

# A Hybrid Simulated Annealing Algorithm for Load Varying Green Vehicle Routing Problem with Stochastic Demands



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Received 1 June 2018; Revised 1 July 2018; Accepted 1 August 2018

**Abstract.** To be environmentally friendly, reducing the energy consumption of vehicles is the trend of solving vehicle routing problems (VRPs). This paper considers the green VRP problem by using a load varying green VRP (LVGVRP) that is generally suitable for both fossil fuel vehicles and alternative fuel vehicles (AFVs), and we also study LVGVRP problems with stochastic demands (LVGVRPSD). We develop a mathematical model to formulate the LVGVRPSD optimization problem based on fact that the energy consumption rate is directly proportional to the total weight of the vehicle, and we use the risk probability to constrain the impact of stochastic customer demands. A hybrid tabu-search improved simulated annealing algorithm (HSAA) is proposed to solve the LVGVRPSD problem, in which k-means clustering, local search, and tabu-list guided searching are used to improve the results of SAA. We conduct experiments on 20 commonly used benchmarks, and the results prove that considering varying vehicle load can obtain better routes with lower energy cost. In addition, the results also prove that HSAA can achieve better objective compared with existing SAA.

**Keywords:** green VRP, simulated annealing, stochastic demands, tabu search, vehicle routing,

## 1 Introduction

The emissions of motor vehicles is one of the main pollutant resources to air quality in urban areas. To reduce the negative impact of vehicles, the governments have made incentives and legislations to promote the adoption of alternative fuel vehicles (AFVs) with environmentally friendly energy supplies, such as electric and hydrogen energies [1]. In recent years, many companies and agencies have greatly increased the adoption of AFVs in their transportation businesses, such as La Poste, Coca-cola and UPS [2]. The adoption of AFVs also imposes a new extension of routing optimization problem, named green vehicle routing problem (GVRP), and it is firstly introduced in [3]. In GVRP, an AFV fleet is used to provide transportation services for a given set of geographically distributed customers. However, due to the limited energy supply of AFVs, the routing optimization model must provide a carefully scheduling of battery recharging, or fuel refilling operations to guarantee the completion of the trips to every customers.

Although the limited energy supply highlights the importance of energy supplement for AFVs, the application is typically suitable for long-haul transportation tasks, but not for short-haul transportation scenarios, such as city transportation scenarios [4]. A basic principle of energy consumption of vehicles is that, the energy consumption rate is directly proportional to the total weights of the vehicles. When the load of a vehicle is changed, the energy consumption rate of both fuel and electric is also changed [5-7]. Therefore, the varying of vehicle loads has significant impact on the energy cost of vehicles, especially in VRP problems with imbalanced customer demands. This is also a principle that consistent to the law of energy conservation. Based on this principle, the routes with total shortest travelling distance will not be equivalent to minimum total energy cost, which is not environmentally friendly. In addition, the reduction of energy cost of AFVs also reduce the transportation cost, which is a win-win situation for

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both economics and environments.

In this paper, we highlight the impact of load weight on the energy consumption of vehicles, and load-varying GVRP (LVGVRP) to minimize the energy consumption of vehicles. In the proposed LVGVRP, the workloads of the AFVs, or vehicles, are varying during the trip of transportation, furtherly the energy consumption rate are also varying according to the workloads of the AFVs. We consider short-distance transportation, and assume that there is sufficient energy reserves when the AFVs departures from the depot station, therefore the battery recharging, or fuel refilling operations are not needed. The proposed LVGVRP is also suitable for scenarios of conventional vehicles with fossil fuels. In this way, LVGVRP can help to reduce the consumption of fuels or electrics, which is also beneficial for reducing travel cost and air pollutions.

In particular, we consider LVGVRP with stochastic demands (LVGVRPSD), which is more consistent to real world situations. In LVGVRPSD, the exact demand of a customer previously unknown to manager, and it can be regarded as a random variable following some historical statistical distribution [8]. With this prerequisite, the scheduling of vehicles must be more careful to avoid the risks that the total workloads of the vehicles exceed their maximum capacities [9]. To cope with the stochastic customer demands, we formulate the capacity constraint by using a risk probability formulation, in which the probability that the maximum workload of a vehicle exceeds its capacity is lower than a predefined probability threshold. By this way, the maximum allowed vehicle workload will be more smaller than its capacity to reduce the risk probability, but the sacrifice is that the total travel cost, or energy consumptions will be increased.

To solve LVGVRPSD, in this paper we propose a hybrid simulated annealing algorithm (HSAA) that is an extension of conventional SAA algorithm [10]. Similar to existing SAA, in HSAA the swap, insertion and reversion actions are used to generate neighborhoods. To improve the quality of the results, HSAA adopts the following improvements:

(1) In the route initialization phase, HSAA adopts a k-means algorithm to generate a route in which the customers of each vehicle route is distributed closely in a local region. This practice is quite suitable for scenarios that the customers are distributed in several clusters.

(2) HSAA adopts local search to control the scope of the reversion action. In this way, the number of involved customers in one reversion action will not be larger than a predefined number, which is helpful the speed up the convergence process, and avoid local optimum traps.

(3) Most importantly, HSAA uses the tabu heuristic to enhance the searching capacity of the neighborhood process. In conventional SAA, the involved nodes is randomly selected in each action, which may lead to suboptimal results compared with exhaustive search that guarantees the optimal results in every searching round. As such, we improve the SAA searching process by using a tabu-list to improve the results of every search round. Experimental results will be provided to prove that HSAA is able to produce better results compared with conventional SAA.

The main contributions of this paper are summarized as follows: we introduce LVGVRP, which is an extension of GVRP. LVGVRP considers the impact of the total vehicle weights on energy consumption rate, and the objective becomes minimizing the energy consumptions but not minimizing travelling distance. We consider the LVGVRP with stochastic demands, i.e. LVGVRPSD, in which workload constraint is more strict to avoid over-loading risks. We propose a HSAA to improve the searching results of the vehicle routes by using the following improvements, including clustering initialization, local search, and tabu heuristic. Experimental results on 20 widely used benchmarks are provided to demonstrate that HSAA can produce better results compared with existing SAAs. More importantly, the results also prove that LVGVRP model can achieve better results compared with conventional models without considering the influences of workload on energy consumption rates.

