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Optimal Design of Training Sequence for MIMO Communication System Based on Information Theory

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Abstract. To further improve the channel estimation performance in Multiple-Input, Multiple-Output (MIMO) communication system, the channel characteristics of MIMO system with the training sequence based on time series method are analyzed. This paper first analyzes the construction method of orthogonal Gold sequence and the complete complementary sequence, and followed by the study of the MIMO training sequence channel estimation method for timing insertion with Gold sequences and complete complementary sequence. Based on this, from the point of view of information theory, to maximize the channel capacity of MIMO system, the training sequence is optimized under the above criterion. The channel capacity of MIMO system based on complete complementary sequence and orthogonal Gold sequence are analyzed. The theory proves that the proposed optimization algorithm transforms the original training sequence into orthogonal matrix and then improves the capacity of the channel. At the same time, the experimental results also verify the feasibility and accuracy of the optimization algorithm.

Keywords: channel capacity, channel estimation, complete complementary sequence, orthogonal Gold sequence, training sequence

1 Introduction

The MIMO channel estimation based on training sequence is one of the research hotspots in the field of communication, integrating training sequence with information data at the transmitting end for transmission and estimating MIMO channels with the orthogonal characteristics of training sequence. Therein, training sequence is an important factor affecting channel estimation, and optimizing it is an effective way to improve the performance of MIMO channel estimation and channel capacity.

At present, the training sequence design has gained wide attention from scholars [1-3]. The difference in the performance of the training sequence directly affects the performance of the MIMO communication system. In the channel estimation algorithm of the MIMO communication system based on the training sequence mode [4-5], the autocorrelation performance is the main index to evaluate the performance of the training sequence. With good autocorrelation and cross-correlation performance, pseudo-random sequence has always been the choice of channel estimation algorithm as training sequence. At present, there are two kinds of MIMO channel estimation methods based on timing insertion training sequence: one is time division processing of training sequence and information data, and this method is using special channels or time slots to transmit training sequence. Since training sequence and information data are transmitted in time division mode, so the advantage of this method is that the training sequence and information data do not interfere with each other, and it has better channel estimation and signal detection performance [6-7]. The channel estimation method in this study is also based on this method. The other is to transmit the training sequence by superimposing the training

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sequence into the information data [8-9], and then separate the training sequence by statistical algorithm at the receiving end for channel estimation. This method is at the cost of sacrificing the signal power and has the advantage of saving the system bandwidth. The compressed sensing is a signal processing method, Literature [10] will use Golay sequence as a training sequence to estimate the channel of sparse channel. The channel is estimated by using the good autocorrelation, good timing synchronization performance and low peak to average power ratio of Golay sequence, and the application results of Golay sequence and Random Gauss sequence are compared. [11-12] uses compressed sensing principle to carry out channel estimation based on OFDM system by optimizing the measurement matrix based on training sequence. An efficient cyclic method to design training sequence is devised in massive MIMO system and the channel estimation performance is also analyzed [13]. While all the above papers only analyze the channel estimation performance of the training sequence, and the MIMO system channel capacity based on training sequence are not researched.

According to the performance of MIMO communication system composed of multiple sets of antennas, this study builds a corresponding channel estimation model based on timing training sequence. Based on this model, this study makes good use of the features such as good autocorrelation function and cross-correlation function of training sequence and combines the theoretical of MIMO channel capacity in information theory to propose an optimization method for timing training sequence with channel capacity as optimization criterion. With the optimization algorithm, under the condition that pseudo-random is used as training sequence, the training sequence is subject to matrix transformation in the way of spectral decomposition to obtain the sequence with better orthogonality and channel estimation performance. The Orthogonal Gold sequences is optimized with the above optimization algorithm, and the simulation results are comparatively analyzed with the performance of pseudo-random sequences such as Orthogonal Gold Sequence, complete complementary sequence (CC-S) and Chu sequence [14].

Notations: We use the lowercase and uppercase boldface letters to describe vectors and matrices. $(\cdot)^H$, $(\cdot)^T$, $(\cdot)^{-1}$, $\text{cov}(\cdot)$ and $\det(\cdot)$ denote the conjugate transpose, transpose, matrix inversion, matrix covariance and matrix determinant respectively.

