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Distributed Implementation of Iterative Kalman Filter Localization with Taylor Expansion for Wireless Sensor Networks

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Abstract. The article integrates the theoretical study, simulation validation and performance analysis to make a deep research on distributed implementation of iterative localization technology in wireless sensor networks. Firstly, we utilize Kalman filter method based on square-root cubature to estimate and correct the node's position in real time. We establish a distributed iterative localization algorithm where the nodes that get localized in the current generation serve as references for remaining nodes to localize and the localization process is repeated. In order to improve the deficiency on location accuracy generated from propagating localization errors, and then we formulate the error feedback control method with Taylor series expansion as condition of evaluating whether a node succeed in localization, which is applied to iterative localization to establish a Taylor feedback Kalman filter localization algorithm. The simulation validation shows that the location accuracy of this kind of algorithm can fully meet the location requirement of the wireless sensor networks. Compared with the traditional method, the results illustrate the performance advantages of the error feedback control method and its contribution to the accuracy of the node position estimation.

Keywords: distributed implementation, iterative localization algorithm, Kalman filter, Taylor feedback, wireless sensor networks

1 Introduction

Wireless Sensor Networks (WSNs) are self-organizing networks within which a large number of nodes that can sense their environment cooperatively are arbitrarily deployed. Recent advancements in the wireless communications and hardware technology field have facilitated the development of WSNs for a wide variety of real-world applications, including health care, environmental monitoring, object tracking, and so on [1]. In order to successfully complete the monitoring task, the randomly scattered sensor nodes need to have the ability to provide their own location information timely and accurately.

Existing localization systems basically consists of three distinct components [2-3]: The first component known as ranging, distance measurement may be completed by using Received Signal Strength Indication (RSSI), Time of Arrival/Time Difference of Arrival (TOA/TDOA). Second, position estimation of the unknown nodes is carried out using the ranging information and positions of reference nodes, which is done by solving a set of simultaneous equations [4]. Such methods include trilateration, multilateration and triangulation. The third and most important one is localization algorithm, which determines how the available information will be manipulated in order to allow most or all of the nodes of a WSN to estimate their positions.

For range-free localization algorithms omitting ranging, such as centroid localization and DV-Hop localization [5-6], have already caused much attention due to their simplicity and robustness to changes in wireless transmission. However, these advantages make them only suitable for the coarse localization of an unknown node. Conversely, range-based localization has higher location accuracy.

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When the case of location estimation of a single node is considered. The single node localization problem can be transformed into the state vector estimation problem, where filtering technique is performed by using the parameters of the node location as the state vector [7], and the real-time correction is made with the increase of the number of sampling points. In [8], it formulates a dynamical system that encodes both the target moving manners and coarse sensor locations in an augmented state by integrating augmented transition and observation models. The paper [9] reviews simultaneous localization and mapping problem based on different filtering techniques used to do the state estimation of the mobile robot.

For the first component and the second component, the derived Kalman filter [10] algorithm is used to estimate and correct the node's position according to RSSI without measuring distance directly. When the system state model and observation model is linear, Kalman filter is optimal and computationally efficient due to its recursive nature. Here we will adopt the Cubature Kalman Filter (CKF) [11] to address the nonlinearity issue aroused in the considered problem. In order to improve the performance of CKF, the square root of the error covariance matrix is added in its filtering process. Thus, the Square-root Cubature Kalman Filter (SCKF) algorithm is formed [12-13].

For the third component, in distributed localization all the relevant computations are done on the sensor nodes themselves and the nodes communicate with each other to get their positions in a network [14-15]. Distributed localization has the advantage of reduced number of transmissions to the base station, which helps the system consume lower energy, run faster and use fewer resources than centralized localizations. In [16], the authors describe a distributed iterative localization algorithm where the nodes that get localized in the current generation act as references for remaining nodes to localize and the result has performed fairly well. Motivated by this, the approach is applied in our design.

However, nodes localization only needs the range information between the nodes and at least three references in a two-dimensional plane. The number of references is less because of the deployment costs, so iterative localization is designed to locate more nodes, the errors of which would be enlarged because of error propagation and accumulation [17-18]. In [19], an error control method for non-Gaussian noisy measurements which clarified that the located errors can be formulated by location or range errors is proposed. It introduces an error estimation method based on non-linear least square residuals and a robust formulation of error control to reduce accumulation error. Therefore, we aim to provide effective efforts to decrease the error of iterative localization in follow-on work.

Taylor series expansion [20] is an iterative algorithm and requires an initial position. It is effective for linearization process for its characteristics of high location accuracy. The location precision of general localization algorithm is not high, because they use the time delay difference measurements regardless of it is good or bad, using the weighted method only partly eliminate the influence of gross time delay error on the location precision. The Taylor series expansion algorithm based on the gross error gray judgment put forward in [21] eliminates the gross time delay error, which improves the convergence speed of iterative search and reduces the calculation amount.

In this article, we will transform the single node localization problem in WSNs into the state vector estimation problem of nonlinear system, where SCKF is performed to estimate and correct the node's position in real time. Then we propose a distributed iterative Kalman filter localization algorithm called IKF. Considering the disadvantage of propagating localization errors, as well as the inaccurate position estimation of one node can be used by other nodes to estimate their positions, we formulate the error feedback control technique with Taylor series expansion as condition of evaluating whether a node succeed in localization, which is applied to IKF algorithm.

The key technical problems that need to be solved are how to improve the deficiency on location accuracy generated from propagating localization errors, and how to simulate the each localizable node's distance from each of its neighboring nodes when the actual location of the unknown node is not known.

The scientific contributions of this work are as follows: (1) Ranging is included in the process of filter by estimating the node's position and the channel attenuation parameter simultaneously, which weaken effect of noises; (2) The distributed implementation and low memory requirement of iterative localization technology make it suitable for highly resource constrained WSN environments; (3) Error feedback control method is formulated for reducing propagating localization errors caused by iteration.