The remainder of this paper is organized as follows. Section 2 provides a brief review of related work, including GVRP, VRP with stochastic demands, and simulated annealing algorithm. The mathematical model of the proposed LVGVRPSD is illustrated in Section 3. Section 4 elaborate the proposed HSAA for solving the LVGVRPSD model. Experimental results are shown in Section 5. Finally, Section 6 concludes this paper.

## 2 Related Work

The VRP has a large variations when considering the different scenario of the applications. A basic VRP is the capacitated VRP (CVRP) that the maximum load of each vehicles is limited. In CVRP, all the vehicles departure from one single depot node, then traverse the customers, and finally return to the depot node. The goal of CVRP is to find optimal routes that minimize the total transportation cost. In general, the objective is to minimize the total travelling cost, such as time or energy consumption. As a basic problem, CVRP has been extensively studied in past years, a detailed introduction can be seen in [11].

As a new emerged extension of CVRP, the GVRP has attracted great research interests in recent years. Aims at reducing Greenhouse Gas (GHG) emissions, AFVs have been adopted as the alternative for goods transportation. Unlike conventional CVRP problem, GVRP considers the travelling distance limitations of AFVs, and the recharging stations must be visited before the energy supply is ran out [3]. In this paper we also consider GVRP, but the goal is to minimize energy consumption for short-haul distance transportation, not matter the fuel vehicles, or AFVs.

Minimizing the vehicle fuel consumption has been previously considered in several literatures. In [12], the author consider the impact of loading weights and transportation speed on the energy consumption rate of vehicle, and uses a typical SAA to find solutions that minimize the total energy consumptions. Considering the load weight capacity of vehicle influences the energy consumption rate, in [13] the authors propose use a fleet with heterogeneous vehicles to minimize CO<sub>2</sub> emissions. In typical CVRP, a vehicle is schedule with one single trip in each transportation round, in [14] a new multi-trip vehicle routing problems (MTVRP) is proposed to reduce GHG emissions by allowing multiple trips of the vehicles. Also, the pickup and delivery of customer demands are frequently discussed in GVRP problems, related works can be seen in [15-16].

In this paper, we consider energy minimization-based GVRP problem with stochastic customer demands. CVRPSD is initially proposed in [17], in which the exact customer demands are not known to the manager, but the probabilistic distribution can be obtained from historical information. CVRPSD has been proved to be useful in many practical applications, such as petroleum products, industrial gases, and home heating oil [18]. In general, the CVRPSD imposes a more strict capacity constraint that require the vehicle to reserve some extra capacity for situations when the total customer demands is larger than its expected value [19]. However, it is still possible that the prescheduled routes fails when the total demands of the vehicle exceeds the vehicle capacity. In this situation, a re-optimization process can be conducted to reduce the travel cost [20].

It is well-known that the CVRP is a NP-hard problem, and the finding the optimal solutions is also a challenge in VRP research. Related methods for solving the CVRP can be simulated annealing [21], tabu-search [22], variable neighborhood search [23], ant colony algorithms [24], and genetic algorithms [25], to name a few. In this paper, we focus on using SAA for solving the VRP problem and its variants. Successful applications can be seen in [12, 26]. In [27], the authors highlight the importance of route initialization process in SAA, and propose a nearest-search-based initialize algorithm to generate initial vehicle routes when using SAA algorithm in solving hybrid VRPs. Recent advances in SAA [27-28] show that using a restart strategy that when the current best solution is not improved with several consecutive temperature decreases can improve the searching ability of SAA. In this paper, the SAA with a restart strategy will be used for performance comparison to demonstrate the performance of the proposed HSAA method.

## 3 Mathematical Formulation

We consider a load-varying green vehicle routing problem with stochastic demands (LVGVRPSD). Before developing the mathematical model, we illustrate the detailed scenario of the model as follows.

(1) There exist one and only one depot node in the whole area. For each vehicle, it departures from the depot node, then traverse the customers on the routes, and finally returns to the deport node. For each customer node, it must be visited once but only once in the whole route.

(2) Before scheduling the route paths of the vehicles, the locations of the nodes, travel costs between two nodes, maximum capacities of each vehicle, are previously known to the manager.

(3) The customer demand can be approximated as a Gaussian distribution, and the mean value and

standard deviation of the distribution can be obtained from historical information.

(4) The energy consumption rate is directly proportional to the total weight of the vehicle. When the workload of the vehicle is increased, the corresponding energy consumption rate also increases.

(5) We consider short-distance transportation applications, and all the vehicles, or AFVs, have sufficient energy storage to complete the whole trip.

### 3.1 Problem Formulation

We depict the scenario by using a complete and undirected graph  $G=(V,U,T)$ , in which  $V=\{v_1,\dots,v_N\}$  is the set of customers, and the corresponding demands are  $c_1,\dots,c_N$ . The depot node is denoted as  $v_0$ , and its corresponding demand  $c_0=0$ . Matrix  $U=[u_{i,j}]_{N\times N}$  denotes the set of edges in the graph. In the route, if  $v_i$  and  $v_j$  are linked, then  $u_{i,j}=1$ , otherwise  $u_{i,j}=0$ . Matrix  $T=[t_{i,j}]_{N\times N}$  denotes weights of an edge, and edge weight  $t_{i,j}$  is equivalent to the energy cost from node  $v_i$  to  $v_j$ . For all capacitated vehicles, we denote their maximum capacities as  $c_{max}$ , and the whole customer set will be divided into  $K$  exhaustive and mutually independent sub-routes  $V=\{V_1^s,V_2^s,\dots,V_K^s\}$ , which holds  $\forall V_k^s,V_l^s\subseteq V$ ,  $V_k^s\cap V_l^s=\emptyset$  and  $V_1^s\cup V_2^s\cup\dots\cup V_K^s=V$ .