2 Training Sequence Design

2.1 Orthogonal Gold Sequence

Gold sequence is a pseudo-random sequence with better correlation function characteristics. It is composed of two optimum m-sequence pairs with equal code length and code clock rate by adding via Mode 2 [15]. The Gold code sequence includes balanced codes and unbalanced codes, and a pattern in which the number of "1" code elements is one more than the number of "0" code elements in the sequence is called a balanced Gold sequence. The Orthogonal Gold Sequence, which is constructed by the balanced Gold sequence, a "0" is added to the last bit of the balanced Gold sequence, so that the number of "0" codes is the same with the "1" code in the Gold sequence, and we call this sequence the Orthogonal Gold Sequence. The purpose of this is also to prevent the influence of DC bias on the channel estimation result.

2.2 Complete Complementary Sequence

A complete complementary sequence is a pseudo-random sequence that consists of a plurality of sets of sequences and in which each set of sequences has multiple sequences [16], and has completely orthogonal autocorrelation function and cross-correlation function, so the CC-S is used as a training sequence to estimate the channel of a MIMO system. Compared with the Orthogonal Gold Sequence, the autocorrelation and cross-correlation performance of completely complementary sequence are better than those of orthogonal Gold sequence [17]. The concept of CC-S is introduced by taking each group of sequences containing two sub-sequences as an example:

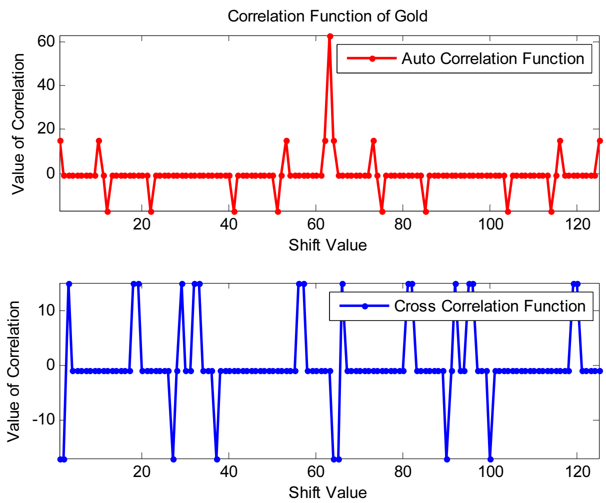
Assume $\{A_0, B_0\}$ as a set of sequences and $\{A_1, B_1\}$ as another set of sequences, and the length of each sequences A_0, B_0, A_1, and B_1 is L, if the following related functions are met:

$$R_{A_0A_0}(\tau) + R_{B_0B_0}(\tau) = \begin{cases} 2L & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \quad (1)$$

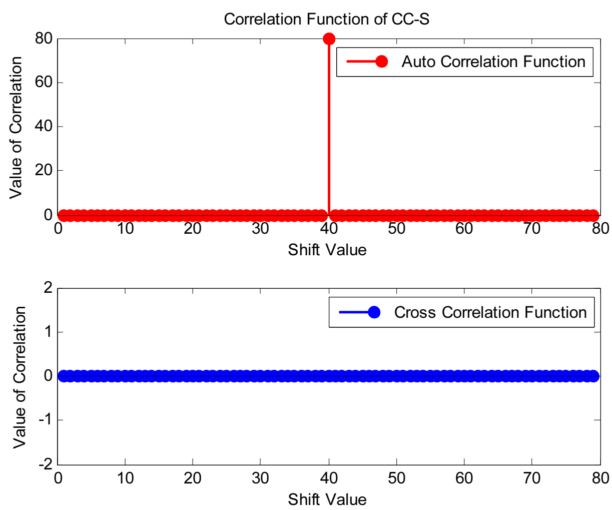
$$R_{A_1A_1}(\tau) + R_{B_1B_1}(\tau) = \begin{cases} 2L & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \quad (2)$$

$$R_{A_1A_0}(\tau) + R_{B_1B_0}(\tau) = 0 \qquad \forall \tau \quad (3)$$

$\{A_1, B_1\}$ and $\{A_0, B_0\}$ make up a pair of CC-S, where $R_{A_1A_1}(\tau)$ is autocorrelation function of Sequence A_1 and $R_{A_1A_0}(\tau)$ is cross-correlation function of Sequences A_1 and A_0. Fig. 1 depicts the autocorrelation and cross-correlation functions of an orthogonal Gold sequence and a CC-S. As can be seen from the figure, the autocorrelation of an orthogonal Gold sequence has certain side lobe value and its cross-correlation also has certain amplitude; compared with the orthogonal Gold sequence, the side lobe value of the autocorrelation function of the CC-S is zero, while that of the cross-correlation is also zero in any shift case, so the CC-S has complete orthogonality.



