Journal of Computers Vol. 30 No. 3, 2019, pp. 192-204
doi:10.3966/199115992019063003014

Efficient and Rational Multi-criteria Group Decision Making Method Based on Vague Set Theory

Kuo-Sui Lin

Department of Information Information Management, Aletheia University, Taiwan
au4234@mail.au.edu.tw

Received 5 April 2018; Revised 10 October 2018; Accepted 18 November 2018

Abstract. There is a need for an efficient and rational multi-criteria group decision making (MCGDM) method to handle situations, in which decision makers from different domains or interested parties are involved in the vague and uncertain decision making processes. Thus, the research purpose of the study was to propose a vague set based MCGDM, which is easily understandable and simple computation to group decision members for handling vague and uncertain decision making processes. The MCGDM method is considered to be an umbrella under which a collective vague value solicitation and aggregation method as well as a numerical transformation method are included. Following the algorithm described in the proposed MCGDM method, an efficient polling method can contribute to soliciting group importance vague values and group performance vague values of alternatives. Besides, the solicited importance and performance vague values were aggregated by using a weighted aggregation function. Furthermore, a new score function can contribute to transform the aggregated vague values into comparable numerical scores for further decision making. Finally, a numerical case study was also conducted to demonstrate that the proposed MCGDM method is efficient and rational to solicit and aggregate vague values as well as to transform aggregated vague values into comparable numerical scores for decision making.

Keywords: multiple-criteria group decision making, polling method, score function, vague set theory

1 Introduction

1.1 Background and Literature Review

Fuzzy sets (FSs) are used to introduce fuzziness by eliminating the abrupt and unambiguous boundaries dividing members from non-members of one specific object. In a fuzzy set, each object is assigned a single value in the interval [0, 1] reflecting its grade of membership. In recent years, fuzzy sets have been applied to a variety of multi-criteria decision making (MCDM) methods under fuzzy environment [11, 21, 28]. However, the single point-based membership value of fuzzy set tells us nothing about its accuracy. From the point of view of real-world practice, it is not sufficiently adequate to assign precisely a single membership value from the interval [0, 1] to each element x without the loss of information. As a generalization of fuzzy sets, Atanassov [12, 23] introduced the concept of the intuitionistic fuzzy sets (IFSs) in 1983 and Gau and Buehrer [26] introduced the notion of vague sets (VSs) in 1993. Bustince and Burillo [7] showed that IFSs and VSs are equivalent.

By using interval-based membership, instead of using point-based membership in a fuzzy set, a vague set (or intuitionistic fuzzy set) has more powerful ability to process vague and uncertain information for multi-criteria decision making problems than the fuzzy set has. Vague set based multi-criteria decision making problems have been originally addressed in [22]. Since then, the vague set or the intuitionistic fuzzy set has been used for building methods to handle multiple criteria decision making problems [3, 6, 10, 16-17, 24-25].

Most of these methods are constructed to support the works of individual decision making problems.

Nevertheless, individual decision making appears to be insufficient, inappropriate or unsuitable in complicated situations, in which expertise from different domains or interested parties need to be taken into consideration during decision making. That is to say that multi-criteria group decision making (MCGDM) is more impartial than individual decision making. MCGDM often involves group decision makers' judgments and preferences, including importance weightings of criteria and performance ratings of alternatives. These weightings and ratings are vague and subjective in nature. However, to the author's knowledge, a comprehensive MCGDM method which includes collective vague value solicitation, aggregation and numerical transformation method for decision making is still lacking in the literature.

1.2 Research Objectives and Contribution

Based on the above analysis, the research purpose of the study was to propose a vague set based MCGDM, which is easily understandable and simple computation to group decision members for handling vague and uncertain decision making processes. Following the algorithm described in the proposed MCGDM method, the objectives of the proposed MCGDM method will contribute to soliciting group importance vague values and group performance vague values of alternatives. Moreover, the solicited vague values will be aggregated and rationally transformed into comparable numerical scores for further decision making. More specifically, the results of this study may contribute to solving the following research questions:

(1) To propose an algorithm of the proposed MCGDM method to group decision members. This algorithm is easily understandable and simple computation under vague and uncertain environment.

(2) To solicit group importance vague values and group performance vague values of alternatives.

(3) To aggregate the group importance vague values and group performance vague values into integrated vague values of alternatives.

(4) To transform the aggregated vague values into comparable numerical scores for further decision making.

1.3 Structure of the Research

The remaining parts of this paper are structured as follows. Section 2 briefly reviews some basic concepts and definitions of MCGDM and vague set theory. In Section 3, a new polling method for vague value solicitation is introduced. In Section 4, a new score function for vague value transformation is proposed. In Section 5, a new vague set based MCGDM method is proposed for soliciting and aggregating group vague values and transforming aggregated vague values into comparable numerical scores for decision making. In Section 6, following the algorithm described in the proposed MCGDM method, a numerical case study is conducted and the results are analyzed and discussed. Finally, conclusions are drawn in Section 7.

2 Preliminaries

2.1 Multi-Criteria Group Decision Making

Multi-criteria group decision making (MCGDM) involves in evaluating a set of decision alternatives with respect to multiple, often conflicting criteria for selecting the most appropriate alternative in a given situation with multiple decision makers' judgments and preferences [23]. Consider a MCGDM problem with m alternatives A_i (i=1,...,m), n independent criteria C_j(j=1,...,n) and a committee (panel) of p decision makers D_k (k=1,...,p). Let r^k_{ij} be the performance rating of alternative A_i when it is examined by the k-th decision maker in terms of criterion C_j. The individual decision matrix (Table 1) shows the evaluation of each decision maker D_k for all the alternatives A_i (i = 1,...,m) with respect to the criteria C_j (j = 1,...,n). The multi-criteria group decision-making problem can be formulated and expressed by a set of individual decision matrix, which includes a set of individual rating matrix: $R^k_{ij} = (r^k_{ij})_{m\times n}$($k$ =1,...,p) and a weighting vector of the criterion: $W=\{\omega_j\}$(j=1,...,n). The rating values of decision matrices can be represented in many forms such as crisp values, vague values or intuitionistic fuzzy values [5-6, 24], linguistic terms [2, 20], fuzzy numbers [19, 30] and intuitionistic fuzzy numbers [9, 13, 18]. Subsequently, the individual decision matrix is weighted and aggregated into a group decision matrix $R_{ij} = (r_{ij})_{m\times n}$. Hence, the decision making group can use the group decision rating matrix to compare alternatives with respect to multiple criteria of different levels of importance.