In VRPSD, the load of each vehicle must not exceed its maximum capacity. However, this constraint may not be satisfied when the demands of customers are stochastic. To deal with this uncertain condition, the constraint can be set as the load of each vehicle is smaller than its maximum capacity with a probability  $P_{th}$ , which can be predefined according to practical requirements. In general, when  $P_{th}$  is relatively smaller, the total travel cost will be lower, but the reliability of the route will be low, i.e. the vehicles have higher risk that the total demands of its sub-route is larger than its capacity. On the contrary, when  $P_{th}$  is relatively higher, the total travel cost will be higher, but the reliability of the route will be high. In practice, a reasonable  $P_{th}$  should be carefully assigned to guarantee the service quality, and balance the tradeoff between reliability and travel cost of the routes.

With the above notations, we have the following mathematical optimization model

$$\text{Minimize } \sum_{k=1}^K \sum_{i,j\in V_k^s,i\neq j} \left(1 + \beta \frac{c_{ki}^w}{c_{th}}\right) u_{kij} t_{ij} \quad (1)$$

Subject to:

$$P\left(\sum_{i\in V_k^s} c_{ki} > c_{th}\right) < P_{th} \quad (2-2)$$

$$\sum_{i\in V_k^s} u_{k0i} = 1 \quad (2-2)$$

$$\sum_{j\in V_k^s} u_{kj0} = 1 \quad (2-3)$$

$$\sum_{k=1}^K \sum_{j\in V_k^s,j\neq i} u_{kji} = 1 \quad (2-4)$$

$$\sum_{k=1}^K \sum_{j\in V_k^s,j\neq i} u_{kij} = 1 \quad (2-5)$$

$$u_{kij} = \{0,1\}, 0 \leq i, j \leq N, i \neq j, k \leq K \quad (2-6)$$

The objective function (1) is to find the optimal route that minimizes the total travel cost of the all the vehicles,  $c_{ki}^w$  is the workload when vehicle  $k$  leaves from node  $i$ ,  $\beta$  is a constant that equals to the ratio of vehicle capacity to vehicle weight. For example, when  $\beta=1$ , the vehicle capacity is equal to the vehicle weight of itself. In this paper,  $\beta$  can be set as a constant ranges from 0.5 to 1. Note that for some vehicles with large capacities,  $\beta$  can be larger than 1. Constraint (2-1) denotes the probability of the load of a vehicle is larger than its capacity should be smaller than the a predefined probability threshold. Constraints (2-2) and (2-3) ensure that the route path of a vehicle should be started from the depot node, and finally back to the depot node. Variable  $u_{kij}$  denotes the binary decision variable determining the whether the vehicle moves from node  $v_i$  to node  $v_j$  in subset  $V_k^s$ . With the two constraints, a vehicle will visit at least one customer to avoid idle vehicles. Constraints (2-4) and (2-5) means that a node must be visited by one but only one vehicle. Also, they ensure that in each sub-route, the vehicle visits and leaves each member node exactly once. Constraint (2-6) implies that the routing decision is restricted as a binary variable.

### 3.2 The Risk Probability Constraint

Given a set of customers with stochastic demands, the risk probability is defined as the probability that the load of a vehicle exceeds its capacity. With the historical information of the customer demands, its corresponding probability distribution functions (PDFs) can be obtained. When the capacity is  $c_{th}$ , it is easy to calculate the target risk probability by using a complementary cumulative distribution function, which is defined as

$$P(c_k > c_{th}) = \int_{c_{th}}^{+\infty} p(c_k) dc_k, \quad (3)$$

where  $c_k = \sum_{i \in V_k^s} c_{ki}$  is the total demand of vehicle  $k$ , and  $p(c_k)$  is the PDF of  $c_k$ . In this paper, we assume that the PDFs of the customers follow the Gaussian distribution, i.e.  $c_k \sim N(\mu_k, \sigma_k^2)$ , where  $\mu_k$  and  $\sigma_k^2$  are the mean value and variance, respectively. By this assumption, the risk probability that the total demands of a sub-route exceed the capacity is

$$P(c_k > c_{th}) = \int_{c_{th}}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(c_{th} - \mu_k)^2}{2\sigma_k^2}\right) dc_k = Q\left(\frac{c_{th} - \mu_k}{\sigma_k}\right), \quad (4)$$

where  $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2} dx$  is the complementary cumulative distribution function of the standard

Gaussian distribution. Given a target risk probability  $P_{th}$ , we have  $Q\left(\frac{c_{th} - \mu_k}{\sigma_k}\right) \leq P_{th}$ , and further we have

$$c_{th} \leq \sigma_k \left(Q^{-1}(P_{th})\right) + \mu_k, \quad (5)$$

which is the maximum planned workload of a vehicle  $k$ . A higher risk probability corresponds to a larger planned maximum capacity, and therefore the vehicle can carry more packets. On the contrary, when the risk probability is relatively smaller, the vehicle must carry less packets.

Note that in real-world applications, the distribution of a random variable may not be ideal Gaussian distribution, but a complex one with multimodal component distributions. In this scenario, using a more flexible way to estimate the PDFs of the demands will increase the reliability of the obtained results. More specifically, a kernel Parzen window estimator, or Gaussian mixture model (GMM), can be applied to obtain more precise estimation results. For example, when the target PDF is estimated by using GMM, we have

$$p(c_k) = \sum_{i=1}^{n_k} \alpha_{ki} p(c_{ki}), \quad (6)$$

where  $n_k$  is the number components in GMM,  $\alpha_{ki}$  and  $p(c_{ki})$  are the prior probability and PDF of the  $i^{\text{th}}$  component, respectively. Then the target risk probability is

$$P(c_k > c_{th}) = \sum_{i=1}^{n_k} \alpha_{ki} \int_{c_{th}}^{+\infty} p(c_{ki}) dc_{ki} = \sum_{i=1}^{n_k} \alpha_{ki} Q\left(\frac{c_{th} - \mu_{ki}}{\sigma_{ki}}\right). \quad (7)$$

However, the inverse function of the above equation can not be directly derived, and finding a proper threshold with a predefined risk probability will cause much higher computation complexity compared with a simple Gaussian distribution.

