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Contributive Degree Evaluation of Links for Capacity Expansion Problem in Urban Traffic Network

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Abstract. With the development of urbanization and motorization, the imbalanced contradiction of urban traffic between supply and demand becomes increasingly sharp. Traffic congestion has become very serious and results in problems such as increase of traffic accidents and rise of fuel depletion. Expanding existing links in the urban traffic network is one of the commonly used methods which can effectively solve the capacity limitation problem and is considered more sensible for road networks. Thus, it is important to evaluate the expansion effect of links, so as to meet the demand for the expansion of traffic network capacity, and the minimum cost of expansion. This study puts forward the definition of a link's contributive degree to evaluate the expansion effect of links for traffic network capacity. According to the stochastic user equilibrium model, a bi-level programming model for determining the maximum contributive degree of link is proposed. Moreover, the efficient-paths-based stochastic traffic assignment algorithm is adopted based on the impact area and the evaluation process for contributive degree of links is put forward. In order to demonstrate the efficiency of the proposed model and algorithm, it is applied to a simple network and some results have been got.

Keywords: bi-level programming, contributive degree, link capacity expansion, traffic assignment, urban traffic network

1 Introduction

With the development of urbanization and motorization, the imbalanced contradiction of urban traffic between supply and demand becomes more and more sharp. Traffic congestion has become a serious urban illness in China, and it results in problems such as travel time delay, increase of traffic accidents, rise of fuel depletion, survive environmental degradation and so on [1]. In order to satisfy the traffic demand, the traffic authorities and policy makers usually pose this problem in one of the following two forms: changing the network topology or introducing traffic control measures [2]. Traditionally, this problem has been referred to as Urban Transportation Network Design Problem (UTNDP). Based on the nature of the decisions considered, UTNDP can be further classified in three groups: (i) Discrete Network Design Problem (DNDP), which only deals with discrete design decisions such as constructing new roads, adding new lanes, determining the directions of one-way streets, and determining the turning restrictions at intersections, (ii) Continuous Network Design Problem (CNDP), which is only concerned with continuous design decisions such as expanding the capacity of links, scheduling traffic lights, and determining tolls for some specific streets, and (iii) Mixed Net-work Design Problem (MNDP), which contains a combination of continuous and discrete decisions [3].

Expanding existing links in the urban traffic network is one of the commonly used methods which can effectively solve the capacity limitation problem and is considered more sensible for road networks. Link capacity expansion is often discussed as CNDP, who concerns about how to determine the set of link capacity expansions and the corresponding equilibrium flows for the network are optimal [4-5]. Link

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capacity expansion has received increased attention and the number of related publications has grown over time. Dantzig, *et al*. [6] presented a convex nonlinear programming problem to deal with network improvements including introduce new links, increase the capacity of existing links, or decrease the free-flow (uncongested) travel time on existing links. Meng and Yang [7] examined the benefit distribution among the network users and the resulting equity associated with the continuous network design problem by using bi-level programming models. Yang, Hao and Zhang [8] considered the optimum capacity problem for a class of bottleneck capacity expansion problem and provided an efficient algorithm to solve this problem. Ordóñez and Zhao [9] considered the problem of expanding arc capacities in a network subject to demand and travel time uncertainty and proposed a robust optimization approach to obtain capacity expansion solutions that are insensitive to this uncertainty. Mathew and Sharma [10] established a bi-level optimization problem: the upper level determines the optimal link capacity expansion vector to minimize the total system cost and the lower level determines the link flows subject to user equilibrium conditions. Miandoabchi et al. [11] investigated a multi-objective DNDP, considering reserve capacity and two newly proposed travel time related objective functions. In their discrete network design problem, four types of decisions were made, including adding lanes to the existing network links, constructing new links, determining the lane allocations on two-way links, and converting some two-way links to one-way links. Baskan [12] employed a bi-level programming model to solve the CNDP problem with link capacity expansion. The upper level objective function was defined as the sum of the total travel time and total investment costs of link capacity expansions on the network, while the lower level problem was formulated as a user equilibrium traffic assignment model. The Harmony Search (HS) algorithm was used to solve the upper level objective function and the Frank-Wolfe method was used to solve the traffic assignment problem at the lower level. Msigwa et al. [13] established a bi-level optimization model with complementarity constraints for the equilibrium transportation problem concerning both capacity expansion and road toll pricing under the user equilibrium conditions. Graham *et al.* [14] studied the effect of road network capacity expansions on aggregate traffic volume and density in U.S. cities using a linear mixed model GPS approach for continuous dose-response estimation. Zhang, *et al*. [15] employed a bi-level mathematical programming modeling for the capacity expansion in the continuous network design problem, and developed a golden ratio based hybrid genetic algorithm to solve the upper-level model of expanded capacity and adopted the classic Frank-Wolfe algorithm to solve the lower-level model.