Table 1. Individual decision matrix

	C_1	…	C_j	…	C_n
W	ω_1		ω_j		ω_n
A_1	$r^k{}_{11}$	…	$r^k{}_{1j}$	…	$r^k{}_{1n}$
A_2	$r^k{}_{21}$	…	$r^k{}_{2j}$	…	$r^k{}_{2n}$
⊥	⊥	…	⊥	…	⊥
A_i	$r^k{}_{i1}$	…	$r^k{}_{ij}$	…	$r^k{}_{in}$
⊥	⊥	…	⊥	…	⊥
A_m	$r^k{}_{m1}$	…	$r^k{}_{mj}$	…	$r^k{}_{mn}$

2.2 Vague Set Theory

The intuitionistic fuzzy set (IFS) on a universe X was introduced by Atanassov [12-13] as a generalization of fuzzy set introduced by Zadeh in 1965 [19]. An intuitionistic fuzzy set A in X is an object having the form $A=\{<x, \mu_A(x), v_A(x)>|x\in X\}$, which is characterized by a membership function (true membership function) μ_A and a non-membership function (false membership function) v_A, where the function μ_A: $X\to[0, 1]$ and $v_A(x)$: $X\to[0, 1]$ define the degree of membership and degree of non-membership of the element $x\in X$ to the set A, respectively. For every $x\in X : 0 \le \mu_A(x) + v_A(x) \le 1$, the value $\pi_A(x) = 1-\mu_A(x)-v_A(x)$ can be determined, called the hesitation margin (hesitancy degree, intuitionistic fuzzy index, degree of uncertainty) of the element $x\in X$ to the IFS A and $\pi_A(x)\in[0,1]$, $\forall x\in X$. Later, in [26] Gau and Buehrer proposed the concept of vague set, where the grade of membership is bounded to a subinterval $[t_A(x), 1-f_A(x)]$ of [0, 1]. Burillo and Bustince [7] proved that the notion of vague sets coincides with that of intuitionistic fuzzy sets. The interval $[t_A(x), 1-f_A(x)]$ is called the vague value of x in A. Relevant definition and operations of vague sets introduced in [4, 8, 26] are briefly reviewed as follows.

Definition 1: vague sets. A vague set A in the universe of discourse X is characterized by a truth membership function, t_A: $X\to$ [0, 1], and a false membership function, f_A: $X\to$ [0, 1], where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x", and $0\le t_A(x) + f_A(x) \le 1$. The grade of membership of x in the vague set is bounded to a subinterval $[t_A(x), 1-f_A(x)]$ of [0, 1]. The interval $[t_A(x), 1-f_A(x)]$ is called the vague value of x in A.

The vague value $[t_A(x), 1-f_A(x)]$ indicates that the exact grade of membership $\mu_A(x)$ of x may be unknown, but is bounded by $t_A(x) \le \mu_A(x) \le 1-f_A(x)$, where $t_A(x) + f_A(x) \le 1$. This interval can be interpreted as an extension to the fuzzy membership function. The precision of uncertainty about x is characterized by the difference between $1-f_A(x)$ and $t_A(x)$, i.e., $1-f_A(x) -t_A(x)$. The value of $\pi_A(x) = 1-f_A(x)-t_A(x)$ *expresses* a hesitation degree of *whether x belongs* to A or *not*. If this value is small, the knowledge about x is relatively precise; if this value is large the knowledge about x is little. If $t_A(x)$ is equal to $1-f_A(x)$, there is no hesitation and the vague set theory reduces to the fuzzy set theory.

Definition 2: union of two vague sets. The union of two vague sets A and B, with respective truth-membership and false-membership functions $t_A(x), f_A(x), t_B(x)$ and $f_B(x)$, is a vague set C, written as $C = A\vee B$, whose truth-membership and false-membership functions are related to those of A and B by: $t_C(x) = \max(t_A(x), t_B(x))$, $1-f_C(x) = \max(1-f_A(x), 1-f_B(x)) = 1-\min(f_A(x), f_B(x))$, for all $x\in X$.

Definition 3: intersection of two vague sets. The intersection of two vague sets A and B, with respective truth-membership and false-membership functions $t_A(x), f_A(x), t_B(x)$ and $f_B(x)$, is a vague set C, written as $C =A\wedge B$, whose truth-membership and false-membership functions are related to those of A and B by: $t_C(x) = \min(t_A(x), t_B(x))$,$1-f_C(x) = \min(1-f_A(x), 1-f_B(x)) =1-\max (f_A(x), f_B(x))$, for all $x\in X$.

3 Polling Method for Vague Value Solicitation

An easily understandable and computationally simple way to generate a group decision is to utilize a polling method to solicit participants' responses. In the case of Yes-No head count polling method, the answer set to a question has only two possible responses, which is usually expressed as A= {Yes, No}, or A={True, False}. A Yes-No head count polling method can be viewed as a pure probability model. It is the same as a Yes-No experiment. For each element x_i, let $N^t(x_i)$ be the number of electorates that voted for, $N^f(x_i)$ be the number of electorates that voted against. Let us consider the ratio

$$\mu(x_i) = N^t(x_i) / (N^t(x_i)+N^f(x_i)), \text{ for all } i.$$

Similarly to probabilities, this ratio belongs to the interval [0, 1]. If every electorate considers all the elements possible, then $\mu(x_i)$ =1 for all i. The values $\mu(x_i)$ for all i are called degrees of possibility, and the function μ which maps each element x_i into the corresponding degree $\mu(x_i)$ is called a truth membership function of a fuzzy set. For instance, $\mu(x_i)$ = 0.7 interprets that 70% of a given population vote for that "the performance on the criterion is good" and 30% of the population vote against that "the performance on the criterion is good".