(a) Correlation Function of Gold Sequence



(b) Correlation Function of CC-S Sequence

Fig. 1. Correlation function of orthogonal Gold sequences and CC-S

3 MIMO Channel Estimation and Sequence Optimization Scheme

3.1 MIMO Channel Estimation Scheme Based on Training Sequence

The purpose of MIMO channel estimation based on training sequence is how to estimate channel characteristics H using received information sequence and training sequence, and Fig. 2 is a timing-based MIMO communication system model, which transmits in a manner that the training sequence is inserted in front of the information sequence. This method is a channel estimation method at the expense of information broadband resources.

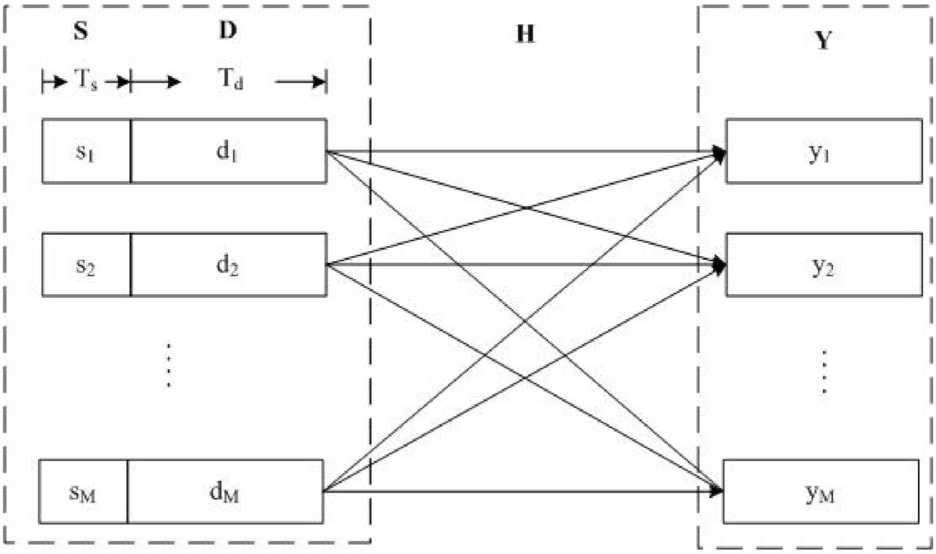


Fig. 2. Transmission architecture of MIMO communication system

In terms of a MIMO communication system, there are M transmitting antennas and N receiving antennas, and the coherent time of the system is T_c, where the symbol lengths of the training sequence and the data sequence are respectively T_s and T_d:

$$T_c = T_s + T_d \tag{4}$$

The training sequence of the $m^{th}, m \in [1,2,\cdots,M]$ transmitting antenna transmitted at k time is $s_m(k)$ and the symbol of the training sequence may be represented by the following vector **s**(k):

$$\mathbf{S}(k) = [s_1(k), s_2(k), \cdots s_M(k)]^T \tag{5}$$

The data received by the receiving antenna is recorded as $\mathbf{y}(k)$:

$$\mathbf{y}(k) = [y_1(k), y_2(k), \cdots y_N(k)]^T \tag{6}$$

(6) is written as a symbol matrix as below:

$$\mathbf{Y} = \begin{bmatrix} y_1(1) & y_1(2) & \cdots & y_1(T_s) \\ y_2(1) & y_2(2) & \cdots & y_2(T_s) \\ \vdots & \vdots & \ddots & \vdots \\ y_N(1) & y_N(2) & \cdots & y_N(T_s) \end{bmatrix} \tag{7}$$

Where $\mathbf{H}$ describes the channel response coefficient between the m^{th} transmitting antenna and the n^{th} $n \in [1,2,\cdots,N]$ receiving antenna of the system:

$$\mathbf{H}=\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix} \quad \textbf{(8)}$$

Therefore, during the training period, we can get:

$$\mathbf{Y}=\sqrt{\frac{\rho}{M}}\mathbf{HS}+\mathbf{V} \quad \textbf{(9)}$$

Where ρ is the average signal-to-noise ratio over a single receiving antenna during the training period and **V** is noise matrix. Assume that the noise is Gauss white noise and power is *P*. Since both the orthogonal Gold sequence and the CC-S have the characteristics of multiple sets, and the orthogonal Gold sequence can be described as:

$$s_G=\left[s_{G1},s_{G2},\cdots,s_{GM}\right]^T \quad \textbf{(10)}$$

Where $s_{Gi},(i=1,2,\cdots M)$ is sub-sequence of the orthogonal Gold sequence. The number of orthogonal Gold sequence is identical to the number of transmitted antennas.