Moreover, some studies have been carried out considering optimal signal timings and capacity expansion plan together. Marcotte [16] considered two network optimization problems, a particular version of the Network Design Problem and traffic signals setting problem, which have the following characteristics: control parameters vary continuously and network users behave according to War-drop's first principle of traffic equilibrium ("user-optimization"). Gao and Song [17] proposed a bi-level programming model to solve signal setting problem with link capacity expansions with user-equilibrium route choice. Chiou [18] considered a signalized road network where the set of link capacity expansions and signal settings were simultaneously determined. Karoonsoontawong and Waller [19] present a robust optimization formulation that simultaneously solved capacity expansion, signal optimization and dynamic traffic assignment problems, considering the long-term O-D demand uncertainty. Baskan and Ozan [20] proposed a bi-level solution methodology combined capacity expansion and signal setting problems. The upper level dealt with minimizing total system travel cost under given budget and signal timing plan while the User Equilibrium link flows were determined by VISUM at the lower level. Chiou [21] presented a robust bi-level model for an urban traffic network system under uncertain travel demand to account for signal settings at junctions and capacity expansion at links simultaneously. Aissi and Vanderpooten [22] dealt with the problem of capacity expansion of a network under independent uncertain demands as a bi-objective minimum cost flow problem.

Typically, the problem of link capacity expansion is usually formulated as a bi-level problem in which the upper level problem is the investment decision-making problem to minimize the total travel cost or maximize the social welfare while the lower level problem is the problem of demand-performance equilibrium for travelers in given investment. In an urban traffic network, one link may be shared by travelers from multiple OD pairs. Thus, expanding one link's capacity may affect the traffic volumes of other links and the implementation of link's capacity expansion will require investment. Therefore, different expansion project will cause different capacity expansion of the network. Accordingly, it is important to evaluate the expansion effect of links, so as to meet the demand for the expansion of traffic network capacity, and the minimum cost of expansion. Therefore, applying the bi-level mathematical modeling method on describing the traffic capacity expansion of network and developing efficient solution algorithm becomes research focus. Many algorithms have been proposed to calculate the lower level model, such as incremental assignment method and Frank-Wolf algorithm, and so forth. Traffic assignment is always the part of the algorithms for traffic network capacity evaluating, and then the vast majority of calculations are spent on finding shortest path during large network traffic assignment. Moreover, a commonly used technique in the literature for evaluating the expansion effect of links is the full network scan approach which will consume a lot of time. In this case, a traffic assignment is firstly conducted to calculate the network capacity under normal condition, and each link in turn is expanded and a traffic assignment is carried out again to account for expansion effects and traveller responses to the link's expansion in an impact area. In view of the above, this study proposes an efficient-paths-based analysis approach for evaluating the contributive degree of links in an impact area of urban traffic network.

The remainder of the paper is structured as follows: In section 2, the definition of the link's contributive degree for an urban traffic network is proposed and a bi-level programming model for determining the maximum contributive degree of a link is presented based on the stochastic user equilibrium. Section 3.1 introduces the efficient-paths-based stochastic traffic assignment algorithm and analyses the impact area of a link. Section 3.2 provides the evaluation process for contributive degree of links. A numerical experiment is tested in section4. Finally, the conclusions are given in Section 5.

2 The Contributive Degree Evaluation of Links

2.1 The Definition of The Link's Contributive Degree

For an urban traffic network, it is important to evaluate the expansion effect of links, so as to meet the demand for the expansion of traffic network capacity, and the minimum cost of expansion. So, this needs to analyze the variation in the service level or the network capacity brought by a link's expansion. In order to measure the level of this change, this paper puts forward the definition of a link's contributive degree.

