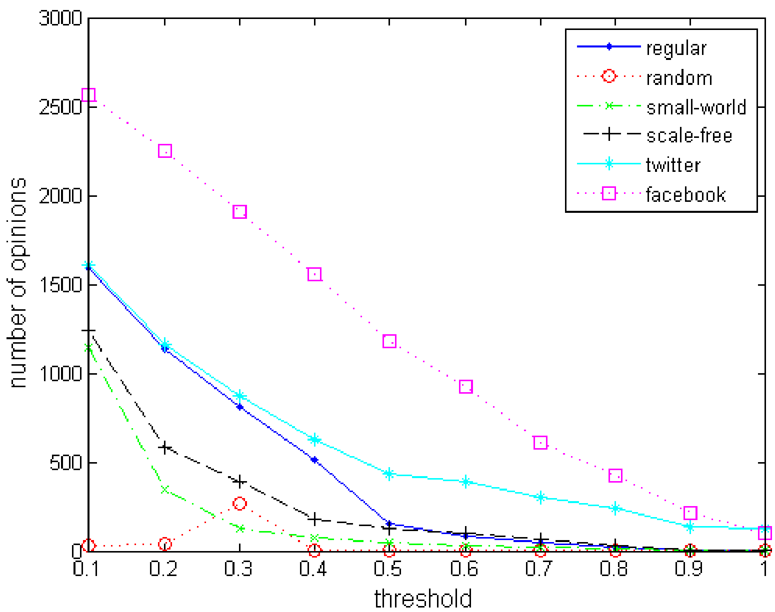

Fig. 2. The number of opinions as a function of confidence threshold *d* on different topologies

To further explore the effects of topologies on opinion evolution and convergence, the evolutions of opinion numbers with time for HK model, with different confidence threshold *d*=0.1 (◊), 0.2 (□), 0.3 (×), 0.5 (◆),0.7 (+),0.9 (○), 1(●), on different topologies depicted in Fig. 3. From top left to bottom right, the 6 sub-figures correspond to opinion evolutions on square lattice, random network, small world network, scale-free network, Twitter friendship network and Facebook friendship network, respectively. Each point is average results of 20 simulations. Obviously, the number of opinions decreases in a certain extent, whenever the opinions evolve on what kind of topologies and what value the confidence threshold is. It is worth noting that, in square lattice, small world network and scale-free network, the evolving process undergoes a phase transition, and the transition point is at d=0.5. When d<0.5, the number of opinions decrease slowly with time, and when d≥0.5, the number of opinions decrease dramatically to about 1, with also a phase transition from fragmentation to consensus. Specially, in random network, the random links make the interaction more sufficient and adequate, so opinions converge better even with a small confidence threshold. In real networks, there is no phase transition in HK model on both Twitter network and Facebook network. Especially in Twitter network, it takes a long time for HK model to decrease the number of opinions to a stable value; the number of opinions even changes a little in Facebook network. This result demonstrates the fact that the real network is a reason for the stalemate in online social network again.

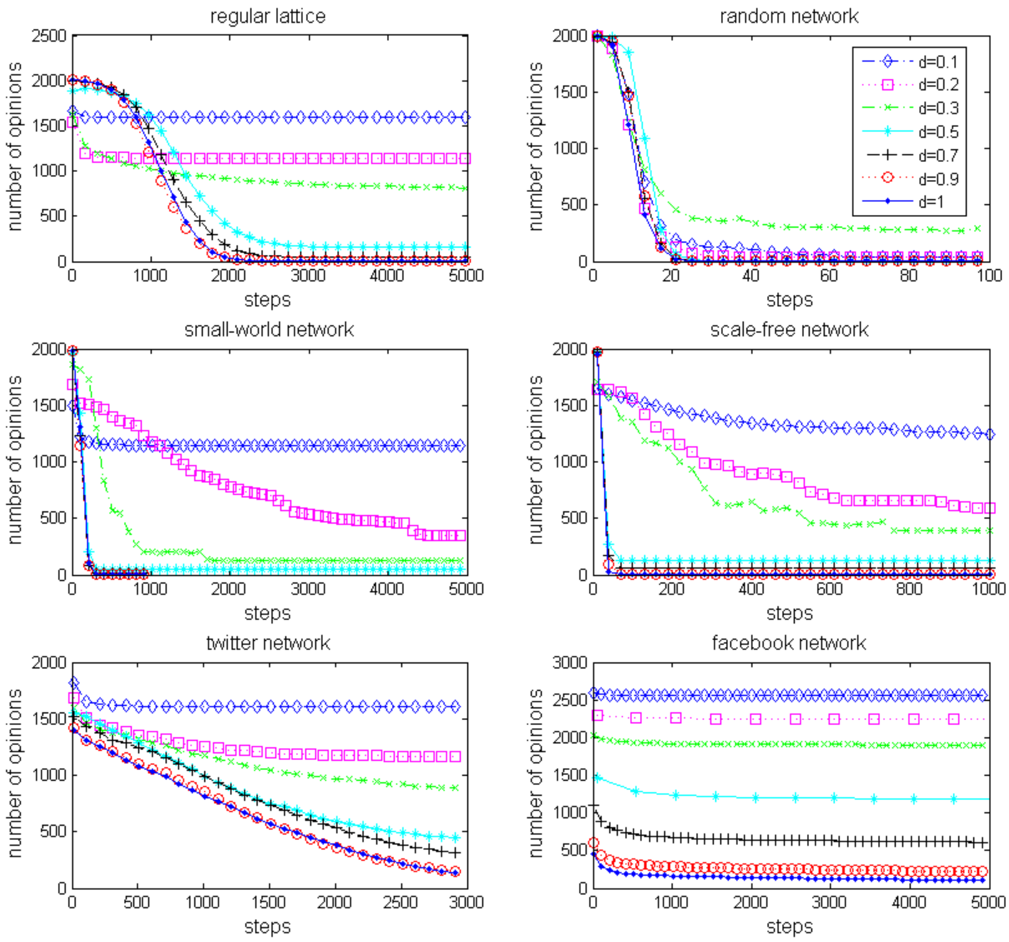

Fig. 3. The number of opinions vs. time step for different confidence thresholds on different topologies

4 Conclusions

In this paper, we employed the HK model on virtual networks (i.e. square lattice, random network, small-world network, and scale-free network) and real networks (i.e. Twitter friendship network and Facebook friendship network) respectively, to investigate the impacts of different topologies on opinion evolution and convergence. We concentrated on the detailed process of system evolution into the stable state, the transition between different steady states with different numbers of communities, and evolution of number of opinions. It is found that, the intrinsic compromise rule of HK model signifies the final steady state of system after some steps of evolution; opinions converge better in random network than in other virtual networks, because the random link contributes to the interaction and convergence, and facilitates the consensus; the real networks (i.e. Facebook network, Twitter network) and the square lattice goes against the convergence of opinions, due to the clustering and locality of the topologies; the topology characteristics of real network is one of the reason of opinion stalemate. However, the further discussion about the evolutions of opinion numbers with time for different confidence threshold on different topologies should be enhanced. For example, the reason for the lack of phase transition should be found. This work needs mathematical proof, the research on characteristics of real networks, and more data of the evolution process, and will be investigated in future. Moreover, the order parameter plays the role of magnetization in magnetic systems. It is sensitive to the unbalance between positive and negative opinions, and will be our object of study measuring the final evolution state of opinion dynamics next time.

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