### 3.3 Simplification of Risk Probability Constraints

When searching the optimal solution for LVGVRPSD, we have to guarantee that the risk probability of every vehicle is lower than the threshold. Actually, this constraint can be simplified if we assume that a vehicle with larger expected customer demands has a larger standard deviation, i.e. when  $\mu_1 \leq \mu_2$ , there is  $\sigma_1 \leq \sigma_2$ . In this way, for two customer demands with mean values  $\mu_i$  and  $\mu_j$ , if  $\mu_i \leq \mu_j$ , we have  $\sigma_i \leq \sigma_j$ , and

$$\frac{c_{th} - \mu_i}{\sigma_i} \geq \frac{c_{th} - \mu_j}{\sigma_j}. \quad (8)$$

Since function  $Q(x)$  is a monotonically decreasing with the increasing of  $x$ , we have

$$Q\left(\frac{c_{th} - \mu_i}{\sigma_i}\right) \leq Q\left(\frac{c_{th} - \mu_j}{\sigma_j}\right). \quad (9)$$

As such, we know that the vehicle with maximum customer needs has largest risk probability, thus constraint (2-1) can be simplified as

$$P\left(\max\left(\sum_{i \in V_k^s} c_{1i}, \dots, \sum_{i \in V_k^s} c_{Ki}\right) > c_{th}\right) < P_{th}. \quad (10)$$

By this way, we just need to check whether the risk probability of vehicle with maximum customer needs is exceed the allowed threshold, and the results will be abandoned if this condition is not satisfied.

## 4 Hybrid Simulated Annealing Algorithm

The simulated annealing algorithm (SAA) is a heuristic that has been successful applied in solving the different VRPs. The conventional SAA adopts a probabilistic perturbation operation in the Monte-Carlo hill-climbing searching process to jump out the local optimal trap and reach global optimum. When a newly randomly generated neighborhood achieves a better objective result, the neighborhood will be accepted as the current solution. Except this solution updating way, the metropolis acceptance criteria is also used to randomly update current solution with a relative worse objective result. By this way, the SAA is able to escape the local optima and find new directions to update the solutions. With each repetition, the temperature will be reduced by using the cooling down operation. At the same time, the metropolis acceptance probability will become smaller, accordingly, a mutation result with worse objective will be less likely to be accepted, the results tends to be converges along with the cooling down of the temperature. Finally, the SAA is terminated when the temperature is below the minimum allow temperature.

#### 4.1 Overview of HSAA

As shown in Fig. 1, we propose a tabu-searching enhanced hybrid SAA to solve the LVGVRPSD problem, in which cooling process and searching process are not stochastic, but static and exhaustive by using a tabu search list. Besides, the k-means-based clustering algorithm is used in the route initialization process, and local search strategy is used in the neighborhood generation operations. At the beginning of the HSAA, the temperature  $T$ , annealing parameter  $r$ , and Metropolis acceptance adjusting parameter  $\lambda$  are specified. The vehicle route  $R_{ini}$ , current route  $R_{current}$  are initialized as the same pattern. The objective differences are also initialized. After the setup process, neighborhood generation and route update will be iterated conducted until the algorithm convergences to an optimal objective. The details of the proposed HSAA will be introduces in the following subsections.

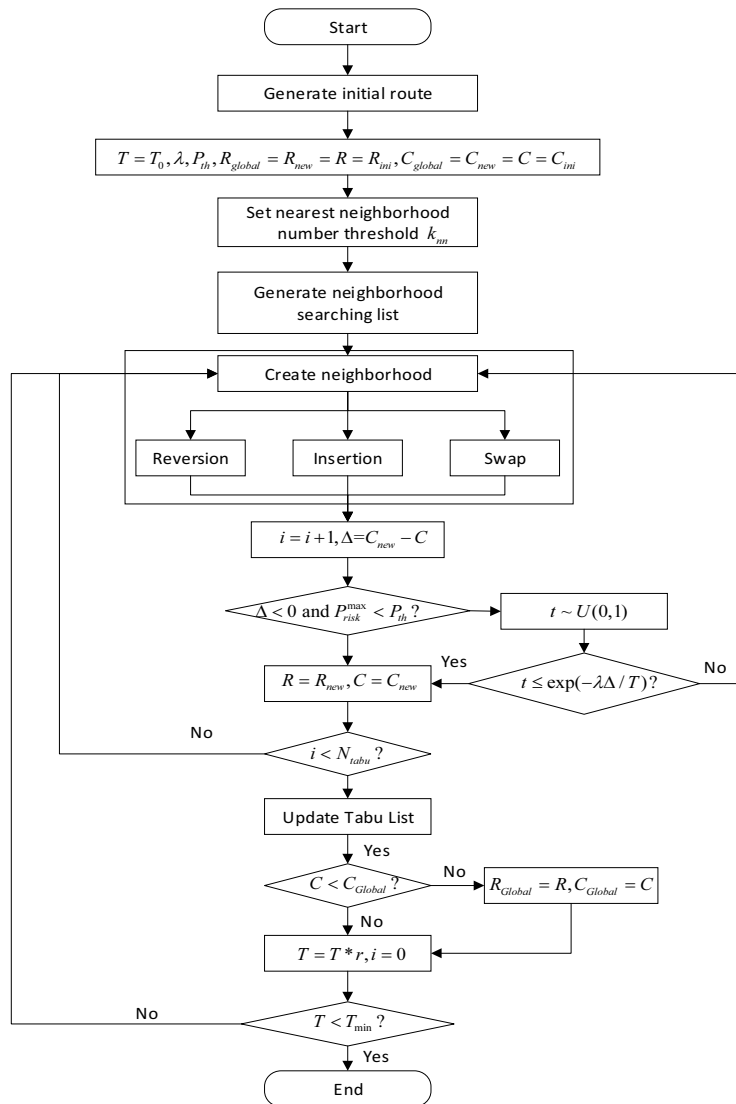


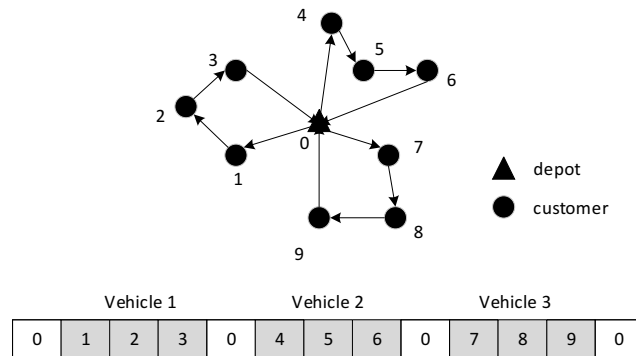
Fig. 1. Flow diagram of the proposed HSAA for solving LVGVRPSD

#### 4.2 Route Representation

The representation pattern of conventional TSP with one single salesman is not suitable in this scenario, thus we adopt a more specified way to construct the route representation, in which the depot node is used to separate the sub-routes of the vehicles. An illustrative example is shown in Fig. 2, in which there are 9 customers, 3 vehicles and one depot node. In this example, as shown in (a), each vehicle visits 3 customers and then returns to the depot node. The visiting sub-route is vehicle 1:  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$ , vehicle 2:  $0 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 0$ , and vehicle 3:  $0 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 0$ . The corresponding route

representation is shown in (b), the route organized in the following serial way:  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 0 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 0$ , in which the sub-routes all the vehicles are divided by the depot node.