The notations used in this paper are given as follows:

(r,s) -- the OD pair on traffic network, where $r \in R$ is the origin, $s \in S$ is the destination and R is the origin set, S is the destination set.

$\overline{q}_{rs}$ --the existing OD demands.

$V = \{v_i \,|\, i = 1,2,...,n\}$ --the intersections set.

$E = \{e_{ij} \,|\, i, j = 1,2,...,n\}$ --the links set.

x_e -- the traffic flow on link *e*.

y_v -- the traffic flow of intersection *v*.

C_e -- the capacity of link *e*.

C_v -- the capacity of intersection *v*.

$T = \{t_e(x_e) \,|\, e \in E\}$ -- the link travel cost set.

$D = \{d(y_v) \,|\, v \in V\}$ -- the intersection cost set.

P_{rs} -- the set of paths from origin *r* to destination *s* in the network.

f_k^{rs} -- the traffic flow on path *k* between *r* and *s*.

c_k^{rs} -- the travel cost of path *k*.

$c_m^{rs} = \min(C_k^{rs} \,|\, f_k^{rs} > 0)$ -- the travel cost of shortest path between *r* and *s*.

δ_{rs}^{ej} , δ_{rs}^{vj} -- 0-1 variables. δ_{rs}^{ej} =1 means link *e* is on path *j* between *r* and *s*, otherwise δ_{rs}^{ej} =0. δ_{rs}^{vj} =1 means intersection *v* is on path *j*, otherwise δ_{rs}^{ej} =0.

Q -- the network capacity before link expansion.

Q_e -- the network capacity after expansion of link *e*.

μ -- the O-D matrix multiplier of the capacity after expansion of link.

α_e -- the level of service for link *e*, indicated as x_e / C_e.

θ be the dispersion parameter.

The traffic network capacity is measured by the maximum common multiplier that can be applied to a given O-D matrix subject to the flow on each link not exceeding its service level when the multiplied O-D matrix is assigned to the network by some equilibrium model [23].

So, the contributive degree of a link for an urban traffic network can be defined as the variation of network capacity brought by the link expansion in a certain investment.

Definition 1. In an urban traffic network, the contributive degree of a link $e(e \in E)$ is defined as

$$Con_e = \frac{Q_e - Q}{B}, \quad \textbf{(1)}$$

where *B* is the investment for link expansion.

2.2 The Mathematical Model

For the investment sector, it is hoped that the investment is used to the most effective place. That is, investing in the link with the largest contributive degree. So, the evaluation of contributive degree for links in urban traffic network is finding the link which causes the maximum variation of the network capacity after the link expansion.

Suppose that the investment *B* is the function of link capacity increment, that is, $B = G_e(\Delta C_e)$ which is a strictly monotone increasing function. So, $\Delta C_e = G_e^{-1}(B)$. Therefore, the objective function for this problem can be expressed the following:

$$\max_{e \in E} \quad Con_e = \frac{Q_e - Q}{B}. \quad \textbf{(2)}$$

For a given investment, *B* is a constant value and greater than zero. In addition, the network capacity before link expansion, *Q* is a determined value. Thus, the objective function can be transformed to the following:

$$\max \quad Q_e = \sum_r \sum_s \mu \overline{q}_{ij}. \quad \textbf{(3)}$$

Assume that the investment is only used for one link, that is:

$$\sum_{e \in E} \lambda_e = 1, \quad \textbf{(4)}$$

where λ_e is 0-1 variable, $\lambda_e = 1$ means link *e* is expanded, otherwise $\lambda_e = 0$.

In this paper, the stochastic user equilibrium (SUE) model is adopted for modeling the travellers' heterogeneous risk-taking behavior. So, we can formulate the mathematical model for determining the maximum contributive degree of a link as follows:

$$\max \quad Q_e = \sum_r \sum_s \mu \bar{q}_{ij}$$

$$s.t.\begin{cases} x_e \le \alpha_e \left(C_e + \lambda_e G_e^{-1}(B) \right), \quad e \in E & (5) \\ \sum_{e \in E} \lambda_e = 1 & (6) \\ \lambda_e \in 0,1, \quad e \in E & (7) \\ \min \quad Z(f) = \dfrac{c_{rs}^m}{\theta} \sum_{r,s} \sum_{k \in P_{rs}} f_k^{rs} \ln f_k^{rs} + \sum_{e \in E} \int_0^{x_e} t_e(w)dw + \sum_{v \in V} \int_0^{y_v} d_v(w)dw & (8) \\ s.t.\begin{cases} x_e = \sum_{r \in R} \sum_{s \in S} \sum_{k \in P_{rs}} f_k^{rs} \delta_{rs}^{ek}, \quad e \in E & (9) \\ y_v = \sum_{r \in R} \sum_{s \in S} \sum_{k \in P_{rs}} f_k^{rs} \delta_{rs}^{vk}, \quad v \in V & (10) \\ \sum_{k \in P_{rs}} f_k^{rs} = \mu \bar{q}_{rs}, \quad r \in R, s \in S & (11) \\ f_k^{rs} \ge 0, \quad r \in R, s \in S, k \in P_{rs} & (12) \end{cases} \end{cases}$$

This bi-level programming model consists of two sub-models. The upper level of the bi-level programming model is to identify the link with maximum contributive degree of an urban traffic network. The lower level represents the user equilibrium assignments considering the influence of intersection. Equation (5) is the service level constraint for links after expansion. Equation (6) and (7) determine which link is expanded. Equations (8) - (12) describe the SUE problem.

3 Solution Algorithm

3.1 Efficient Paths-Based Stochastic Traffic Assignment Algorithm

The key problem to solve the bi-level programming model is to find the reaction function through solving the lower-level problem [24].

The lower level of this bi-level programming model is a traffic assignment model. In this paper, the efficient-paths-based stochastic traffic assignment algorithm proposed by Yang *et al*. [25] is adopted for stochastic traffic assignment.

(1) Efficient Paths Searching Algorithm

Generally speaking, there are some paths between an O-D pair which are not chosen by travelers whatever congestion the traffic network is from result of traffic assignment and practical observation. If an amount of traffic flow is assigned to a new path, the choice probability of other paths which already have got traffic flows will be influenced. Herein the conception of path effect degree, proposed by Yang *et al*. [25], is used to denote the degree of one path affecting traffic flow.

Definition 2 (see [25]). For an OD pair (r,s), let P_{rs} be the path set which is chosen by travellers. The effect degree of path *j* for P_{rs} is defined as:

$$inf_{P_{rs}}^{j} = \sum_{i \in P_{rs}} \left| pos_{rs}^{i} - pos'^{i}_{rs} \right|, \tag{13}$$

where pos_{rs}^{i} and pos'^{i}_{rs} are the choice probability of path *i* before and after adding a new path *j* to the path set P_{rs}.

Besides, the choice probability of path *i* is:

$$pos_{rs}^{i} = \frac{\exp(-\theta(c_i^{rs} / c_m^{rs}))}{\sum_{l \in P_{rs}} \exp(-\theta(c_l^{rs} / c_m^{rs}))}, \tag{14}$$

where c_i^{rs} is the travel cost of path *i* and c_m^{rs} is the travel cost of shortest path between *r* and *s*.

Definition 3 (see [25]). The path k in efficient path set A_{rs} for (r,s) pair must not have a loop, and should satisfy the following:

$$\overline{q}_{rs} inf_{A_{rs}}^{k} > \varepsilon, \quad \textbf{(15)}$$

where ε is a small positive number to represent efficient path parameter.

So, an algorithm to search for efficient paths is designed based on the definitions above, which is stated as follows:

Step1. Compute shortest path length r_i from the origin *r* to node *i*.

Step2. Compute likelihood value l_{ij} for each link e_{ij} according to equation (16).

$$l_{ij} = e^{\theta(r_j - r_i - t_{ij})}. \quad \textbf{(16)}$$

Step3. Forward pass. Consider nodes in ascending order of r_i from the origin *r*. For each node *i*, calculate the link weight w_{ij} for each link e_{ij}, where

$$w_{ij} = \begin{cases} l_{ij}, i = r \\ l_{ij} \sum\limits_{m \in D(i)} w_{mi}, i \neq r \end{cases}. \quad \textbf{(17)}$$

Step4. Generate path.

Step4-1. Search path by step from the origin *r*, and let $m = r, i = 0$.