The article is organized as follows. In the next section, related work has been presented. Section 3 presents the formulation of distributed iterative localization algorithm. Numerical simulation and results analysis is illustrated in section 4. In Section 5, conclusions are given.

2 Related Work

2.1 Wireless Channel Propagation Model

In range-based schemes, RSSI technique has been widely used for distance measurement as it is an economical and cost effective measurement technique. Theoretically, a sender node sends a signal with a determined strength that fades as the signal propagates, and a known wireless propagation model can be used to convert the signal strength received by various known nodes into distance. In order to establish the wireless propagation channel, we assume that the propagation model obeys the log-normal shadow fading. This propagation model at a given transmission distance can be described as:

$$P_r(d)[dBm] = P_r(d_0) - 10\eta \lg\left(\frac{d}{d_0}\right) + v_\alpha , \tag{1}$$

where d is the transmission distance, $P_r(d_0)$ is the received signal power at reference distance d_0 ($d_0 = 1m$), $P_r(d)$ is the received signal power from the sender. η is the channel attenuation parameter, and v_α is a zero-mean Gaussian noise with standard deviation α .

The relationship between RSSI values and the received signal power from the receiving node is expressed by:

$$P_r(d)[dBm] = RSSI(d) + OFFSET , \tag{2}$$

where $OFFSET$ is a constant value. $RSSI(d)$ is the received signal strength from the sender. It follows from (1) and (2) that

$$RSSI(d)[dBm] = RSSI(1) - 10\eta \lg(d) + v_\alpha . \tag{3}$$

The RSSI can be estimated when the unknown nodes receive the RF signal from the beacon nodes, and the distance can be calculated via (3).

2.2 State-Space Model

It is assumed that wireless communication can be formed between two nodes if and only if they are within their communication range. In order to simplify the real application, we assume that all nodes have the same communication range. The relationship about the distance d_i and RSSI value between the unknown node and the i^{th} beacon node can be written as

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} , \tag{4}$$

$$RSSI_i(\gamma)[dBm] = RSSI_{d_0} - 10\eta \lg(d_i) + v_i, 1 \le i \le n , \tag{5}$$

where $\gamma = \left[x, y, \eta, RSSI_{d_0}\right]^T$ is the state vector to be estimated, $RSSI_i(\gamma)$ received by the unknown node is the signal strength from the i^{th} beacon node, $RSSI_{d_0}$ is the received signal strength at reference distance d_0, v_i denotes the value obtained from the noisy range measurements corresponding to the i^{th} beacon node.

Noise-free received signal strength $H_i(\gamma)$ derived from Eq. (4) and (5) is given as

$$H_i(\gamma) = RSSI_{d_0} - 10\eta \lg\sqrt{(x_i - x)^2 + (y_i - y)^2}, 1 \le i \le n . \tag{6}$$

Eq. (5) is represented as a vector form: $RSSI = H(\gamma) + v$, where $RSSI = [RSSI_1, RSSI_2, \cdots RSSI_n]^T$, $H(\gamma) = [H_1(\gamma), H_2(\gamma), \cdots H_n(\gamma)]^T$, $v = [v_1, v_2, \cdots v_n]^T$.

Let the state vector of blind node at time k^{th} be $\gamma_k = \left[x_k, y_k, \eta_k, RSSI_{d_0,k}\right]^T$, we can obtain a nonlinear system of the localization problem with additive noise, whose state-space model is defined by the pair of difference equations in discrete time:

$$\text{Process equation: } \gamma_k = A\gamma_{k-1} + w_{k-1}, \quad \textbf{(7)}$$

$$\text{Measurement equation: } \mathrm{RSSI}_k = H(\gamma)_k + v_k, \quad \textbf{(8)}$$

where $A_{4\times4}$ is the unit state transition matrix; $\{w_{k-1}\}$ and $\{v_k\}$ are independent process and measurement gaussian noise sequences with zero means and covariance matrices $Q_{k-1} = \mathrm{cov}(w_{k-1}) = diag\left(\sigma_x^2, \sigma_y^2, \sigma_\eta^2, \sigma_{RSSI_{d_0}}^2\right)$ and $R_k = \mathrm{cov}(v_k) = diag\left(\sigma_{v,1}^2, \sigma_{v,2}^2, \cdots, \sigma_{v,n}^2\right)$, respectively.

2.3 RSSI State Estimation Based on SCKF

For the state-space model depicted in Eq. (7) and (8), it is ready to apply SCKF which consists of two consequent stages at step $k(k = 1,2,\cdots N)$.

Time update

(1) Evaluate the cubature points $(i = 1,2,\cdots m)$: $\Gamma_{i,k-1|k-1} = S_{k-1|k-1}\varepsilon_i + \hat{\gamma}_{k-1|k-1}$, where $m = 2n_\gamma$.

(2) Evaluate the propagated cubature points $(i = 1,2,\cdots m)$: $\Gamma^*_{i,k|k-1} = A\Gamma_{i,k-1|k-1}$.

(3) Estimate the predicted state: $\hat{\gamma}_{k|k-1} = \sum_{i=1}^{m}\frac{1}{m}\Gamma^*_{i,k|k-1}$.

(3) Estimate the square-root factor of the predicted error covariance: $S_{k|k-1} = tria\left(\gamma^*_{k|k-1} \quad S_{Q,\mathrm{k}-1}\right)$, where $S_{Q,\mathrm{k}-1}$ denotes a square-root factor of Q_{k-1} such that $Q_{k-1} = S_{Q,\mathrm{k}-1}S^T_{Q,\mathrm{k}-1}$ and the weighted, centered matrix $\gamma^*_{k|k-1} = \frac{1}{\sqrt{m}}\left[\Gamma^*_{1,k|k-1} - \hat{\gamma}_{k|k-1}, \cdots, \Gamma^*_{m,k|k-1} - \hat{\gamma}_{k|k-1}\right]$.