The CC-S can be described as follows:

$$s_c=\begin{bmatrix} s_{c1}^1 & s_{c2}^1 & \cdots & s_{cM}^1 \\ s_{c1}^2 & s_{c2}^2 & \cdots & s_{cM}^2 \end{bmatrix}^T \quad \textbf{(11)}$$

Its channel training sequence model is shown in Fig. 3.

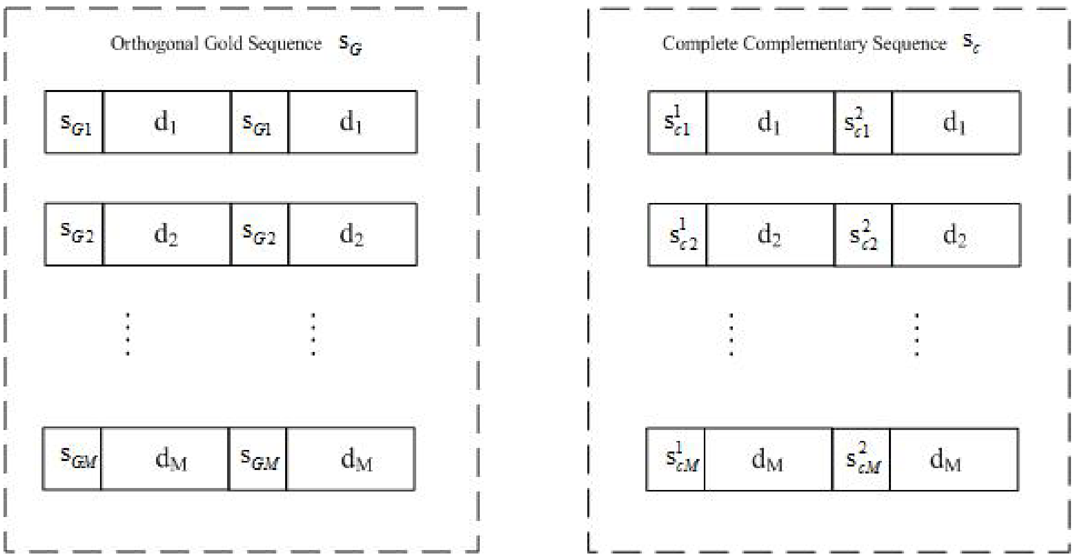


Fig. 3. Training sequence model

According to the constructed orthogonal Gold sequence and CC-S, the orthogonal Gold sequences and CC-S consistent with the number of transmitting antennas are selected as training sequences.

3.2 Optimization of Training Sequence Algorithm

This section focuses on optimizing training sequence algorithm. According to Formula (10), the effect of the coefficient $\sqrt{\frac{\rho}{M}}$ is temporarily ignored to simplify the flow of the optimization algorithm based on the training sequence is as follows:

Procedure:

Step 1: Given initial training sequence **S**

Step 2: Calculate the maximum mutual information of **Y** and **H**:

$$C = I_{\max}(\mathbf{H},\mathbf{Y}) = \{G(\mathbf{Y}) - G(\mathbf{Y}|\mathbf{H})\}_{\max}$$

Because H and V are statistically independent:

$$I(\mathbf{H},\mathbf{Y}) = G(\mathbf{Y}) - G(\mathbf{Y}|\mathbf{H}) = G(\mathbf{Y}) - G(\mathbf{V})$$

Step 3: Calculate the entropy $G(\mathbf{Y})$ of the random matrix **Y**: Assume $E(\mathbf{H}) = 0$ is mean of **H** and $\mathrm{cov}(H) = \delta_h^2 \boldsymbol{I}_{M\times N}$ is covariance matrix of **H**:

$$G(\mathbf{Y}) = \lg\left[\det(\delta_h^2 \cdot \mathbf{S}\mathbf{S}^H + \mathbf{P})\right]$$