Note that there must be at least one customer in each sub-route. This can be achieved by finding the intervals of the locations of the depot node in the obtained route, and if the locations between two adjacent depot is smaller than 2, the neighborhood should be abandoned since this condition is not satisfied.



**Fig. 2.** Example of route representation

### 4.3 K-means Clustering-based Route Initialization

In VRP with multiple capacity limited vehicles, the “cluster first, route second” is a popular method for its easy implementation and fast speed for VRP with large amount of vehicles [29]. However, this method is not competitive compared with heuristic method in achieving the optimal results. In this paper, we propose to use the k-means clustering algorithm [29] to initialize the vehicle routes. This initialization process is able to speed up the searching process, as the initial routes of each vehicle lies in a certain sub-region, and large scale neighborhood searching operation can be avoided.

In practice, the results of a simple k-means clustering process is usually not able produce an initial that satisfies all the constraints. To deal with this problem, a refinement process needs to be conducted to adjust the initial route. In particular, the algorithm reassigns the customers of a vehicle with largest customer demands to the vehicle with least customer demands, and this reassignment process is repeated until all the constraint conditions are satisfied.

When the number of vehicle is too small, the repetition number of the above reassigning process will be larger than a predefined value, which means that the algorithm is failed to find an initialized route. In this situation, more vehicles can be used until the algorithm can stopped with a proper initial route.

### 4.4 Neighborhood Generation

The neighborhood searching process aims at finding a new better solution that achieves lower cost, or a perturbation that may help the current solution to escape from the local optima. Note that in each neighborhood searching operation, some conditions need to be specified to ensure the new generated neighborhood satisfy the constraints in equation 2. In addition, the neighborhoods are not randomly generated, but by using a predefined tabu-search list. This practice will traverse all the actions in the tabu list, which leads to a larger computation complexity. However, this is helpful to find the best neighborhood of the current solution, which is helpful to avoid sub-optimal search directions.

We adopt the following three commonly used operations for generating new neighborhoods, including swap, insertion, and reversion. The details of above three operations are introduced as follows.

**Swap:** This operation is conducted by swapping the locations of the selected nodes. Usually, when two nodes are selected, their locations are swapped.

**Insertion:** This operation is realized by randomly selecting a node, and then insert it into another one randomly selected location. Except finding a new solution, this Insertion operation also has critical influences on assigning a reasonable number of customers for each vehicle.

**Reversion:** This operation is conducted by randomly selected two nodes from the current route, and the inverse the substrings between these two selected nodes. When the solution is trapped into a local



optima, this Reversion is very helpful to generate a larger perturbation compared with Swap and Insertion operations.

Note that when a new neighborhood is generated, it must satisfy all the constraints of the optimization model. If constraints are not satisfied, the obtained neighborhood will be abandoned and start next neighborhood generation operation.

#### 4.5 Local Search Strategy

It has been proved the local searching is helpful in improving the quality of metaheuristic algorithms [30]. In neighborhood generation process of the proposed HSAA, all of the aforementioned three neighborhood operations can be improved by using a local search approach. Particularly, the reversion operation is strongly influenced by the length of the selected substring. The reason is that, when the length of the randomly selected substring is too large, the possibility to obtain a better solution is very low. If a relative worse neighborhood is accepted, it will take much more extra operations to make the route converges again, which add the complexity of the HSAA. Besides, if this local search strategy is adopted, some unnecessary operations will be skipped, and the efficiency of reversion operation is slightly improved.

An efficient local searching process can be realized by using the following process: In the algorithm initialization phase, we find out the nearest neighbors of each customer. When generate the tabu-searching list, when two nodes are not satisfy the maximum nearest neighbor number threshold, they will be accepted in the list, otherwise it is abandoned.

#### 4.6 Tabu-Search List

As aforementioned, in this paper the neighborhoods process are not randomly generated. Similar to Tabu search algorithm, before the iterative neighbor generation process, a tabu-search list is firstly generated. The three neighborhood operations, i.e. swap, insertion and reversion, are all can be different actions with two selected nodes. Therefore, to generate all possible better result to the current solution, we also adopt a tabu-search list in the propose HSAA. However, although guided search using a tabu search table is helpful to produce better results, the complexity will be increased when the annealing parameter  $r$  is given. Apparently, using the tabu-search list to guide the generation of HSAA will impose a large time complexity compared with the conventional SAA.

In the tabu-search list process, the local search strategy will be used to guide the generation of the permutation actions. If the two nodes are not closely nearest neighbors, the permutation actions of the two nodes will be abandoned.

In conventional Tabu Search algorithm, when an action produces a better result, this action will be skipped in the next few neighborhood generation rounds. This practice also can be adopt here, since it is useful to speed up the convergence process and reduce time complexity.

#### 4.7 Metropolis Acceptance Criteria

The metropolis acceptance criteria is the most distinguished operation of SAA, in which a solution with a worse objective is still possible can be accepted as the new solution, but not only the solutions with better results. More specifically, when a new neighborhood route  $R_{new}$  is obtained, it is accepted as the new solution by using the following Metropolis criteria

$$P(R = R_{new}) = \begin{cases} 1, & \text{if } \nabla E > 0 \\ \exp\left(-\frac{\lambda \nabla E}{T}\right), & \text{if } \nabla E \leq 0 \end{cases}, \quad (11)$$

where  $\nabla E = E_{new} - E$  is the difference value of travel cost between the new solution and the current best solution, and  $\lambda$  is a parameter adjusting the accepting probability. In the whole annealing process, a relative larger  $\lambda$  will increase the convergence speed of the solution, since random perturbation is not helpful when the system in a chaotic state. When the temperature cools down, the system tend to be steady state, and  $\lambda$  can be relatively smaller to generate random perturbation to escape local minima.