Measurement update

(1) Evaluate the cubature points $(i = 1,2,\cdots m)$: $\Gamma_{i,k|k-1} = S_{k|k-1}\varepsilon_i + \hat{\gamma}_{k|k-1}$.

(2) Evaluate the propagated cubature points $(i = 1,2,\cdots m)$: $Z_{i,k|k-1} = H\left(\Gamma_{i,k|k-1}\right)$.

(3) Estimate the predicted measurement: $\hat{z}_{k|k-1} = \sum_{i=1}^{m}\frac{1}{m}Z_{i,k|k-1}$.

(4) Estimate the square-root of the innovation covariance matrix:
$S_{zz,k|k-1} = tria\left(P^*_{k|k-1} \quad S_{R,\mathrm{k}}\right)$, where $S_{R,\mathrm{k}}$ denotes a square-root factor of R_k such that $R_k = S_{R,\mathrm{k}}S^T_{R,\mathrm{k}}$ and the weighted, centered matrix $P^*_{k|k-1} = \frac{1}{\sqrt{m}}\left[Z_{1,k|k-1} - \hat{z}_{k|k-1}, \cdots, Z_{m,k|k-1} - \hat{z}_{k|k-1}\right]$.

(5) Estimate the cross-covariance matrix: $M_{xz,k|k-1} = A_{k|k-1}{P^*_{k|k-1}}^T$, where the weighted, centered matrix $A_{k|k-1} = \frac{1}{\sqrt{m}}\left[\Gamma_{1,k|k-1} - \hat{\gamma}_{k|k-1}, \cdots, \Gamma_{m,k|k-1} - \hat{\gamma}_{k|k-1}\right]$.

(6) Estimate the Kalman gain: $W_k = \left(M_{xz,k|k-1} / {S_{zz,k|k-1}}^T\right) / S_{zz,k|k-1}$.

(7) Estimate the updated state: $\hat{\gamma}_{k|k} = \hat{\gamma}_{k|k-1} + W_k\left(\mathrm{RSSI}_k - \hat{z}_{k|k-1}\right)$.

(8) Estimate the square-root factor of the corresponding error covariance $S_{k|k} = tria\left(A_{k|k-1} - W_kP^*_{k|k-1} \quad W_kS_{R,\mathrm{k}}\right)$

3 Distributed Iterative Localization Algorithm

In this section, we firstly utilize the RSSI parameter estimation algorithm based on SCKF to estimate and correct the node's position in real time. Then we establish a distributed iterative kalman filter localization algorithm called IKF where the nodes that get localized in the current generation serve as references for remaining nodes to localize. Considering the disadvantage of propagating localization errors, we formulate the error feedback control technique with Taylor series expansion as condition of evaluating whether a node succeed in localization, which is applied to IKF algorithm motivated by different error control techniques.

3.1 Iterative Kalman Filter Localization Algorithm

3.1.1 Performance Evaluation of SCKF

In the case of location estimation of a single node, we set $\hat{\gamma}_{0|0}=\left[0.2,2,3,-20\right]^T$, $S_{0|0}=diag\left(0,0,0,0\right)$, the actual location of the unknown node is (1,4). RSSI values collected at four known nodes are simulated as Eq. (5) where $d_i=\sqrt{\left(x_i-1\right)^2+\left(y_i-4\right)^2}$, v_i is 0.8 times of random noise, $i=1,2,3,4$.

The experimental results shown in Fig. 1 to Fig. 2 demonstrate that the RSSI state estimation algorithm based on SCKF is available, feasible and efficient in the wireless localization.

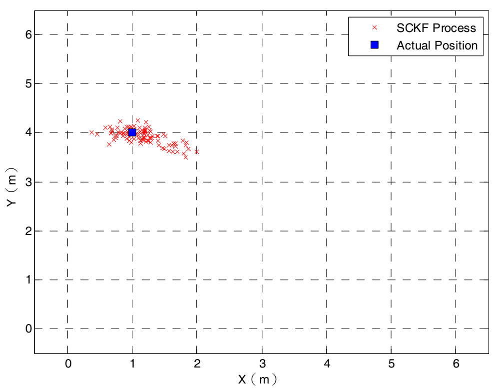


Fig. 1. Location estimation of a single node

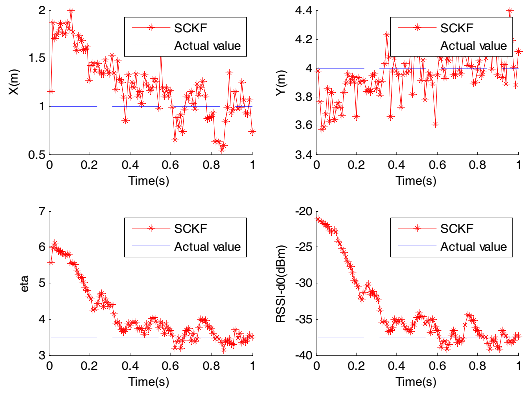


Fig. 2. Comparison of estimated state and actual value

From the Fig. 2, we can find that estimated parameter tends to be the true value with the increase in the number of samples. Therefore estimated coordinate through the last sampling is considered to be the estimated position determined by SCKF.

3.1.2 Iterative Kalman Filter Localization

The distributed implementation proposed here is the realization of the overall localization of large-scale nodes in the basis of the successive iteration of a single node. First of all, the distributed Iterative Kalman Filter (IKF) localization algorithm without error feedback control is designed as follows:

Algorithm 1. [Iterative Kalman Filter Localization, IKF]:

(1) n unknown nodes and m beacon nodes are randomly deployed in a 2-dimensional plane. After each iteration, the nodes that get settled can be used as references for the next step. They transmit their location information as the beacon nodes do.

(2) Each node that falls within transmission radii of 3 or more non-collinear references (beacon nodes or settled nodes) is referred to as a localizable node [16]. Calculate the number of localizable nodes lk.