Step 4: Calculate the entropy $G(V)$ of the random matrix **V**: $\mathrm{cov}(\boldsymbol{v}) = \mathbf{P}$ is covariance matrix of $\boldsymbol{v}$: $G(\mathbf{V}) = \lg[\det(\mathbf{P})]$

Step 5: Calculate the objective function of the optimization and get according to Step 2:

$$\begin{aligned} I(\mathbf{H},\mathbf{Y}) = G(\mathbf{Y}) - G(\mathbf{V}) &= \lg\left[\det(\mathbf{I}_M + \delta_h^2 \mathbf{P}^{-1}\mathbf{S}\mathbf{S}^H)\right] \\ &= \lg\left[\det(\mathbf{I}_M + \delta_h^2 \mathbf{S}^H \mathbf{P}^{-1}\mathbf{S})\right] \end{aligned}$$

Step 6: Optimize the training sequence S, and according to Step 5, enable $I(\mathbf{H},\mathbf{Y})$ to get the maximum value $I_{\max}(\mathbf{H},\mathbf{Y})$.

When $\delta_h^2 \mathbf{S}^H \mathbf{P}^{-1}\mathbf{S}$ is diagonal matrix, mutual information quantity $I(\mathbf{H},\mathbf{Y})$ gets the maximum value, that's, the channel capacity. Assume that the noise is Gauss white noise, $\mathbf{P} = \delta_n^2 \boldsymbol{I}$.

(1) When the training sequence $\mathbf{S}$ is a CC-S $\mathbf{S}_c$,

$$\begin{aligned} \mathbf{S}_c{}^H \mathbf{P}^{-1}\mathbf{S}_c &= (\mathbf{P}^{-1/2}\mathbf{S}_c)^H (\mathbf{P}^{-1/2}\mathbf{S}_c) \\ &= \delta_n^2 \boldsymbol{I} = \mathbf{\Lambda} \end{aligned} \quad \textbf{(12)}$$

Then, $I(\mathbf{H},\mathbf{Y})$ gets the maximum value, since the CC-S satisfies complete orthogonality, that's,

$$\mathbf{S}_c{}^H \mathbf{S}_c = \mathbf{\Lambda} \quad \textbf{(13)}$$

As can be seen from Formula (13), the CC-S is the optimal training sequence.

(2) When the training sequence is an orthogonal Gold sequence, since the orthogonal Gold sequence doesn't satisfy complete orthogonality, that's, $\mathbf{S}_G{}^H \mathbf{S}_G \neq \mathbf{\Lambda}$ isn't diagonal matrix, and since Product $\mathbf{S}_G{}^H \mathbf{S}_G$ is diagonal matrix, the spectral transformation can be taken to set $\mathbf{\Theta} = \mathbf{S}_G{}^H \mathbf{S}_G$, it is spectrally decomposed to get:

$$\mathbf{\Theta} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q} \quad \textbf{(14)}$$

Where, $\mathbf{Q}$ is orthogonal matrix, $\mathbf{S}_G$ is subject to matrix transformation $\mathbf{S}_G\mathbf{Q}$, enabling $(\mathbf{S}_G\mathbf{Q})^H \mathbf{P}^{-1}(\mathbf{S}_G\mathbf{Q})$ as diagonal matrix, that's,

$$\begin{aligned} (\mathbf{S}_G\mathbf{Q})^H \mathbf{P}^{-1}(\mathbf{S}_G\mathbf{Q}) &= \mathbf{Q}^H (\mathbf{S}_G^H \mathbf{P}^{-1}\mathbf{S}_G)\mathbf{Q} \\ &= \mathbf{\Lambda} \end{aligned} \quad \textbf{(15)}$$

Formula (15) indicates that after the optimization of Matrix $\mathbf{Q}$, $\mathbf{S}_G\mathbf{Q}$ is the optimal signal and since m sequence is the same as the orthogonal Gold sequence, both of which both of which don't satisfy the characteristics of complete orthogonality, the optimization of m sequence is the same as that of orthogonal sequence. As can be seen from the above formula, for any pseudo-random sequence, as long as its autocorrelation characteristic is not completely orthogonal, the channel capacity and channel estimation performance of the system can both be improved by the above optimization scheme.