## 5 Computational Results

The computational experiments are conducted in a laptop computer with Intel (R) Core (TM) i7 CPU at 2.40 GHz, 8 Gb RAM. The proposed algorithm is coded in a MATLAB R2017a platform, and in a 64bit Windows 10 system.

The experiments include the following three parts, one is the performance comparison of the existing SAA and the proposed HSAA, and our goal is to prove that the proposed HSAA outperforms conventional SAA method. The second part is the performance comparison of energy cost between conventional CVRP and the proposed LVGVRP, and our goal is to show that LVGVRP can lead to lower energy cost compared with CVRP. The last one the test results of VRPSD that aim at demonstrate the proposed improvement of the proposed LVGVRPSD model. In addition, the impact of the ratio  $\beta$  will be provided, and some examples will be illustrated to show the differences between results of VRP and LVGVRP.

In all the experiments, the initial temperature is set as 1, and the minimum temperature is set a 0.01. The annealing parameter  $r$  is set from 0.99 to 0.995. The Metropolis acceptance adjusting parameter  $\lambda$  is set as 0.1. When local search is adopted, the nearest neighbor threshold is set from 5 to 20. Particularly, in the test of SAA, the restart strategy that restart to the current best solution when the algorithm fails to find new better solution after a certain iterations is used to avoid local optima trap, and the restart iteration number is set a 10.

### 5.1 Performance Comparison of SAA and the Proposed HSAA

The experimental results in this subsection is used to demonstrate the performance improvement of the proposed HSAA compared with existing SAA. In this test, 20 commonly used CVRP benchmarks [31] are used. The results are shown in the Table 1, which contains the optimal minimum achievable results of the all instances, the achieved best result, the difference gap between optimal minimum achievable results and the corresponding best solutions, and the execution time of the algorithm. To provide an overall performance comparison, the average performance of the results are provided in the last row of the table.

**Table 1.** Comparison of SAA and the proposed HTSA on 20 benchmark datasets

No.	Instances	Optimal	SAA			HSSA (propsoed)		
			Best	diff.	time(s)	best	diff.	time(s)
1	A-n32-k5	787	787.08	0%	26.18	787.08	0%	11.07
2	A-n33-k5	662	662.26	0%	26.84	662.26	0%	13.02
3	A-n53-k7	1010	1020.15	0.99%	36.24	1012.25	0.19%	40.02
4	A-n69-k9	1166	1170.82	0.26%	35.84	1165.9	0.26%	42.1
5	A-n80-k10	1766	1788.86	1.30%	44.56	1774.9	0.51%	101.5
6	B-n31-k5	672	676.09	0.59%	8.73	676.09	0.59%	10.13
7	B-n43-k6	742	748.47	0.81%	27.65	746.98	0.67%	25.18
8	B-n67-k10	1033	1066.57	3.29%	50.72	1041.11	0.77%	72.7
9	B-n78-k10	1221	1254.32	2.70%	53.48	1227.9	1.39%	87.63
10	E-n33-k4	835	837.67	0.36%	18.61	837.67	0.36%	11.15
11	E-n51-k5	521	524.93	0.77%	31.96	524.61	0.77%	32.45
12	E-n76-k10	832	866.31	4.09%	74.85	835.26	0.36%	90.25
13	F-n72-k4	238	242.44	1.68%	13.78	241.97	1.68%	53.27
14	F-n135-k7	1162	1208.56	3.78%	50.35	1163.08	0%	222.82
15	M-n101-k10	820	838.54	2.32%	45.51	819.56	0%	135.82
16	M-n121-k7	1034	1045.21	1.06%	194.03	1043.55	0.97%	169.74
17	P-n19-k2	212	212.66	0.47%	5.36	212.66	0.47%	8.31
18	P-n55-k7	570	573.16	0.53%	33.08	570.26	0%	36.57
19	P-n76-k4	598	613.41	2.51%	33.65	601.45	0.50%	61.08
20	P-n101-k4	691	703.28	1.74%	42.43	693.94	0.43%	105.45
	Average	828.6	842.0	1.48%	42.69	831.95	0.45%	66.51

From the results in Table 1, it can be observed that the achieved best results of the proposed HSAA on the 20 benchmarks are all not larger than existing SAA. In terms of average best results, the SAA achieves 842.0, which is 1.48% larger than 829.6, the average best objective of HSAA is 831.95, and performance gap is 0.45%. Apparently, the proposed HSAA outperforms SAA. However, it can be seen that execution time of HSAA (average: 66.51s) is larger than SAA (average 42.69s), which is resulted by the larger complexity of tabu list guided search process. However, if the performance on producing better objective is improved, the sacrifice on such a time increasing is acceptable.

### 5.2 Performance Comparison of CVRP and LVGVRP

The conventional CVRP aims at minimizing the travelling distance ignores the impact of energy consumption rate that varies with the difference of vehicle loads, thus the obtained solution is not optimal in a LVGCVRP scenario. This subsection provides comparison results of CVRP and LVGCVRP on achieved minimal energy consumptions with different ratio parameters  $\beta$ . Recall that  $\beta$  is the ratio of the maximum vehicle load capacity to vehicle weight.

The detailed results are shown in Table 2. The scenario of  $\beta=0$  means that the maximum vehicle load capacity has no effect on the energy consumption rate. The results of  $\beta=0$  are also provided to give a comparison on how parameter affects the obtained results. In this test, we test the results with three different  $\beta$  values, i.e.  $\beta=0.5$ ,  $\beta=0.8$  and  $\beta=1$ . From the results in the above table, it can be seen that LVGVRP always achieves lower energy consumptions compared with conventional CVRP, which means that LVGVRP is more advanced in reducing energy cost. More specifically, when  $\beta=0.5$ , the average consumptions are CVRP: 1031.8 and LVGVRP 1019.0. When  $\beta=0.8$ , the average consumptions are CVRP: 1049.4 and LVGVRP 1039.1. When  $\beta=1$ , the average consumptions are CVRP: 1227.8 and LVGVRP 1199.6. Apparently, we can see that with the increasing of parameter  $\beta$ , the energy cost of both CVRP and LVGVRP also increase. However, average energy cost of LVGVRP is lower than CVRP.