(3) Let the related data of each node be sampled 100 times according to the time interval of 0.01s. RSSI parameter estimation algorithm based on SCKF is then applied to estimate and correct the target's

position $(\hat{x},\hat{y})$ and the channel attenuation parameter simultaneously.

(4) After all the lk localizable nodes determine their positions, the localization error is computed as (9).

$$RelError_k = \frac{\sum_{l=1}^{lk}\sqrt{(x_l-\hat{x}_l)^2+(y_l-\hat{y}_l)^2}}{lk*r}. \tag{9}$$

where (x_l,y_l) and $(\hat{x}_l,\hat{y}_l)$ are the actual and estimated locations of localizable node l, $l=1,2,\cdots,lk$.

(5) Steps 2 to 4 are repeated until either all unknown nodes get localized or no more nodes can be localized.

As the iterations progress, the number of settled nodes have been increasing, which increases the number of reference nodes, reduces the possibility of flipping ambiguity, but also extends the time required for localization. In this paper, we restrict the number of reference nodes to a maximum value of 6, which is chosen by the 6 nearest distances of references apart from localizable nodes.

3.2 Error Feedback Control Based on Taylor Expansion

In order to overcome the weakness of iterative localization, we construct the following method.

Assuming that (x,y) is the actual location of the unknown node and $(\hat{x},\hat{y})$ represents the estimated position determined by SCKF, the distance d_i between the unknown node and beacon node i can be written as

$$d_i=\sqrt{(x_i-x)^2+(y_i-y)^2}=f(x,y),i=1,2,\cdots n, \tag{10}$$

Similarly, the estimated distance $\hat{d}_i$ can be written as

$$\hat{d}_i=\sqrt{(x_i-\hat{x})^2+(y_i-\hat{y})^2}=f(\hat{x},\hat{y}),i=1,2,\cdots n, \tag{11}$$

Denote the location error of (x,y) and $(\hat{x},\hat{y})$ is $(\Delta x,\Delta y)$, so the actual location can be written as $x=\hat{x}+\Delta x, y=\hat{y}+\Delta y$. Thus,

$$f(x,y)=f(\hat{x}+\Delta x,\hat{y}+\Delta y), \tag{12}$$

Expanding Eq. (13) with Taylor formula at $(\hat{x},\hat{y})$, then we obtain

$$\begin{aligned} f(\hat{x}+\Delta x,\hat{y}+\Delta y) &= f(\hat{x},\hat{y})+\frac{\partial f(\hat{x},\hat{y})}{\partial\hat{x}}\Delta x+\frac{\partial f(\hat{x},\hat{y})}{\partial\hat{y}}\Delta y \\ &= f(\hat{x},\hat{y})-\frac{x_i-\hat{x}}{\hat{r}_i}\Delta x-\frac{y_i-\hat{y}}{\hat{r}_i}\Delta y \end{aligned}, \tag{13}$$

where $\hat{r}_i=\sqrt{(x_i-\hat{x})^2+(y_i-\hat{y})^2}$.

So d_i can be obtained from Eq. (12) and (13), and is written as

$$d_i=\hat{d}_i-\frac{x_i-\hat{x}}{\hat{r}_i}\Delta x-\frac{y_i-\hat{y}}{\hat{r}_i}\Delta y, \tag{14}$$

Let $\Delta d_i = \hat{d}_i - d_i$, $a_{x_i} = \frac{x_i - \hat{x}}{\hat{r}_i}, a_{y_i} = \frac{y_i - \hat{y}}{\hat{r}_i}$. Eq. (14) can be simplified as

$$\Delta d_i = a_{x_i}\Delta x + a_{y_i}\Delta y, i = 1, 2, \cdots n, \tag{15}$$

Let $H = \begin{bmatrix} a_{x_1} & a_{y_1} \\ a_{x_2} & a_{y_2} \\ \vdots & \vdots \\ a_{x_n} & a_{y_n} \end{bmatrix}, \Delta d = \begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \vdots \\ \Delta d_n \end{bmatrix}$, $\Delta\rho = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$, we obtain

$$H\Delta\rho = \Delta d, \tag{16}$$

The position of the location error can be obtained by using the least-squares algorithm:

$$\Delta\rho = \left(H^T H\right)^{-1} H^T \Delta d, \tag{17}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} \dfrac{\sum_{i=1}^{n} a_{x_i}\Delta d_i \sum_{i=1}^{n} a_{y_i}^2 - \sum_{i=1}^{n} a_{y_i}\Delta d_i \sum_{i=1}^{n} a_{x_i} a_{y_i}}{\sum_{i=1}^{n} a_{x_i}^2 \sum_{i=1}^{n} a_{y_i}^2 - \left(\sum_{i=1}^{n} a_{x_i} a_{y_i}\right)^2} \\ \dfrac{\sum_{i=1}^{n} a_{y_i}\Delta d_i \sum_{i=1}^{n} a_{x_i}^2 - \sum_{i=1}^{n} a_{x_i}\Delta d_i \sum_{i=1}^{n} a_{x_i} a_{y_i}}{\sum_{i=1}^{n} a_{x_i}^2 \sum_{i=1}^{n} a_{y_i}^2 - \left(\sum_{i=1}^{n} a_{x_i} a_{y_i}\right)^2} \end{bmatrix}. \tag{18}$$

3.3 Taylor Feedback Kalman Filter Localization Algorithm

Below, we summarize the iterative SCKF based on Taylor expansion localization algorithm writing the steps explicitly when they differ from the IKF algorithm. Taylor Feedback Kalman Filter (TFKF) algorithm is as follows:

Algorithm 2. [Taylor Feedback Kalman Filter Localization, TFKF]:

(1) n unknown nodes and m beacon nodes are randomly deployed in a two-dimensional sensor field.

(2) Each node that falls within transmission radii of 3 or more non-collinear references is referred to as a localizable node. Calculate the number of localizable nodes lk.