However, group decision makers' judgments and preferences are imprecise, subjective and complicated in nature. The binomial or dichotomous response to the above question is not sufficient to catch the vagueness of human judgement. In that case, the answer set to the above question will include three elements which are usually expressed as A ={Yes, Abstention, No}, or A ={True, Uncertainty, False}. An "Abstention" vote to the question is another way of resolution for a respondent who is reluctant or difficult to reply "Yes" or "No" response. Thus, vague sets are used in this study for tackling imprecise, subjective and complicated decision making problems. Based on vague set theory, in [14-15] the author proposed a polling method for respondents to simultaneously reflect their intensities of support, hesitation and opposition of the evaluations or judgments about some specific events. The author's vague set based polling method also used by Huang et al. in their case study on implementations of information security risk assessment to chlorine processing systems of water treatment plants [1]. The proposed vague set based polling method allows group decision makers to solicit vague values for fine tuning of group judgments and preferences. The procedure of the polling method for soliciting vague values can be depicted as follows:

Step 1: proposing a fuzzy question. Suppose $X=\{x_1, x_2,..., x_n\}$ is a set of subject, or any element of the domain X. A survey respondent (or decision maker) D_k of a panel $D=\{D_k\}$, k=1,…, p, is asked to express his preference strength of resolution using a 100-point scale by reply the following fuzzy proposition P:

P: x_i is A, where x_i is the subject and A is a predicate that characterizes the subject x_i.

For example in the fuzzy proposition P: The performance on the criterion is good. "The performance on the criterion" is the subject and "is good" is the predicate. Thus, the evaluation result given by the decision maker k on a set of n criteria C_j (j =1,…, n) form a vector $M_k = (m_{1k}, m_{2k},…, m_{nk})$, where each $m_{jk}\in[0,100]$ is the allocated marks by the decision maker k to the criterion C_j.

Step 2: inducing response from respondent. The response to the posed-question is represented by allocating 100 points (marks) to different votes, e.g., True-Abstention-False votes, or Yes-Hesitation-No votes. The allocated points on a specific vote reflect the degree of intensity that the subject would like to assert the proposition with respect to the vote. If all the 100 points are allocated on one specific vote, say, 100/True, then the reply to the question is "definitely true". This step can also be repeated in consecutive rounds until acceptable level of consensus amongst respondents has been reached.

Step 3: recording and calculating the responses from respondent group. The allocated points on different votes from respondent group (decision making group) can be recoded and calculated as follows:

Step 4: translating the calculated result into vague value. Interpreted by the vague set theory, the strength of preference can be reflected and the corresponding values of membership and non-membership functions are obtained based on questionnaire completed by all members of the respondent group. Thus, membership function of the solicited vague value are defined as

Respondent	Response (allocated point)			Given points
	Yes	No	Hesitation	
#1	34	42	24	100
#2	50	15	35	100
#3	45	35	20	100
⊥	⊥	⊥	⊥	⊥
#10	50	28	22	100
Total Point	725	145	130	1000

$$t_A(x_i) = N^t(x_i)/(N^t(x_i) + N^f(x_i) + N^\pi(x_i)),$$
$$f_A(x_i) = N^f(x_i)/(N^t(x_i) + N^f(x_i) + N^\pi(x_i)),$$

where $t_A(x_i)$ is the truth-membership function and $f_A(x_i)$ is the false-membership function for x_i; $N^t(x_i)$ is the total number of points allocated on "Yes" vote for x_i; $N^f(x_i)$ is the total number of points allocated on "No" vote for x_i,

The uncertainty in a vague set is represented by the difference $1-f_A(x_i)-t_A(x_i)$. Thus, $N^\pi(x_i)$ is the total number of points allocated on "Indeterminacy" vote for x_i. In addition to the truth-membership function $t_A(x_i)$ and the false-membership function $f_A(x_i)$, the author further defines a hesitation-membership function $\pi_A(x_i)$ for reflecting the equivocal area of decision making,

$$\pi_A(x_i) = N^\pi(x_i)/(N^t(x_i)+N^f(x_i)+N^\pi(x_i)),$$

where $\pi_A(x_i)$ is the hesitation membership function for x_i.

According to the calculated result, the degree of belief that the "The Taste is Good" is 0.725, i.e., $t_A(x_i) = 0.725$; the degree of disbelief that "The Taste is Good" is 0.145, i.e., $f_A(x_i) =0.145$; the degree of uncertainly that "The Taste is good" is 0.13, i.e., $a_A(x_i) =0.13$. By using vague set theory, the vague value can be obtained as $[t_A(x_i), 1-f_A(x_i)] = [t_A(x_i), t^*_A(x_i)] = [0.725, 0.855]$. Since more than two options are counted, respondents are more than willing to reveal more details about strength of preference of their resolutions with respect to "Yes", "No" or "Abstention" responses. The proposed vague set based polling method provides an easily understandable and computationally simple soft scale with continuously-valued logic to allow fine tuning of group responses.

4 New Score Function for Vague Value Transformation

The score function was first proposed by Chen and Tan [22] to measure a vague value for multi-criteria fuzzy decision making. It is a most widely used score function (or net membership) based on the membership function and non-membership function: $S_A = t_A-f_A$, where A is a vague value. Let $A= [t_A, 1-f_A)]$ and $B =[t_A(x_i), 1-f_A]$ be two vague values. It intuitively expressed that if $S_A>S_B$ then A should be greater (better) than B. Several famous research works on score functions for vague value transformation have been developed to rank vague values or intuitionistic fuzzy values [4, 27, 29]. However, these score functions do not capture sufficient information about vague values of alternatives because they only consider the truth membership degree and the false membership degree, but do not account for the hesitation degree. It is therefore important to develop a new score function which is rational to provide a new option to capture decision makers' preferences, considering not only the truth membership degree and the false membership degree but also the hesitation degree.

As shown in Fig. 1, the author presented a new 3D representation for visualizing the vague set A in the universe of discourse X, $A=\{(x, [t_A(x), 1-f_A(x)])|x\in X\}$, and the grade of membership $\mu_A(x)$ of x. In the third dimension, a corresponding second membership function $m_A(x, \mu_A(x))$ maps the grade of membership of the elements in the interval $[0, t_A(x)]$, $[t_A(x), 1-f_A(x)]$ and $[1-f_A(x), 1]$ respectively. The value $m_A(x, \mu_A(x))$ is a random value from the interval [0, 1]. It means that the second membership function $m_A(x, \mu_A(x))$ indicates to what degree of support an element in its respective interval falls under "the concept x is true". If an element has a grade of second membership function $m_A(x, \mu_A(x))$ equal to 1, this reflects a complete fitness between the element and "the concept x is true"; if an element has a grade of support membership function $m_A(x, \mu_A(x))$ equal to 0, then the element does not belong to that "the concept x is true".