**Table 2.** Comparison of CVRP and LVGCVRP on 20 benchmark datasets

No.	Instances	CVRP				LVGCVRP		
		$\beta=0$	$\beta=0.5$	$\beta=0.8$	$\beta=1$	$\beta=0.5$	$\beta=0.8$	$\beta=1$
1	A-n32-k5	787.08	979.37	1094.7	1171.7	946.4	1030.7	1090.1
2	A-n33-k5	662.26	833.7	936.4	1004.9	800.1	871.3	923.3
3	A-n53-k7	1012.25	1250.6	1389.6	1482.3	1243.4	1376.6	1476.7
4	A-n69-k9	1165.9	1450.3	1621.0	1734.7	1427.2	1588.3	1676.2
5	A-n80-k10	1774.9	2223.3	2485.2	2659.8	2197.4	2473.6	2620.4
6	B-n31-k5	676.09	808.7	888.2	941.2	804.2	881.1	932.3
7	B-n43-k6	746.98	901.0	993.5	1055.1	896.6	984.8	1043.5
8	B-n67-k10	1041.11	1279.0	1418.0	1510.6	1263.8	1401.2	1485.4
9	B-n78-k10	1227.9	1542.5	1718.5	1835.7	1536.7	1896.3	1818.9
10	E-n33-k4	837.67	1025.6	1138.3	1213.5	1016.3	1125.3	1191.8
11	E-n51-k5	524.61	648.6	723.0	772.6	644.5	719.4	769.1
12	E-n76-k10	835.26	1049.0	1174.0	1257.3	1045.3	1165.9	1235.4
13	F-n72-k4	241.97	307.58	346.9	373.18	290.9	329.7	335.9
14	F-n135-k7	1163.08	1445.3	1614.7	1727.6	1436.7	1598.2	1698.9
15	M-n101-k10	819.56	1016.5	1134.6	1213.4	1000.2	1105.1	1178.5
16	M-n121-k7	1043.55	1300.0	1449.5	1549.1	1287.2	1431.7	1528.9
17	P-n19-k2	212.66	267.0	299.6	321.4	261.1	290.1	309.5
18	P-n55-k7	570.26	707.1	784.8	836.7	693.8	759.4	801.1
19	P-n76-k4	601.45	752.2	839.4	897.5	745.6	825.5	891.3
20	P-n101-k4	693.94	847.7	937.5	997.3	843.1	928.5	985.3
	Average	831.9	1031.8	1149.4	1227.8	1019.0	1139.1	1199.6

### 5.3 Performance Comparison of CVRPSD and LVGVRPSD

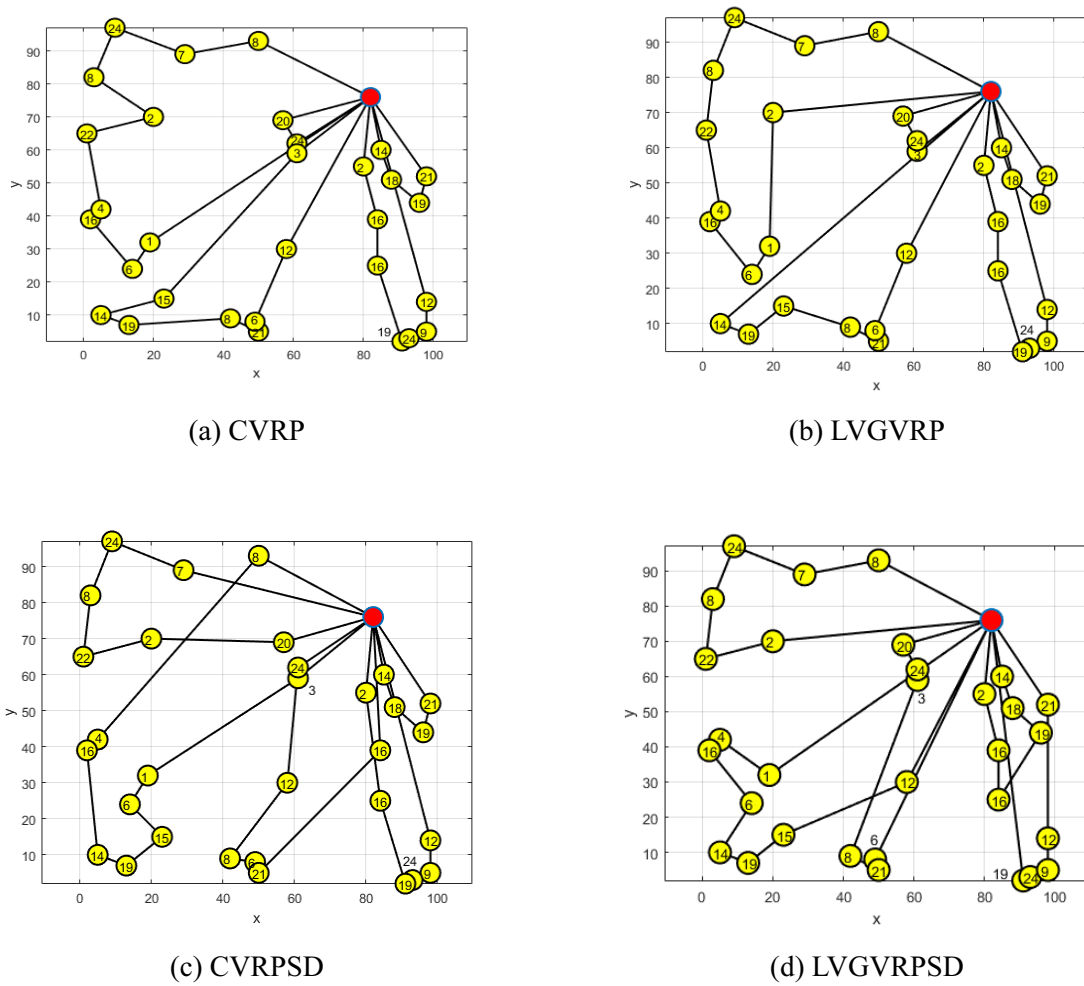
To satisfy the VRP problems with stochastic demands, the risk probability constraint requires that each vehicle to be un-full loaded to avoid overloaded risk. In this way, more vehicles may be needed since the allowed capacity is smaller compared with its real capacity. In this subsection, we test the difference of CVRPSD and LVGVRPSD. In all the tests, the ratio  $\beta$  is fixed as 1, and the variance of each customer is set as 0.1 times of its need. In this way, a customer with larger demands will be assigned with a larger variance. We also show the results of two risk probabilities, one is  $P_{rh}=0.2$ , and another one is  $P_{rh}=0.1$ .