4 Simulation Verification

In the computer simulation, the performance of MIMO channel capacity with optimization algorithm is analyzed in this study. The number of transmitting antennas in the simulation parameters is 1, 2, 4 and 8; the number of receiving antennas is 1, 2, 4 and 8; the order of the channel is 3. The modulation mode adopts QPSK modulation. The length of the selected orthogonal Gold sequences is 16, 32 and 64, respectively. The length of the CC-S is 40 and the length of the data frame is 256.

Fig. 4 simulates channel capacity based on CC-S and orthogonal Gold sequences under different length conditions and different signal-to-noise ratio conditions.

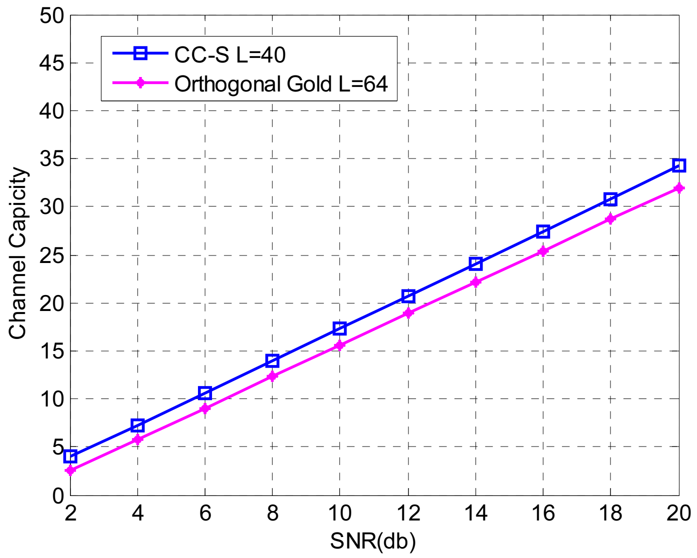


Fig. 4. Channel capacity based on CC-S and orthogonal Gold sequences

As can be seen from Fig. 4, although the length of the orthogonal Gold sequence is longer than that of the CC-S, the orthogonal Gold sequence may have a slightly lower channel capacity than the CC-S because the orthogonal Gold sequence doesn't have the characteristic of complete orthogonality.

To verify the performance of the optimization algorithm, Fig. 5 simulates the channel estimation based on the optimized orthogonal Gold sequence and the original orthogonal sequence. As a result, the orthogonality is improved by optimizing the orthogonal Gold sequence, so that the channel capacity is improved to some extent.

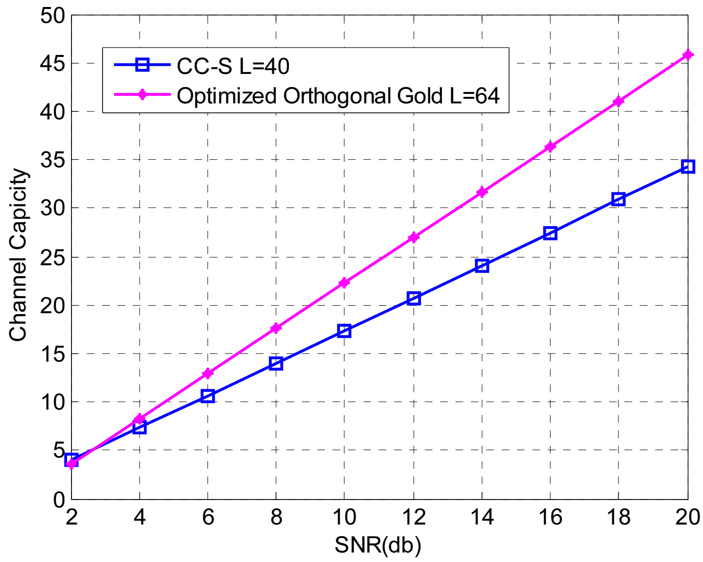


Fig. 5. Channel capacity based on complementary sequence and optimized orthogonal Gold sequence

To further compare the performance of the channel capacity under different antenna configuration, Fig. 6 simulates the channel capacity of the optimized orthogonal sequence with different numbers of transmitting antennas and receiving antennas. As the increase in the number of antennas brings a certain diversity gain, it can be seen from Fig. 6 that as the number of antennas increases, the channel capacity is also increasing.