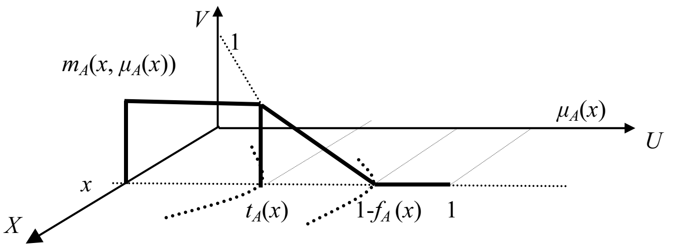

Fig. 1. Distribution of secondary membership function of vague value

For the elements $\mu_A(x)$ in the interval $[0, t_A(x)]$, the property of "being true" is completely satisfied. For the element $\mu_A(x)$ between $t_A(x)$ and $(1-f_A(x))$, the property of "being true" is partially satisfied. The second membership value of each element in the interval $[t_A(x), 1-f_A(x)]$ can be read as follows: the second membership function $m_A(x, \mu_A(x))$ takes numerical values "Equal to 1 and is continuous and strictly decreasing to 0 as the $\mu_A(x)$ value increases between $t_A(x)$ and $(1-f_A(x))$". Therefore, the support membership function $m_A(x, \mu_A(x))$ are plausibly to be strictly decreasing on the interval $[t_A(x), 1-f_A(x)]$. Using this function, the second membership value $m_A(x, \mu_A(x))$ on the interval $[t_A(x), 1-f_A(x)]$ is linearly mapped to a value in range [1, 0]. For each element $\mu_A(x)$ in the interval $[1-f_A(x), 1]$, the element does not belong to that "the concept x is true" and the element has a grade of support membership function $m_A(x, \mu_A(x))$ equal to 0.

Definition 4: Lin's new score function. If X is a collection of objects denoted generically by x, then A is defined to be a vague set of the universe of discourse X, written as $A=\{(x, [t_A(x), 1-f_A(x)])|x\in X\}$. The vague value $[t_A(x), 1-f_A(x)]$ indicates that the exact grade of membership $\mu_A(x)$ of x may be unknown but it is bounded by $t_A(x)\leq\mu_A(x)\leq 1-f_A(x)$. Therefore, the vague set A and its secondary membership function can be presented as $m_A(x, \mu_A(x))$. This implies that the value of the primary membership of x, $\mu_A(x)$, is also referred to as the secondary domain of the secondary membership function $m_A(x, \mu_A(x))$. In this case, X is referred to as the primary domain; U is referred to as the secondary domain, as well as the value of the primary membership of x; $V_A(x)$ is referred to as the secondary membership value of x. As shown in Fig. 1, $\mu_A(x)$ of [0, 1] is the primary membership function, whose primary domain is the universe of discourse X; $m_A(x, \mu_A(x))$ of [0, 1] is the secondary membership function, whose secondary domain is the vague value U.

The secondary membership value corresponding to each primary membership value in its respective interval can be represented as follows:

$$m_A(x, \mu_A(x)) = \begin{cases} 0 & \text{for} \leq \mu_A(x) < t_A(x), \\ (1\text{-}f_A(x)-\mu_A(x))/(1-f_A(x)\text{-}t_A(x)), & \text{for } t_A(x)\leq\mu_A(x)\leq 1\text{-}f_A(x), \\ 0, & \text{for } 1-f_A(x)<\mu_A(x)\leq 1, \end{cases}$$

where $m_A(x, \mu_A(x))$: $X \times [0,1]\rightarrow[0,1]$.

By above definition, the primary membership function $\mu_A(x)$ and the secondary membership distribution $m_A(x, \mu_A(x))$ define a rectangular fuzzy number RFN and a right triangular fuzzy number TFN, which are both "normal" and "convex". Each data object in the interval $[0, t_A(x)]$ and its secondary membership distribution define a rectangular fuzzy number denoted as RFN(0, $t_A(x)$). Each data object in the interval $[t_A(x), 1-f_A(x)]$ or $[t_A(x), t^*_A(x)]$ is characterized by its degree of secondary membership function $m_A(x, \mu_A(x))$, linearly decreasing from 1 to 0. The data object and its secondary membership distribution defines a right triangular fuzzy number denoted as TFN($t_A(x)$, $t_A(x)$, $1-f_A(x)$). For the elements in the interval $[1-f_A(x), 1]$, the secondary membership distribution $m_A(x, \mu_A(x))$ is equal zero.

Each data object in the intervals $[0, t_A(x)]$, $[t_A(x), 1-f_A(x)]$ and $[1-f_A(x), 1]$ is characterized by its respective membership function distribution. By adding up the areas under the curves of their respective subintervals, the numerical score of the vague value can be derived. Thus the following score function can be used to transform vague value $V_A(x)$ into numerical score:

$$S_L(V_A(x)) = t_A(x)\times 1+ (1-f_A(x)-t_A(x))\times 1/2+ (1-(1-f_A(x)))\times 0= t_A(x)/2+(1-f_A(x))/2=(t_A(x)+t_A^*(x))/2. \quad \textbf{(1)}$$

For example, at x, the primary membership values in the interval $[0, t_A(x)]$ is [0, 0.5], $[t_A(x), 1-f_A(x)]$ is [0.5, 0.85] and $[1-f_A(x),1]$ is [0.85, 1] respectively. As shown in Fig. 2, the primary membership function $\mu_A(x)$ and the secondary membership function $m_A(x, \mu_A(x))$ define a rectangular fuzzy number RFN(0,

$t_A(x)$) =RFN(0, 0.5) and a right triangular fuzzy number TFN($t(x)$, $t_A(x)$,$1-f_A(x)$)=TFN(0.5, 0.5, 0.85). By using the authors' new score function in (Eq. 1), the numerical score of the vague value $S_L(V_A(x))$ can be transformed as:

$$S_L(V_A(x)) = (0.5 + 0.85)/2 = 0.675 .$$

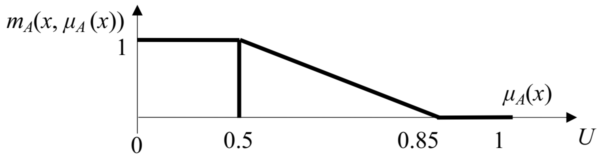

Fig. 2. Membership function of a vague value $V(x) = [t_A(x), 1-f_A(x)]$

5 Vague Set Based Multi-Criteria Group Decision Making Method

The single point-based membership value of fuzzy set tells us nothing about its accuracy. By using interval-based membership, instead of using point-based membership in fuzzy set, vague set has more powerful ability in dealing with uncertain and imprecise judgments of decision makers than fuzzy set has. In this study, the author proposed an easily understandable and reasonable vague set based multi-criteria decision-making method for making selection or ranking alternatives. A stepwise depicted algorithm of the multi-criteria group decision making method is described in the following steps.