(3) Each localizable node measures its distance from each of its neighboring nodes as $\tilde{d}_i = d_i + d_i\zeta$, where d_i is the actual distance given by $d_i = \sqrt{\left(x_i - x\right)^2 + \left(y_i - y\right)^2}$ and ζ represents range error which obeys the normal distribution of the mean ranging error rate δ as the expectation and the variance is 0.01. ($\tilde{d}_i$ can be obtained by RSSI in practical application)

(4) Sensor nodes sample only 100 times a second and apply SCKF to estimate and correct the target's position $\left(\hat{x}, \hat{y}\right)$.

(5) Calculate the estimated distance $\hat{d}_i = \sqrt{\left(x_i - \hat{x}\right)^2 + \left(y_i - \hat{y}\right)^2}$ and substitute distance error $\Delta d_i = \hat{d}_i - \tilde{d}_i$ of reference node i into (19) yields $\left(\Delta x, \Delta y\right)$.

(6) After all the lk localizable nodes determine their positions, each localizable node that meets

precision requirement expressed by $\frac{1}{r}\sqrt{(\Delta x)^2+(\Delta y)^2}<\varepsilon$ is referred to as a successful node. Otherwise it fails to locate. The nodes that get settled (achieves success in location) at the current iteration serve as references. Calculate the number of successful nodes co.

(7) Then the localization error is computed as the mean of relative error of distances between actual locations (x_c, y_c) and the locations $(\hat{x}_c, \hat{y}_c)$, $c=1,2,\cdots,co$ determined by SCKF. This is computed as (19).

$$RelError_k=\frac{\sum_{c=1}^{co}\sqrt{(x_c-\hat{x}_c)^2+(y_c-\hat{y}_c)^2}}{co*r}, \quad \textbf{(19)}$$

(8) Steps 2 to 7 are repeated until either all unknown nodes get localized or no more nodes can be localized.

4 Numerical Simulation and Results Analysis

4.1 Description of the Simulation Environment

The algorithms described above are implemented in MATLAB to evaluate their performance by measuring the average localization error over all the unknown nodes and comparing the result of all algorithms. All experiments were performed at a PC with CPU of 2.40 GHz and RAM of 8.00 GB. To ease our illustration, we focus on the two-dimensional simulated sensor network which consists of unknown and beacon nodes that are well-distributed on a square area. Each sensor node has a transmission radius of r = 30m. For comparison, the relative value of root mean square error described by (19) is used to determine the distance error of the estimated position per iteration. The mean of overall $RelError$ is also calculated as (20) to determine the system performance.

$$MRelError=\frac{\sum_{k=1}^{K}(RelError_k*co)}{\sum co}, \quad \textbf{(20)}$$

where K is the number of iterations of a trial.

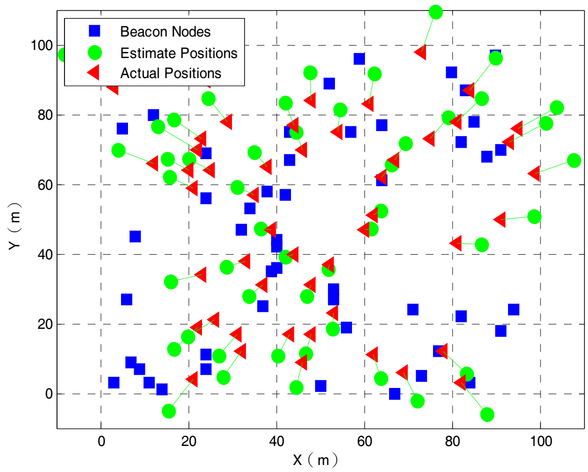
We describe an implementation of the iterative localization and present simulation results for the large-scale sensor network location problems solved by the distributed TFKF, and compare it with the distributed IKF and LS in the numerical experiments. The values in all the figures and tables denote average value over 50 runs.

4.2 Performance Evaluation

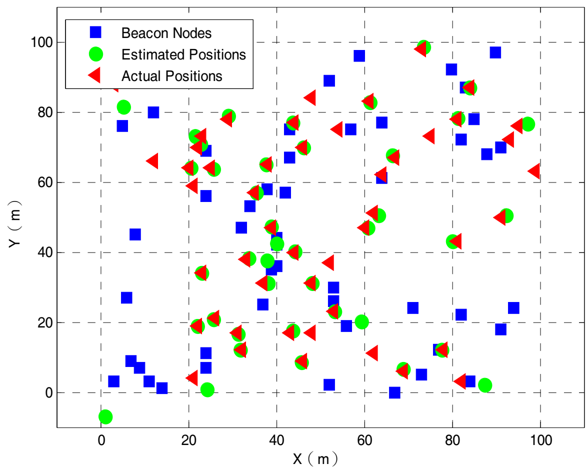
4.2.1 Cases of Fixed Number of Beacon Nodes

Let the geographical region be marked by a 100m×100m area. Now consider the case where 100 sensor nodes are randomly placed in the square area while the number of beacon nodes is fixed as 50 for the simulation. Since we know the accurate positions of all the beacons, and for each unknown node, beacons can be detected within its communication range.

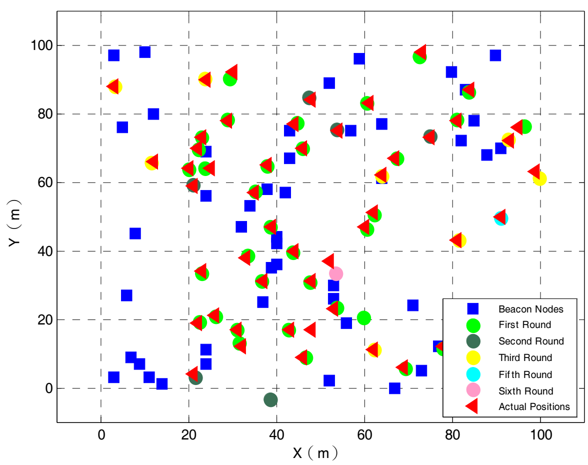
The actual positions of all nodes and the coordinates of the unknown nodes estimated by three different approaches in a trial run are shown in Fig. 3. We aim to find how the accuracy of the estimation relies on the error feedback control for each localization algorithm.