Step4-2. If $O_m = \varnothing$ where O_m denotes the set of eligible node of all links leaving node *m*, then terminate. Otherwise, compute the choice probability $p(m,j) = \dfrac{w_{mj}}{\sum\limits_{k \in O_m} w_{mk}}$ of link e_{mj} for $j \in O_m$ where $\sum\limits_{j \in O_m} p(m,j) = 1$ and calculate cumulative probability $P_k = \sum\limits_{j=1}^{k} p(m,j), k = 1,2,\cdots,|O_m|$.

Step4-3. Generate a random number $p \in (0,1)$. If $p \leq P_1$, then first node is chosen. If $p \in [P_{k-1}, P_k)$, $k = 2,\cdots,|O_m|$, then the *k*th node is chosen and let *n* be the node number.

Step4-4. If *n*=*s*, then terminate; else let $m = n, i = i+1$ and go to Step4-2.

Path *k* is an efficient path if it satisfies the condition $q_{rs} inf_{A_{rs}}^{k} > \varepsilon$ from definition 2, while the effect degree

$$inf_{A_{rs}}^{k} = pos_{rs}^{k} = \frac{\exp\left(-\theta(c_{rs}^{k} / c_{rs}^{m})\right)}{\sum\limits_{l \in A_{rs}} \exp\left(-\theta(c_{rs}^{l} / c_{rs}^{m})\right) + \exp\left(-\theta(c_{rs}^{k} / c_{rs}^{m})\right)}. \quad \textbf{(18)}$$

Then, SUE traffic assignment could be applied on the efficient path set.

(2) Efficient Paths-Based Stochastic Traffic Assignment Algorithm

In this paper, a solution scheme based on the method of successive averages (MSA) is developed based on the efficient path set. The compute procedure is stated as follows:

Step1. Initialize the OD demands $\overline{q}_{rs}$ and the certain equilibrium criterion ξ. Let *n*= 1, perform the assignment and yield the initial link flows x_e^n and the traffic flow of intersection y_v^n.

Step2. According to the current link flows, calculate the link travel cost $t_e(x_e^n)$ and intersection delay $d(y_v^n)$, and update the travel cost of efficient paths.

Step3. Perform the assignment in terms of the current travel cost of efficient paths and then get path's flow and the auxiliary link flow $\hat{x}_e^n$.

Step4. Update link flow by the following equation:

$$x_e^{n+1} = x_e^n + (1/n)(\hat{x}_e^n - x_e^n), e \in E. \quad \textbf{(19)}$$

Step5. If the certain equilibrium criterion, the following inequation, is satisfied, stop and output the solution; otherwise, let n=n+1 and go to Step 2.

$$\sqrt{\sum_{e\in E}(x_e^{n+1}-x_e^n)^2} \Big/ (\sum_{e\in E}x_e^n) < \xi \tag{20}$$

(3) Impact area

Obviously, a link's expansion will influence the path choice probability of travelers. The impact area of a link's expansion is the area that the selectable paths, chosen by travelers, passing by for an OD demand. Actually, the choice probability of paths in efficient path set will be impacted for the efficient path set whose path includes expanded link. The effect of the link expansion will then impose restrictions only on its impact area and not disperse throughout the whole network. So, the efficient path set whose path includes expanded link a is the impact area of link a. If link a belongs to more than one efficient path set, the impact area of link a is all the efficient path sets whose path includes link a.

For example, a simple network is shown in Fig. 1. The efficient paths from 1 to 6 and 2 to 5 are shown in Table 1. The flow on the link 1 in Fig. 1 contains the traffic flow from point 2 to 5. Therefore, the impact area of expansion of link 1 is the area that the paths passing by in the efficient path set for 2-5. Accordingly, the impact area of expansion of link 1 is the area that path 2-1-3-5, path 2-1-3-4-6-5, path 2-4-6-5 and path2-4-3-5 passing by.