The comparison results are shown in Tab. 3. Firstly we compare the results with Tab. 2, and the observation is that when considering stochastic customer demands by using risk probability model, the total energy consumption will be increased. The reason is that the allowed maximum workload of each vehicle is reduced, and we need more vehicles to pick up the same amount of customer demands. Next we compare the results of CVRPSD and LVGVRP, the conclusion is that LVGVRP always produce lower energy consumption, which is similar to the results in Tables 1 and 2. At last, it can be seen that, when the risk probability become lower, the energy consumption will be higher, which is within to the expectations. As a conclusion, it can be seen the proposed LVGVRPSD model is more capable to achieve vehicle routes with lower energy cost. In addition, it can be seen that the proposed HSAA outperforms conventional SAA in achieving better optimization results.

**Table 3.** Comparison of CVRPSD and LVGCVRPSD on 20 benchmark datasets, parameter  $\beta$  is set as 1, and variance  $\sigma_i^2 = 0.1c_i$

No.	Instances	CVRPSD				LVGVRPSD			
		vehicle number	$P_{rh} = 0.2$	vehicle number	$P_{rh} = 0.1$	vehicle number	$P_{rh} = 0.2$	vehicle number	$P_{rh} = 0.1$
1	A-n32-k5	5	1195.3	5	1265.5	5	1184.2	5	1226.7
2	A-n33-k5	5	1054.7	6	1036.1	5	1030.7	6	985.8
3	A-n53-k7	8	1609.9	8	1630.0	8	1568.8	8	1610.7
4	A-n69-k9	10	2158.6	10	2220.8	10	2145.5	10	2188.1
5	A-n80-k10	11	2895.2	11	3089.3	11	2861.8	11	3050.3
6	B-n31-k5	5	985.3	5	1015.3	5	970.4	5	997.7
7	B-n43-k6	6	1153.2	7	1183.4	6	1121.8	7	1162.7
8	B-n67-k10	11	1601.7	11	1702.2	11	1584.9	11	1680.2
9	B-n78-k10	11	1964.8	11	2043.5	11	1919.7	11	2014.1
10	E-n33-k4	5	1296.5	5	1377.9	5	1282.8	5	1348.4
11	E-n51-k5	6	795.4	6	811.3	6	777.1	6	798.9
12	E-n76-k10	11	1311.6	12	1345.1	11	1294.6	12	1310.1
13	F-n72-k4	5	375.6	5	398.4	5	357.3	5	383.5
14	F-n135-k7	8	1925.6	8	1976.3	8	1886.3	8	1940.6
15	M-n101-k10	11	1368.4	11	1432.8	11	1354.9	11	1403.1
16	M-n121-k7	8	1757.3	8	1829.6	8	1747.9	8	1820.2
17	P-n19-k2	3	330.7	3	335.6	3	321.6	3	325.2
18	P-n55-k7	7	886.9	8	872.6	7	873.8	8	862.8
19	P-n76-k4	5	911.1	5	933.4	5	886.7	5	890.7
20	P-n101-k4	5	1071.1	5	1113.3	5	1053.5	5	1090.9
	Average		1332.4		1380.6		1311.2		1354.5

To further illustrate the impact of load varying vehicles in vehicle routing, we use the “A-32-k5” benchmark to give an example to show the differences, as shown in Fig. 3. Subfigures (a) and (b) show the different routes obtained by using CVRP and LVGVRP. The exact demand of each customer is displayed in each node. In (a), we can be the route in the upper left region is:  $0 \rightarrow 1 \rightarrow 6 \rightarrow 16 \rightarrow 4 \rightarrow 22 \rightarrow 2 \rightarrow 5 \rightarrow 24 \rightarrow 7 \rightarrow 8 \rightarrow 0$ , while in (b), the corresponding route is:  $0 \rightarrow 2 \rightarrow 1 \rightarrow 6 \rightarrow 16 \rightarrow 4 \rightarrow 22 \rightarrow 5 \rightarrow 24 \rightarrow 7 \rightarrow 8 \rightarrow 0$ . The difference is that, in (b), the customer with demand 2 is firstly visited, while it is visited after the customer with demand 22. In this way, the long travelling distance with large load will be avoided, and the energy consumption is reduced.



**Fig. 3.** Example of obtained routes by using different models

Another comparison can be seen in subfigures (c) and (d), which plot the results of CVRPSD and LVGVRPSD, respectively. It can be seen that in (c), the customer with location (50,93) and demand 8, denoted as customer A, is visited before the customer with location (4,22) and demand 4, denoted as customer B. In (d), customer A is assigned to be visited after the node with location (29,89) and demand 7, but not before B. By this way, energy cost will be lower since long distance travelling with the demand of customer A is avoided. In a word, the LVGVRP model seeks to find an optimal solution that minimizing energy cost of load varying vehicles, but not the travelling distance in existing models.

## 6 Conclusion

In this paper, we propose a LVGVRP model to formulate the vehicle routing problem by minimizing energy consumption of load varying vehicles. The scenarios of customers with stochastic demands are also considered, and the probability risk is developed to constrain the vehicle loads. We also present a hybrid tabu-search improved simulated annealing algorithm (HSAA) to solve the proposed model, in which k-means clustering, local search, and tabu-list guided searching are used to improve the results of the SAA. Experimental on 20 commonly used benchmarks are conducted, and the results prove that LVGVRP can produce results that consumes less energies compared with conventional VRP models. In addition, the results also prove that the proposed HSAA can produce better results compared with existing SAA method.

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