(a) Location estimated by LS



(b) Location estimated by IKF



(c) Location estimated by TFKF

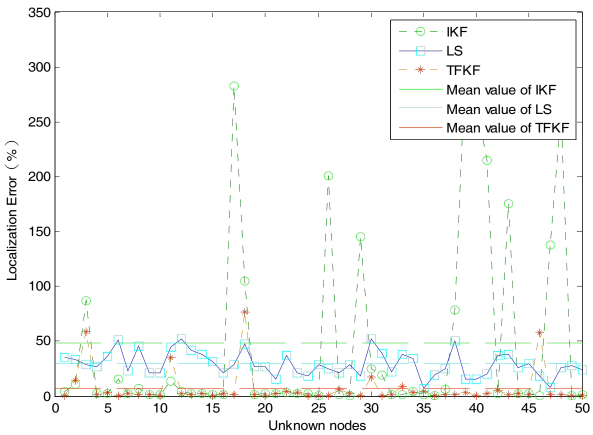
Fig. 3. Comparisons of localization results determined by LS, IKF and TFKF

As shown in Fig. 3(a) and Fig. 3(b) of Fig. 3, when error feedback control isn't introduced, the LS used here has performed better than IKF in localization, while in Fig. 3(c), we introduce error feedback control, and then the iterative localization results determined by TFKF estimate the location of the nodes almost 100% accurately. In order to show the detailed difference, we present the localization error and elapsed time of TFKF as following Table 1. It can be seen from Table 1 that the sensor nodes labeled as successful are over 96% after the third round iteration.

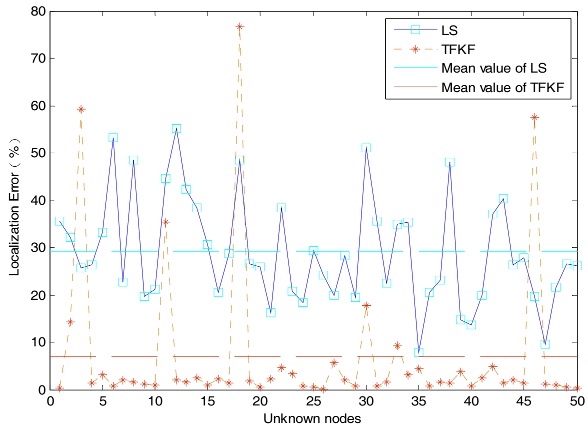
Table 1. Process of iterative localization determined by TFKF

Iteration	Localizable nodes	Successful nodes	Localization error	Elapsed time (s)
1	50	34	0.0319	3.5084
2	16	6	0.2515	4.6302
3	10	8	0.0211	5.3623
4	2	0	0	5.6279
5	2	1	0.0157	5.8967
6	1	1	1.3892	6.1091

Next, we give the deviation of each unknown node in iterative localization, which can be calculated as the distance between actual node's position and the estimated position determined by three algorithms respectively. We can know that the localization error of three algorithms have obvious distinction, TFKF has a high location performance. Results are presented in Fig. 4.



(a) Comparison of IKF, LS and TFKF



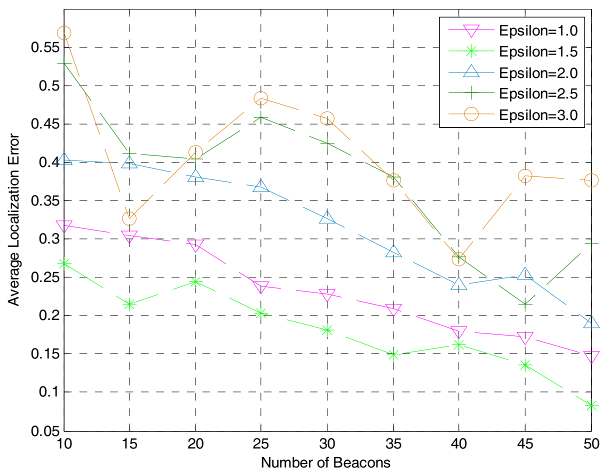
(b) Comparison of LS and TFKF

Fig. 4. Localization error at different unknown node

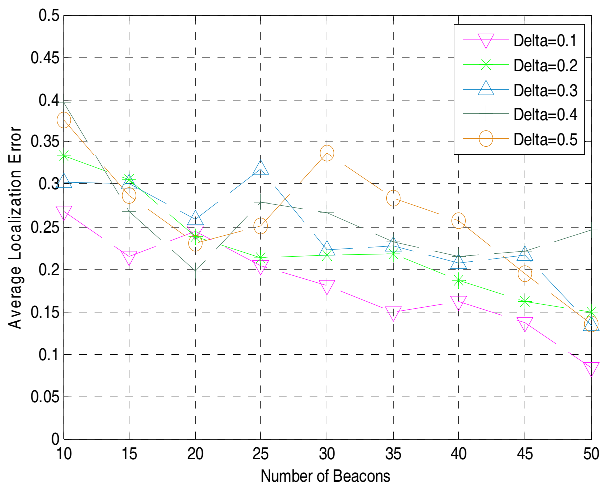
4.2.2 Cases of Changing Number of Beacon Nodes

To further show the advantages of the error feedback control method, we evaluate its performance by measuring the average localization error over all the unknown nodes and comparing the result of all algorithms when they are operating under the same conditions, with different numbers of beacon nodes when the total number of sensor nodes is fixed as 100.