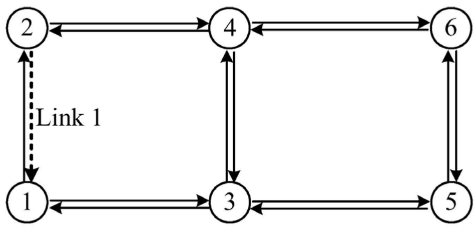


Fig. 1. A transportation network G

Table 1. Efficient paths

1-6		2-5	
No	Path	No	Path
1	1-3-5-6	5	2-4-6-5
2	1-3-4-6	6	2-4-3-5
3	1-2-4-6	7	2-1-3-5
4	1-2-4-3-5-6	8	2-1-3-4-6-5

3.2 Solution Algorithm for Evaluation the Contributive Degree of Links

Based on above stochastic traffic assignment method, the evaluation process for contributive degree of links is stated as follows:

Step1. Search the efficient path set A_{rs} .

Step2. Let $Q_e = 0$, $EX = \varphi$;

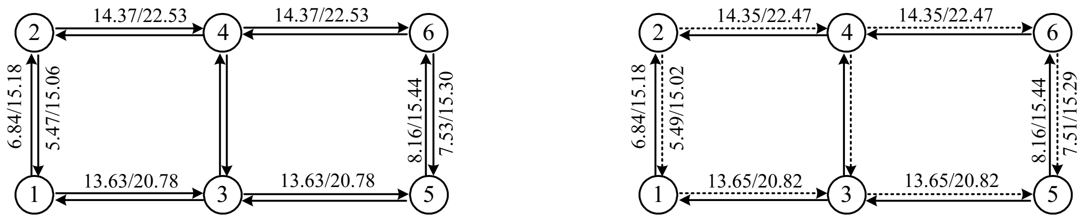
Step3. If $E/EX = \varphi$, then terminate. Otherwise, select a link $e \in E/EX$, let $EX = EX + \{e\}$, $u_e = 1$ and update the travel cost of link e and the travel cost of efficient path.

Step4. Perform the efficient paths-based stochastic traffic assignment in terms of the travel cost and efficient paths in the impact area of link e. Get path's flow and the auxiliary link flow $\hat{x}_e^n$.

Step5. Calculate the level of service for all links $\alpha_e = \hat{x}_e^n / C_e$, let $\alpha^* = \max(\alpha_e)$.

Step6. Set a small positive number γ. If $\left|\alpha - \alpha^*\right| < \gamma$, output Q_e and go to Step3. Otherwise, let the OD demands be $\mu\bar{q}_{rs}\alpha/\alpha^*$, go to Step4.

Now we use the simple network in Fig. 1 as example to illustrate the calculation process for contributive degree of links. The traffic demands from 1 to 6 and 2 to 5 are 15 and 13, respectively. According to the calculation process above, the stochastic traffic assignment can be perform firstly as shown in Fig. 2(a) where the numbers give the $x_e / t_e(x_e)$ of link. Select a link $a \in E$ for expanding, such as link 1 from point 2 to 1. The the impact area of expansion of link 1 is the area that path 2-1-3-5, path 2-1-3-4-6-5, path 2-4-6-5 and path2-4-3-5 passing by, which is shown in Fig. 2(b) with broken lines. In this impact area, the efficient paths-based stochastic traffic assignment is performed again. Then the result of traffic assignment after link expansion is shown in Fig. 2(b). After all the links are calculated, the contributive degree of links can be obtained.



(a) The result of traffic assignment

(b) the result of traffic assignment after link expansion

Fig. 2.

4 An Application to a Simple Network

A simple network is shown in Fig. 3. Assume that the unit expansion investment is same for all the links. The investment function $B = \Delta C \times \psi$, and let $\psi = 1$. The travel cost function of link is $t_e(x_e) = t_e(0)\left[1 + 2.62(\frac{x_e}{C_e})^5\right]$ and the intersection cost function is used the following equation (21) referring to Highway Capacity Manual 2000 where C_v, g/C are the intersection capacity and the weighted average green-to-cycle-length ratio for through movements. The parameters of link and intersection are given in Table 2 and Table 3, and the traffic demands from 1 to 12 and 3 to 10 are 27 and 31.

$$d(y_v) = \frac{0.5C\left(1 - \frac{g}{C}\right)^2}{1 - \left[\min(1, \frac{y_v}{C_v})\frac{g}{C}\right]} + 225\left[(\frac{y_v}{C_v} - 1) + \sqrt{(\frac{y_v}{C_v} - 1)^2 + \frac{3.2y_v}{0.25C_v^2}}\right]. \quad \textbf{(21)}$$