Step 1: Conducting Vague Values Solicitation

Consider a multi-criteria group decision-making problem with a set of m alternatives A_i (i=1,…, m), a set of n criteria C_j (j=1,…, n) and a panel of p decision makers D_k (k=1,…, p). The polling method introduced in Section 3 can be used to precisely and efficiently solicit importance and performance vague values. For every decision maker, the assessed rating value to the criterion is represented by allocating 100 points to different True-Abstention-False votes, or Yes-Abstention-No votes. The allocated points on a specific vote reflect the degree of intensity that the decision maker would like to assert the proposition with respect to the vote. For soliciting performance ratings on criterion C_j for alternative A_i, if N^t_{rij} is the total number of points allocated on one specific vote for "agree on performance rating", N^f_{rij} is the total number of points allocated on one specific vote for "disagree on performance rating", and N^π_{rij} is the total number of points allocated on one specific vote for "undecided performance rating", then $t_{rij} = N^t_{rij}/N_{rij}$, $f_{rij} = N^f_{rij}/N_{rij}$, $\pi_{rij} = N^\pi_{rij}/N_{rij}$ can be derived, where $N_{rij} = N^t_{rij} + N^f_{rij} + N^\pi_{rij}$.

The solicited performance rating on criterion C_j for alternative A_i can be expressed as $r_{ij} = [t_{rij}, 1-f_{rij}]$, where r_{ij} is the rating of the alternative A_i to the criterion C_j . Then the performance of A_i can be characterized by the vague set shown as following criterion-rating vector: R_i={(C_1, r_{i1}), (C_2, r_{i2}),…,(C_n, r_{in})}={(C_1, [t_{ri1}, $1-f_{ri1}$]), (C_2, [t_{ri2}, $1-f_{ri2}$]),…,(C_n, [t_{rin},$1-f_{rin}$])}. Let $1-f_{rij} = t^*_{rij}$, then r_{ij} can be rewritten as $r_{ij} = [t_{rij}, t^*_{rij}]$ and the group performance rating R_i can be rewritten as following criterion-rating vector: R_i ={r_{ij}}={(C_1, [t_{ri1}, t^*_{ri1}],(C_2, [t_{ri2}, t^*_{ri2}]),…,(C_n, [t_{rin}, t^*_{rin}])). As expressed in Table 2, $R = (r_{ij})_{m\times n}$ = ([t_{rij}, t^*_{rij}])$_{m\times n}$ (i =1,…, m; j =1,…, n) is called as a vague set based group rating matrix.

Table 2. Vague set based group decision matrix

	C_1	…	C_j	…	C_n
W	$[t_{w1}, t^*_{w1}]$,		$[t_{wj}, t^*_{wj}]$		$[t_{wn}, t^*_{wn}]$
A_1	$[t_{r11}, t^*_{r11}]$	…	$[t_{r1j}, t^*_{r1j}]$	…	$[t_{r1n}, t^*_{r1n}]$
A_2	$[t_{r21}, t^*_{r21}]$	…	$[t_{r2j}, t^*_{r2j}]$	…	$[t_{r2n}, t^*_{r2n}]$
⊥	⊥	…	⊥	…	⊥
A_i	$[t_{ri1}, t^*_{ri1}]$	…	$[t_{rij}, t^*_{rij}]$	…	$[t_{rin}, t^*_{rin}]$
⊥	⊥	…	⊥	…	⊥
A_m	$[t_{rm1}, t^*_{rm1}]$	…	$[t_{rmj}, t^*_{rm1}]$	…	$[t_{rmn}, t^*_{rmn}]$

Similarly, for soliciting importance weightings on criterion C_j, if N^t_{wj} is the total number of points allocated on one specific criterion for "agree on importance weighting", N^f_{wj} is the total number of points allocated on one specific criterion for "disagree on importance weighting", and N^π_{wj} is the total number of points allocated on one specific criterion for "undecided on importance weighting", then $t_w = N^t_{wj} / N_{wj}, f_{wj} = N^f_{wj} / N_{wj}$, $\pi_{rj} = N^t_{wj} / N_{wj}$ can also be derived. The solicited importance weighting on criterion C_j can be expressed by a vague value $\omega_j = [t_{wj}, 1- f_{wj}]$.

Then the importance of each alternative A_i can be characterized by the vague set shown as following criterion-weighting vector: $W = \{(C_1, \omega_1), (C_2, \omega_2), \ldots, (C_n, \omega_n)\} = \{(C_1, [t_{w1}, 1-f_{w1}]), (C_2, [t_{w2}, 1-f_{w2}]), \ldots, (C_n, [t_{wn}, 1-f_{wn}])\}$. Let $1- f_{wj} = t^*_{wj}$, then ω_j can be rewritten as $\omega_j = [t_{wj}, t^*_{wj}]$ and the importance weighting W can be rewritten as following criterion-weighting vector: $W = \{\omega_j\} = \{(C_1, [t_{r1}, t^*_{w1}], (C_2, [t_{w2}, t^*_{w2}]), \ldots, (C_n, [t_{rn}, t^*_{wn}])]$. As shown in Table 2, $W = \{\omega_j\}$ (j=1,…, n) is called as a vague set based group weighting vector.

Step 2: Performing Weighted Aggregation on Vague Values

Suppose that there is a decision making group who wants to evaluate a set of alternatives which satisfies the criteria $C_1, C_2, \ldots$, and C_n *or* which satisfies the criteria C_s. This decision making group's requirement is represented by the following expression: C_1 *AND* C_2 *AND*,..., *AND* C_n *OR* C_s.