Fig. 5(a) shows the variation of average localization error by increasing the number of beacons while varying the preset precision parameter ε from 1.0 to 3.0, and the mean ranging error rate δ in the distance measurements is fixed at 0.1. As can be seen from the results, for random nodes placement, the influence of the beacon node proportion on the estimated results is limited, even in very low requirement of precision. Even for 10% beacons, the average localization error can be kept below 0.35 for $\varepsilon \le 1.5$. As $\varepsilon \le 1.5$ keeps decreasing, average localization error increases, the reason for which might be that the inaccuracy of ranging plays a larger role. So that the number of successful nodes every time is small but the quantity is not accurate. Fig. 5(b) shows the variation of average localization error when the number of beacons increases with different δ, while ε is fixed at 1.5. When $\delta = 0.1$ and the beacon node proportion is greater than or equal to 30% for the test problems with 100 sensors, the average localization error can be kept below 0.2.



(a) Varying ε with fixed δ



(b) Varying δ with fixed ε

Fig. 5. Variation of average localization error for different numbers of beacons

Fig. 6 shows that TFKF outperforms all other methods in average localization error, and it is clear that the use of error feedback control contributes favorably in the improvement of the position estimation.

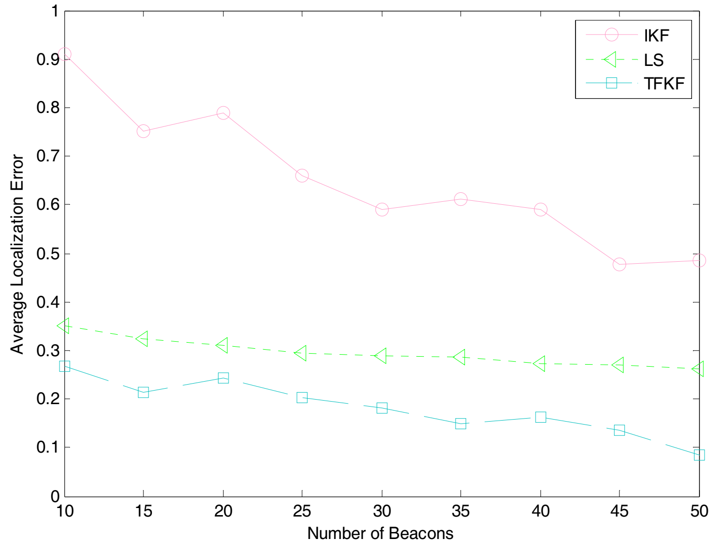


Fig. 6. Variation of average localization error for different numbers of beacons with three algorithms

We can observe a slight reduction in the entire criterions when the number of beacon nodes increases from Table 2. This is because localization with more beacons leads that the uncertainty of node location are weakened, so the undetermined nodes gradually obtain the opportunity to be identified and calibrated.

Table 2. A summary of results of 50 trial runs LS, IKF and TFKF localization with different numbers of beacons

100 nodes		Average number of iterations		Average localization error			Average elapsed time (s)		
Unkonwn	Beacon	IKF	TFKF	IKF	LS	TFKF	IKF	LS	TFKF
90	10	4	13	0.9104	0.3511	0.2672	6.3449	0.8311	13.4496
85	15	3	12	0.7525	0.3247	0.2143	5.7307	0.7564	12.8988
80	20	3	12	0.7893	0.3095	0.2444	5.5782	0.7826	13.0229
75	25	3	11	0.6602	0.2954	0.2036	5.3086	0.6980	12.8472
70	30	2	11	0.5891	0.2901	0.1806	4.8974	0.4423	11.1938
65	35	2	12	0.6109	0.2856	0.1492	4.5934	0.2612	9.2109
60	40	1	13	0.5891	0.2733	0.1621	4.1964	0.4122	10.0851
55	45	2	9	0.4784	0.2704	0.1363	4.0747	0.6315	7.9509
50	50	1	8	0.4727	0.2624	0.0857	3.6271	0.8534	9.1518

4.2.3 Comparison with Other Related Work

Let the geographical region be marked by a 100m×100m area. Now consider the case where 100 sensor nodes are randomly placed while the number of unknown nodes is fixed as 50. Then we provide a comparative table to indicate the contribution of this paper.

We compare them with the Particle Swarm Optimization (PSO) [16], Bacterial Foraging Algorithm (BFA) [16], Maximum Likelihood Estimation (MLE) [22] and Strong Adaptive SCKF [23] in the numerical experiments. It's also worth mentioning that derivative iterative Strong Adaptive SCKF is a new distributed iterative kalman filter localization mechanism, which is implemented by combining the ordinary nonlinear filtering algorithm with Kalman filter for improved estimation results. The mechanism performs Kalman filter for further processing on the basis of SCKF or EKF state vector estimation. The article about Strong Adaptive SCKF has been accepted and waiting to be published.

The values in Table 3 denote average value over 50 runs. It can be seen from Table 3 that the TFKF algorithm outperforms all other methods. As shown in Table 3, we can observe a slight reduction in the entire criterions so as to distinguish ours and others in more detail.

Table 3. A summary of results of 50 trial runs five algorithms for 50 unknown nodes

Algorithms	Average results of 50 trials			
	number of nonlocalized nodes	number of iterations	localization error	Elapsed time(s)
Particle Swarm Optimization (PSO)	0.3210	49	0.3511	288.4117
Bacterial Foraging Algorithm (BFA)	0.1956	49.4	0.2137	902.1132
Maximum Likelihood Estimation (MLE)	/	/	0.2624	0.8534
Iterative Strong Adaptive SCKF	0	3	0.1727	3.6271
Taylor Feedback Kalman Filter (TFKF)	1	8	0.0857	9.1518

5 Conclusion

In this article we have developed a new distributed iterative Kalman filter localization technique TFKF, which is implemented by controlling the error with Taylor expansion, to improve the deficiency of the distributed IKF and LS methods. A comparison of the performances in terms of number of iterations, localization error and elapsed time is presented, which shows that the improved algorithm has high location accuracy.