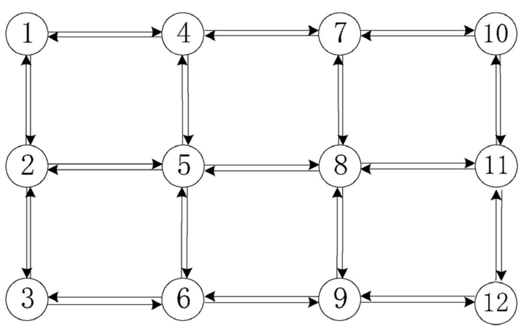


Fig. 3. A simple traffic network

Table 2. The parameters of link

e	i, j	$t_e(0)$ /min	C_e	e	i, j	$t_e(0)$ /min	C_e	e	i, j	$t_e(0)$ /min	C_e
1	1, 2	15	50	13	4, 7	25	50	25	8, 9	15	50
2	2, 1	15	50	14	7, 4	25	50	26	9, 8	15	50
3	1, 4	15	50	15	5, 6	20	20	27	8, 11	18	40
4	4, 1	15	50	16	6, 5	20	20	28	11, 8	18	40
5	2, 5	15	50	17	5, 8	20	30	29	9, 12	22	30
6	5, 2	15	50	18	8, 5	20	30	30	12, 9	22	30
7	2, 3	25	50	19	6, 9	20	40	31	10, 11	26	50
8	3, 2	25	50	20	9, 6	20	40	32	11, 10	26	50
9	3, 6	10	30	21	7, 8	20	60	33	11, 12	16	20
10	6, 3	10	30	22	8, 7	20	60	34	12, 11	16	20
11	4, 5	10	20	23	7, 10	24	40				
12	5, 4	10	20	24	10, 7	24	40				

Table 3. The parameters of intersection

v	C_v	g	C	v	C_v	g	C	v	C_v	g	C
1	100	50	50	5	100	50	60	9	80	40	45
2	90	45	50	6	80	40	60	10	90	45	40
3	100	50	50	7	80	40	40	11	80	40	30
4	80	40	50	8	90	45	50	12	80	40	30

Let $\theta = 30$, $\varepsilon = 0.01$. The efficient path sets from 1 to 12 and 3 to 10 are attained as shown in Table 4 by use of efficient paths searching algorithm.

Table 4. Efficient path set

1-12		1-12		3-10		3-10	
Path	travel cost	Path	travel cost	Path	travel cost	Path	travel cost
1-4-5-8-11-12	93.29	1-2-3-6-9-12	106.29	3-6-9-8-7-10	102.21	3-6-5-8-7-10	109.54
1-4-5-8-9-12	97.71	1-4-7-8-11-12	107.41	3-6-5-4-7-10	102.57	3-2-5-4-7-10	113.29
1-2-5-8-11-12	98.47	1-2-5-6-9-12	107.54	3-6-9-8-11-10	103.63	3-2-1-4-7-10	116.5
1-4-5-6-9-12	100.57	1-4-7-8-9-12	110.21	3-6-9-12-11-10	106.86	3-2-5-8-11-10	118.27
1-2-5-8-9-12	102.89	1-4-7-10-11-12	173.45	3-6-5-8-11-10	109.34	3-2-5-8-7-10	118.47

Set the level of service $\alpha = 70\%$, and the investment $B = 2, 4, 6, 8, 10$. The link's contributive degree and variation of network capacity are shown in Table 5. The results show that the contributive degrees of one link with different investment are different. Generally, the higher the service level value, the bigger the contributive degree will be. From Table 5, it can be seen that the contributive degrees of some links, such as link1, 12, 16, 19 and 29, are negative. This shows that expansion of some links may actually reduce the capacity of the network. For example, link 1 belongs to path 1-2-3-6-9-12, and link 9, the most congested link, also belongs to this path. When link 1 is expanded, travelers choosing this path will increase, and then link 9 will become more crowded. In addition, the expanding effect of some links with small service level value is not obvious. For example, the contributive degrees of link 2, 4, 6, 7, 10, 14, 18, 20, 21, 22, 24, 25, 26, 28, 30, 31, 32 and 34, which the service level values are no more than 11%, are zero.