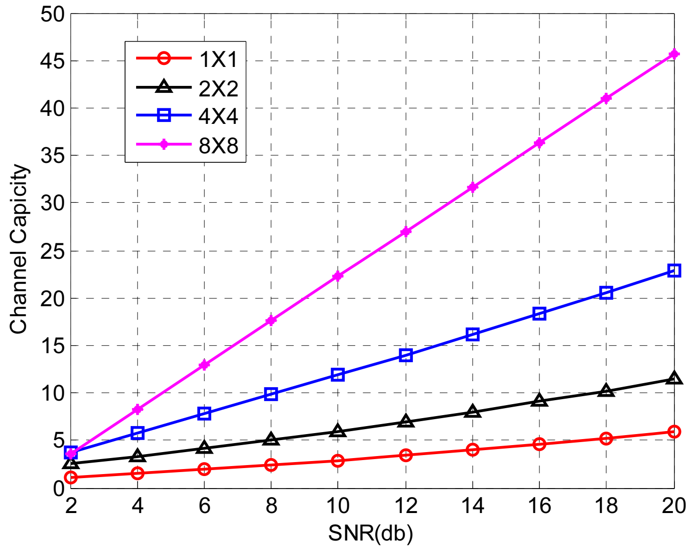


Fig. 6. Relationship between the channel capacity and the number of antennas

In addition, the orthogonal gold sequence, the optimized orthogonal sequence and the CC-S are applied in the MIMO system to perform channel estimation in this study, with the number of both the transmitting antennas and the receiving antenna is 2. Fig. 7 is a simulation result of channel estimation errors (normalized minimum mean squared error) of MIMO systems with different sequences under different signal-to-noise ratios. It can be seen from the result that the optimized orthogonal Gold sequence, since its orthogonality is satisfied, has a lower channel estimation error than the orthogonal Gold sequences. Therefore, the performance of the system is improved. Because the length of the optimized orthogonal sequence is larger than that of the CC-S and Chu sequence used, its channel estimation performance is slightly better than the channel estimation results of the two sequences. Compared with method proposed by literature [12], the channel estimation error of this paper is much lower than that because of the perfect orthogonality obtained by the optimization method.

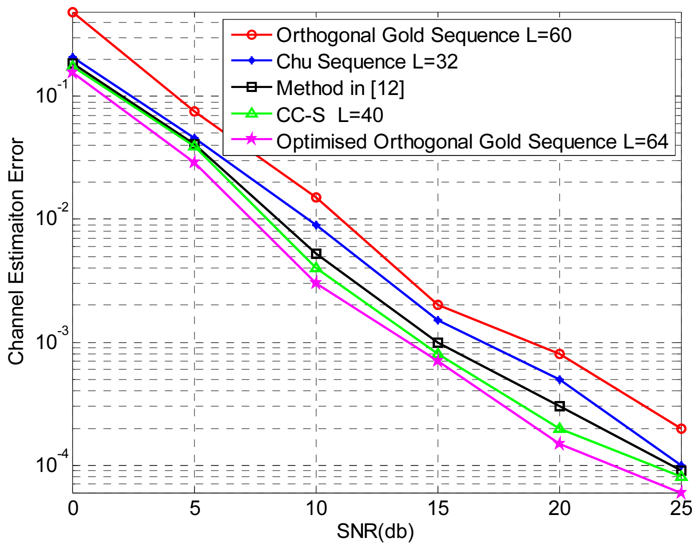


Fig. 7. Relationship between bit error rate and signal-to-noise ratio under different sequences

5 Conclusions

This study sets up a MIMO channel estimation model based on timing training sequence and proposes a training sequence optimization algorithm with channel capacity as optimization criteria. An optimization scheme based on information theory is used to raise the channel capacity of MIMO communication system. With the CC-S and the orthogonal Gold sequence as basic sequences, this study analyzes the channel capacity. From the results, it is found that under the white Gaussian noise, since the CC-S has characteristics of complete orthogonal autocorrelation and cross-correlation, the MIMO channel capacity based on the CC-S can still maximize the channel capacity without an optimization algorithm. This study also compares it with the traditional Chu sequence and finds that orthogonal Gold sequences with the proposed scheme can still get better effect in channel estimation after optimization. This paper only analyzes the optimization under the information theory, there are still many key in theory and in practice to be research further, such as correlation function criterion, peak to average power ratio and the application of the proposed method in massive MIMO system.

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