In summary, there are three distinguishing technical novelty of this article: (1) The error feedback control based on Taylor expansion is applied to the iterative Kalman filter localization algorithm so as to improve the deficiency on location accuracy generated from propagating localization errors; (2) we formulate the error feedback control technique as condition of evaluating whether a node succeed in localization, which is applied to IKF algorithm motivated by different error control techniques; (3) In order to simulate the each localizable node's distance from each of its neighboring nodes when the actual location of the unknown node is not known, we obtain measuring distance between single localizable node and each of its neighboring nodes by RSSI in practical application.

Although the study of the iterative Kalman filter localization technique TFKF in this paper can solve the locating problem under some specific circumstances, there are still many deficiencies that need further study: (1) When the range of the location area remains unchanged, the positioning accuracy achieved by SCKF can't be greatly improved by increasing the number of beacon nodes, this is because the beacon node can only bring more measurement information for more nodes remains to be positioned, which improves the measurement accuracy to some extent, but the effect on the positioning accuracy is limited, so the maximum number of reference nodes is limited to 6, and the nearest 6 from the target; (2) The number of references is less because of the deployment costs in the actual monitoring environment, so iterative localization is designed to locate more nodes. However, the error propagation and accumulation can't be completely eliminated only by repositioning, so the future work can be extended to the research about error control method.

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References

[1] N. Patwari, J. Ash, S. Kyperountas, A.O. Hero, R.L. Moses, N.S. Correal, Locating the nodes: cooperative localization in wireless sensor networks, IEEE Signal Processing Magazine 22(4)(2005) 54-69.

[2] A. Boukerche, H.A.B. Oliveira, E.F. Nakamura, A.A.F. Loureiro, Localization systems for wireless sensor networks, IEEE Wireless Communications 14(6)(2007) 6-12.

[3] T. Hamel, C. Samson, Position estimation from direction or range measurements, Automatica: A Journal of IFAC the International Federation of Automatic Control 82(2017) 137-144.

[4] X.-Z. Cheng, D.-R. Zhu, S. Zhang, G. Zhu, RSSI-based differential correction least-squares-quasi-newton positioning algorithm, Chinese Journal of Sensors and Actuators 27(1)(2014) 1-15.

[5] A. Kaur, P. Kumar, G. P. Gupta, A weighted centroid localization algorithm for randomly deployed wireless sensor networks. Journal of King Saud University - Computer and Information Sciences, 2017.

[6] L Gui, X Zhang, Q Ding, F. Shu, A. Wei, Reference anchor selection and global optimized solution for DV-hop localization in wireless sensor networks, Wireless Personal Communications 96(4)(2017) 5995-6005.

[7] D. Veillard, F. Mailly, P. Fraisse, EKF-based state estimation for train localization, in: Proc. 2016 IEEE SENSORS, 2016.

[8] X.Y. Jiang, B.Z. Lu, P. Ren, Augmented filtering based on information weighted consensus fusion for simultaneous localization and tracking via wireless sensor networks, International Journal of Distributed Sensor Networks 11(2)(2015) 1-7.

[9] T.S. Ho, Y.C. Fai, E.S.L. Ming, Simultaneous localization and mapping survey based on filtering techniques, in: Proc. IEEE 10th Asian Control Conference, 2015.

[10] R.E. Kalman, A new approach to linear filtering and prediction problems, Transactions of the AMSE Journal of Basic Engineering 82(1)(1960) 35-45.

[11] I. Arasaratnam, S. Haykin, Cubature kalman filters, IEEE Transactions on Automatic Control 54(6)(2009) 1254-1269.

[12] Y. Liu, J.F. Su, M.Q. Zhu, Received signal strength indicator parameter estimation algorithm based on square-root cubature kalman filter, Journal of System Simulation 26(1)(2014) 119-124.

[13] X. Song, Y. Zhang, D. Liang, Input forces estimation for nonlinear systems by applying a square-root cubature kalman filter, Materials 10(10)(2017) 1162.

[14] H. Jia, J. Zheng, G. Wang, Y. Chen, D. Huang, H. Yuan, On the localization algorithm of wireless sensor network and its application, International Journal of Online Engineering 13(3)(2017) 40-51.

[15] A. Pal, Localization algorithms in wireless sensor networks: current approaches and future challenges, Network Protocols and Algorithms 2(1)(2010) 45-73.

[16] R.V. Kulkarni, G.K. Venayagamoorthy, M.X. Cheng, Bio-inspired node localization in wireless sensor networks, in: Proc. the 2009 IEEE International Conference on Systems, Man, and Cybernetics, 2009.

[17] L. Wang, T.K. Hon, J.D. Reiss, A. Carallaro, An ierative approach to source counting and localization using two distant microphones, IEEE/ACM Transactions on Audio Speech & Language Processing 24(6)(2016) 1079-1093.

[18] X. Qu, L. Xie, W. Tan, Iterative constrained weighted least squares source lcalization using TDOA and FDOA measurements, IEEE Transactions on Signal Processing 65(15)(2017) 3990-4003.

[19] X.P. Wu, S.L. Tan, Y.Y. He, Effective error control of iterative localization for wireless sensor networks, International Journal of Electronics and Communications(AEÜ) 67(2013) 397-405.

[20] N.A.N. Ariffin, N. Rosli, Stochastic Taylor expansion of derivative-free method for stochastic differential equations, Malaysian Journal of Fundamental and Applied Sciences 13(3)(2017) 159-164.

[21] Y. Fan, Y.L. Chu, P Taylor series expansion vibratory source localization algorithm based on the gross error gray judgment, Journal of Information & Computational Science 12(2)(2015) 485-493.

[22] L.H. Zhong, C.Q. Hu, J.J. Jin, Analysis and implementation of maximum likelihood estimation positioning algorithm based on RSSI, Journal of Jilin University (Science Edition) 52(3)(2014) 556-560.

[23] Y.-J. Wang, S.-Y. Liu, Z. Ma, Z. Zhang, X.-H. Tang, Nonlinear system state estimation mechanism based on kalman filter for wireless sensor networks localization, Journal of Computers 29(3)(2018) 94-108.