Assume that the performance rating r_{ij} of alternative A_i against C_j have been solicited and represented by the following rating vector expressed as vague values: $R_i = \{r_{i1}, r_{i2}, \ldots, r_{in}\} = \{[t_{ri1}, 1-f_{ri1}], [t_{ri2}, 1-f_{ri2}], \ldots, [t_{rin}, 1-f_{rin}]\} = \{[t_{ri1}, t^*_{ri1}], [t_{ri2}, t^*_{ri2}], \ldots, [t_{rin}, t^*_{rin}]\}$.

Assume that the importance weightings against C_j (j=1,…,n) have been solicited and represented by the following weighting vector expressed as vague values:

$$\begin{aligned} W &= \{(\omega_1, \omega_2, \ldots, \omega_n) \\ &= \{[t_{w1}, 1-f_{w1}], [t_{w2}, 1-f_{w2}], \ldots, [t_{wn}, 1-f_{wn}]\} \\ &= \{[t_{w1}, t^*_{w1}], [t_{w2}, t^*_{w2}], \ldots, [t_{wn}, t^*_{wn}]\}. \end{aligned}$$

By using intersection operation and union operation of vague sets (Definition.2 and 3), the weighted aggregation on vague ratings that the alternative A_i satisfies and does not satisfy the decision-making group's requirement can be determined to obtain a weighted aggregated vague value. The weighted aggregated vague value can be obtained by using following weighted aggregation function $W(E(A_i))$:

$$\begin{aligned} & W(E(A_i)) = V_{Ai} \\ & = [t_{ri1}, t^*_{ri1}] \wedge [t_{w1}, t^*_{w1}] \vee [t_{ri2}, t^*_{ri2}] \wedge [t_{w2}, t^*_{w2}] \vee \ldots \vee [t_{rij}, t^*_{rij}] \wedge [t_{wj}, t^*_{wj}] \ldots [t_{rin}, t^*_{rin}] \wedge [t_{wn}, t^*_{wn}] \vee [t_{ris}, t^*_{ris}] \\ & = [(t_{ri1} \wedge t_{w1}) \vee (t_{ri2} \wedge t_{w2}) \vee \ldots \vee (t_{rin} \wedge t_{wn}), (t^*_{ri1} \wedge t^*_{w1}) \vee (t^*_{ri2} \wedge t^*_{w2}) \vee \ldots \vee (t^*_{rin} \wedge t^*_{wn})] \\ & = [\max(\min(t_{ri1}, t_{w1}), \min(t_{ri2}, t_{w2}), \ldots, \min(t_{rin}, t_{wn}), t_{ris}), \max(\min(t^*_{ri1}, t^*_{w1}), \min(t^*_{ri2}, t^*_{w2}), \ldots, \min(t^*_{rin}, t^*_{wn})), t^*_{ris}]] \\ & = [t_i, t^*_i], \end{aligned} \quad \textbf{(2)}$$

where $\wedge$ denotes the minimum operator and $\vee$ stands for maximum operator of the vague values.

By using the weighted aggregation function (Eq. 2), weighted aggregated vague value for each alternative can be derived by multiplying the vague weighting vector with vague rating vector. The weighted aggregated vague value is also a vague value.

Step 3: Performing Score Transformation on Vague Values

In vague set based MCGDM method, a score function is a widely used approach to transform vague values into comparable crisp values. However, several deficiencies remain evident when using these vague based score functions for ranking the vague values to handle multi-criteria decision-making problems. Thus, a new score function is required to measure the degree of suitability of each alternative, with respect to a set of criteria characterized by vague values. By applying the new score function (Eq.1), the weighted aggregated vague values of alternative A_i (i=1,…,m) derived in Sect. 5 can be transformed into comparable crisp scores $S(V_{Ai})$ (i=1,…,m). The greater the score of $S(V_{Ai})$, the higher the degree of appropriateness that the alternative A_i satisfies some given criteria.

Step 4: Ranking or Selecting Alternatives

By comparing the transformed scores $S(V_{Ai})(i=1,\ldots, m)$, the ranking order of alternatives can be obtained. The larger score denotes the better alternative. The alternative with the largest score is selected as the best alternative.

6 Numerical Case Study

6.1 Implementation of the Case Study

Following the algorithm described in the proposed MCGDM method, a case study was conducted to evaluate the ranking order of three software products that satisfy a set of six main quality characteristics as well as to demonstrate the efficiency and rationality of the proposed MCGDM method. For the MCGDM problem, let $D=\{D_k\}$, $k=1,\ldots, p$, be a panel of five decision makers, $A=\{A_i\},i =1,...,3$, be a set of three software products (alternatives), and $C =\{C_j\}$, $j=1,...,6$, be a set of six characteristics (criteria). The set of software quality characteristics includes: (C_1) Functionality, (C_2) Reliability, (C_3) Usability, (C_4) Efficiency, (C_5) Maintainability, and (C_6) Portability.

In the first step, by using the polling method for conducting vague values solicitation, the group importance weighting values of the decision criteria and the group performance rating values with respect to above criteria are solicited and interpreted by answering the following fuzzy sentences:

	Yes	Abstention	No
Do you agree that the quality performance of the software product A_i with respect to the criterion C_j is good?	( )	( )	( )
Do you agree that the criterion C_j of the software product A_i is important?	( )	( )	( )

As shown in Table 3, the solicited vague values for the multi-criteria group decision-making problem can be expressed by a vague value based group decision matrix: $R =(r_{ij})_{m\times n}= ([t_{rij}, t^*_{rij}])_{m\times n}$ $(i =1,\ldots, m; j =1,\ldots, n)$ and a vague value based group weighting vector: $W=\{\omega_j\} =\{[t_{wj}, t^*_{wj}]\}$ $(j =1,..., n)\}$.

Table 3. Solicited vague values for group decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
W	[0.6, 0.7]	[0.7, 0.8]	[0.8, 0.9]	[0.6, 0.8]	[0.7, 0.8]	[0.6, 0.7]
A_1	[0.5, 0.8]	[0.5, 0.7]	[0.5, 0.7]	[0.5, 0.6]	[0.5, 0.8]	[0.5, 0.9]
A_2	[0.8, 0.9]	[0.6, 0.9]	[0.6, 0.9]	[0.7, 0.8]	[0.8, 0.9]	[0.4, 0.6]
A_3	[0.5, 0.8]	[0.5, 0.9]	[0.5, 0.9]	[0.5, 0.9]	[0.5, 0.8]	[0.5, 0.7]

In the second step, by applying the proposed weighted aggregation function (Eq. 2), the weighting values and performance rating values of alternatives with respect to the criteria can be aggregated into following weighted aggregated vague values of alternatives:

$$W(E(A_i)) = V_{Ai} = \begin{matrix} A_1 & A_2 & A_3 \\ \|[0.5, 0.8], & [0.7, 0.9], & [0.5, 0.9]\| \end{matrix}.$$

In the third step, by using the new score function (Eq. 1), the aggregated vague values of alternatives V_{Ai} can be transformed into following comparable scores, by which the ranking of all the given alternatives can be found.

$$S_L(V_{A1}) = (0.5+0.8)/2 = 0.65,$$
$$S_L(V_{A2}) = (0.7+0.9)/2 = 0.80,$$
$$S_L(V_{A3}) = (0.5+0.9)/2 = 0.70.$$

Consequently, the ranking order of the three software products is given as follows: $A_2 \succ A_3 \succ A_1$.

6.2 Findings and Implications of the Case Study

An efficient and rational MCGDM method was proposed in this study to handle situations where several decision makers are involved in the vague and uncertain decision making processes. The vague set based MCGDM method is considered to be an umbrella under which a collective vague value solicitation and aggregation method as well as a numerical transformation method are included. Following the algorithm described in the proposed MCGDM method, a new efficient polling method can contribute to solicit group importance vague values and group performance vague values of alternatives. Besides, the solicited importance and performance vague values are aggregated by using a weighted aggregation function. Furthermore, a new score function was proposed to transform the aggregated vague values of alternatives into comparable numerical scores.

To the author's knowledge, the comprehensive MCGDM method which includes collective vague value solicitation, aggregation and numerical transformation method for decision making is still lacking in the literature. Thus, one-to-one comparisons among the proposed MCGDM method and the relative works are difficult to make. However, the resultant ranking order of the proposed MCGDM method can be compared with the resultant ranking order using existing score functions and decision making methods. In other words, the ranking order derived in the previous section can be compared with the ranking orders derived as mentioned below to demonstrate its efficiency and rationality, which is explained as follows:

The previously mentioned aggregated vague values of alternatives can be transformed into following comparable scores by using Chen and Tan's decision making method and score function $S_{CT}(V_{Ai})$ [22], Hong and Choi's decision making method and score function $S_{HC}(V_{Ai})$ [4], Xu's decision making method and score function $S_X(V_{Ai})$ [29] as well as Zhang and Xu's decision making method and score function $S_{ZX}(V_{Ai})$ [27], respectively:

$$S_{CT}(V_{A1}) = (t_{A1}-f_{A1}) = (0.5-0.2) = 0.3,$$
$$S_{CT}(V_{A2}) = (t_{A2}-f_{A2}) = (0.7-0.1) = 0.6,$$
$$S_{CT}(V_{A3}) = (t_{A3}-f_{A3}) = (0.5-0.1) = 0.4,$$

$$S_{HC}(V_{A1}) = (t_{A1}+f_{A1}) = (0.5+0.2) = 0.7,$$
$$S_{HC}(V_{A2}) = (t_{A2}+f_{A2}) = (0.7+0.1) = 0.8,$$
$$S_{HC}(V_{A3}) = (t_{A3}+f_{A3}) = (0.5+0.1) = 0.6,$$

$$S_X(V_{A1}) = (t_{A1}-f_{A1}) = (0.5-0.2) = 0.3,$$
$$S_X(V_{A2}) = (t_{A2}-f_{A2}) = (0.7-0.1) = 0.6,$$
$$S_X(V_{A3}) = (t_{A3}-f_{A3}) = (0.5-0.1) = 0.4,$$

$$S_{ZX}(V_{A1}) = (t_{A1}-f_{A1}) = (0.5-0.2) = 0.3,$$
$$S_{ZX}(V_{A2}) = (t_{A2}-f_{A2}) = (0.7-0.1) = 0.6,$$
$$S_{ZX}(V_{A3}) = (t_{A3}-f_{A3}) = (0.5-0.1) = 0.4.$$

Table 4 summarizes the comparison results of the ranking orders of the three software products derived from existing decision making method and famous score functions and from proposed new score function. The best selection made by the proposed method is identical with the other four existing famous score functions. Except the result of the ranking order derived from Hong and Choi's score function $S_{HC}(V_{Ai})$, the proposed score function obtains the same rank as the other existing famous score functions.

Table 4. Score functions and ranking orders

Score function	Ranking order
Chen and Tan's score function $S_{CT}(V_{Ai})$ [22]	$A_2 \succ A_3 \succ A_1$
Hong and Choi's score function $S_{HC}(V_{Ai})$ [4]	$A_2 \succ A_1 \succ A_3$
Xu's order relation score function $S_X(V_{Ai})$ [29]	$A_2 \succ A_3 \succ A_1$
Zhang and Xu's score function $S_{ZX}(V_{Ai})$ [27]	$A_2 \succ A_3 \succ A_1$
Proposed score function in this study, $S_L(V_{Ai})$	$A_2 \succ A_3 \succ A_1$

Nevertheless, the ranking order derived from Hong and Choi's score function $S_{HC}(V_{Ai})$ is not reasonable if explained in voting model. In terms of Hong and Choi's score function $S_{HC}(V_{Ai})$, the alternative A_1 with aggregated vague value V_{A1}= [0.5, 0.8] is better than the alternative A_3 with weighted aggregated vague value V_{A3} = [0.5, 0.9]. The alternative A_1 has 5 votes in favor and 2 votes against it. Alternative A_3 has 5 votes in favor and 1 vote against it. Most rational person should choose the alternative A_3 as the better choice, which is conflict with the result drawn by Hong and Choi's score function. That is to say that Hong and Choi's score function applied in this example is inadequate. It is observed that the best alternative is A_2 and the ranking order derived by the proposed score function is consistent with the ranking orders derived by other score functions.

The calculation procedure and the comparison results of the case study suggest that the proposed MCGDM method is efficient and rational. The proposed method can not only contribute an easily understandable and computationally simple polling method to decision makers for efficiently soliciting vague values, but it can also contribute a sound theoretical and reasonable score function for rationally transforming aggregated vague values into comparable numerical scores.

7 Conclusions

In the subjective and uncertain decision making processes, an efficient and rational MCGDM method was proposed to group decision makers, who are from different domains or interested parties for making selection or ranking alternatives. The vague set based MCGDM method is considered to be an umbrella under which the collective vague value solicitation method, the aggregation method, and the numerical transformation method are included. A numerical case study was conducted to demonstrate the main contributions of this study. More specifically, the proposed MCGDM method can contribute to solving the following research questions:

(1) The research proposed an algorithm for a vague set based MCGDM method, which is easily understandable and simple computation under vague and uncertain environment.

(2) The polling method can contribute to soliciting group importance vague values and group performance vague values of alternative.

(3) The solicited vague values can be aggregated into integrated vague values of alternatives.

(4) The aggregated vague values can be transformed into comparable numerical scores for decision making.

Future research efforts are likely to proceed in several directions. Firstly, the proposed MCGDM method should be applied for conducting more empirical studies in business or industry fields. Secondly, a further study could also be carried out to extend the proposed MCGDM method for aggregating decision makers' judgments presented as values of other extensions of fuzzy set.

References

[1] C.-C. Huang, K.-J. Farn, Y.-S. Lin, A study on implementations of information security risk assessment: application to chlorine processing system of water treatment, International Journal of Network Security 16(4)(2014) 241-248.

[2] C.-H. Cheng, Y. Lin, Evaluating the best main battle tank using fuzzy decision theory with linguistic criteria evaluation, European Journal of Operational Research 142(2002) 174-86.

[3] D.-F. Li, Multiattribute decision making models and methods using intuitionistic fuzzy sets, Journal of Computer and System Sciences 70(2005) 73-85.

[4] D.-H. Hong, C.-H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 114(2000) 103-113.

[5] D. Stanujkic, E.K. Zavadskas, J. Tamošaitienė, An approach to measuring website quality in the rural tourism industry based on Atanassov intuitionistic fuzzy sets, E&M Ekonomie a Management 18(4)(2015) 184-99.

[6] F. Shen, J. Xu, Z. Xu, An automatic ranking approach for multi-criteria group decision making under intuitionistic fuzzy environment, Fuzzy Optimization and Decision Making 14(3)(2015) 311-334.

[7] H. Bustince, P. Burillo, Vague sets are intuitionistic fuzzy sets, Fuzzy Sets and Systems 79(1996) 403-405.

[8] J. Ye, Improved method of multi-criteria fuzzy decision-making based on vague sets, Fuzzy Sets and Systems 39(2)(2007) 164-169.

[9] J. Ye, Using an improved measure function of vague sets for multicriteria fuzzy decision-making, Expert Systems with Applications 37(6)(2010) 4706-4709.

[10] J.-Q. Wang, H.-Y. Zhang, Multicriteria decision-making approach based on Atanassov's intuitionistic fuzzy sets with incomplete certain information on weights, IEEE Transactions on Fuzzy Systems 21(3)(2013) 510-515.

[11] K. Renganath, M. Suresh, Supplier selection using fuzzy MCDM techniques: A literature review, in: Proc. 2016 IEEE International Conference on Computational Intelligence and Computing Research (ICCIC), 2016.

[12] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986) 87-96.

[13] K.T. Atanassov, On Intuitionistic Fuzzy Sets Theory, Springer-Verlag, Berlin, 2012.

[14] K.-S. Lin, K.-J. Farn, A group decision making method for evaluating information incident management capability, in: Proc. 18th Cryptology and Information Security Conference (CISC'08), 2008.

[15] K.-S. Lin, W. McEwan, K.-J. Farn, A fuzzy knowledge-based approach to security protection classification problem of an information system, in: Proc. 11 International Conference on Information Networking, 1997.

[16] L. Dymova, P. Pavel Sevastjanov, Operations on intuitionistic fuzzy values in multiple criteria decision making, Scientific Research of the Institute of Mathematics and Computer Science 10(1)(2011) 41-48.

[17] L. Lin, X.-H. Yuan, Z.-Q. Xia, Multicriteria fuzzy decision making methods based on intuitionistic fuzzy sets, Journal of Computer and System Sciences 73(2007) 84-88.

[18] L. Zhou, On Atanassov's intuitionistic fuzzy sets in the complex plane and the field of intuitionistic fuzzy numbers, IEEE Transactions on Fuzzy Systems 24(2)(2016) 253-259.

[19] L.A. Zadeh, Fuzzy sets, Information Control 8(1965) 338-356.

[20] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Information Sciences 8(3)(1975) 199-249.

[21] A. Mardani, A. Jusoh, E.K. Zavadskas, Fuzzy multiple criteria decision-making techniques and applications– Two decades review from 1994 to 2014, Expert Systems with Applications 42(8)(2015) 4126-4148.

[22] S.-M. Chen, J.-M. Tan, Handling multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Sets and Systems 67(1994) 163-172.

[23] S. Wibowo, H. Deng, A fuzzy rule-based approach for screening international distribution centres, Computers and Mathematics with Applications 64(5)(2012) 1084-1092.

[24] T. Chen, A comparative analysis of score functions for multiple criteria decision making in intuitionistic fuzzy settings, Information Sciences 181(17)(2011) 3652-3676

[25] V.L.G. Nayagam, S. Muralikrishnan, G. Sivaraman, Multi-criteria decision making method based on interval valued intuitionistic fuzzy sets, Expert Systems with Applications 38(3)(2011) 1464-467.

[26] W.-L. Gau, D.J. Buehrer, Vague sets, IEEE Transactions on Systems Man, and Cybernetics 23(1993) 610-614.

[27] X.-M. Zhang, Z.-S. Xu, A new method for ranking intuitionistic fuzzy values and its application in multi-attribute decision making, Fuzzy Optimization and Decision Making 12(2012) 135-146.

[28] Z. Guo, H. Liu, D. Zhang, J. Yang, Green supplier evaluation and selection in apparel manufacturing using a fuzzy multi-criteria decision-making approach, Sustainability 9(4)(2017) 1-13.

[29] Z.-S. Xu, Intuitionistic preference relations and their application in group decision making. Information Sciences 177(2007) 2363-2379

[30] Z.-T. Gong, X. Feng, Alpha beta-statistical convergence and strong alpha beta-convergence of order gamma for a sequence of fuzzy numbers, Journal of Computational Analysis and Applications 21(2)(2016) 228-236.