The link's contributive degree is also observed in different service levels. For the sake of convenience, link 1, 9, 11, 17, 29 are taken for example when $B = 2$. The results are displayed in Table 6. As shown in Table 6, the contribution degrees of a link with different service level values are different when expanded. Generally, the contributive degree is a little bigger when the link with higher value of service level and the link with maximal contributive degree is the restricted link which has the biggest value of the service level. Therefore, the restricted link should be priority expanded.

Table 5. The contributive degree and variation of network capacity of link ($\alpha = 70\%$)

e	B	ΔQ	Con_e	e	B	ΔQ	Con_e	e	B	ΔQ	Con_e
2, 4, 6, 7, 10,	2	0	0		2	0.09	0.045		2	-0.036	-0.018
14, 18, 20, 21,	4	0	0		4	0.14	0.035		4	-0.065	-0.0163
22, 24, 25, 26,	6	0	0	11	6	0.18	0.03	19	6	-0.087	-0.0145
28, 30, 31,	8	0	0		8	0.205	0.0256		8	-0.105	-0.0131
32, 34	10	0	0		10	0.22	0.022		10	-0.119	-0.0119
	2	-0.0008	-0.0004		2	-0.025	-0.0125		2	0.0014	0.0007
	4	-0.0014	-0.00035		4	-0.04	-0.01		4	0.0024	0.0006
1	6	-0.002	-0.00033	12	6	-0.05	-0.0083	23	6	0.0034	0.00056
	8	-0.0024	-0.0003		8	-0.055	-0.007		8	0.004	0.0005
	10	-0.0027	-0.00027		10	-0.06	-0.006		10	0.0045	0.00045
	2	0.01	0.005		2	0.002	0.001		2	0.0001	0.00005
	4	0.02	0.005		4	0.004	0.001		4	0.0002	0.00005
3	6	0.03	0.005	13	6	0.006	0.001	27	6	0.0002	0.00003
	8	0.04	0.005		8	0.0065	0.0008		8	0.00023	0.000025
	10	0.045	0.0045		10	0.007	0.0007		10	0.00024	0.000024
	2	0.0006	0.0003		2	0.003	0.0015		2	-0.0154	-0.0077
	4	0.001	0.00025		4	0.005	0.0013		4	-0.0276	-0.0069
5	6	0.0014	0.00023	15	6	0.006	0.001	29	6	-0.037	-0.0062
	8	0.0016	0.0002		8	0.0065	0.0008		8	-0.044	-0.0055
	10	0.0018	0.00018		10	0.007	0.0007		10	-0.05	-0.005
	2	0.0005	0.00025		2	-0.061	-0.0305		2	0.0004	0.0002
	4	0.0009	0.00023		4	-0.1	-0.025		4	0.0006	0.00015
8	6	0.0012	0.0002	16	6	-0.123	-0.0205	33	6	0.0009	0.00015
	8	0.0014	0.00018		8	-0.138	-0.017		8	0.0012	0.00015
	10	0.0016	0.00016		10	-0.149	-0.015		10	0.0014	0.00014
	2	1.5	0.75		2	0.035	0.0175				
	4	3	0.75		4	0.062	0.0155				
9	6	4.47	0.745	17	6	0.084	0.014				
	8	5.94	0.7425		8	0.1	0.0125				
	10	7.39	0.739		10	0.114	0.0114				

Table 6. The contributive degree in different service levels

e	α	ΔQ	Con_e	e	α	ΔQ	Con_e	e	α	ΔQ	Con_e
	70%	-0.0008	-0.0004		70%	0.0900	0.0450		70%	-0.0154	-0.0077
	75%	-0.0015	-0.0008		75%	0.1163	0.0580		75%	-0.0183	-0.0091
1	80%	-0.0027	-0.0014	11	80%	0.1414	0.0707	29	80%	-0.0200	-0.0100
	90%	-0.0056	-0.0028		90%	0.1496	0.0748		90%	-0.0327	-0.0164
	100%	-0.0083	-0.0042		100%	0.1185	0.0593		100%	-0.0136	-0.0068
	70%	1.5000	0.7500		70%	0.0350	0.0175				
	75%	1.5443	0.7721		75%	0.0420	0.0210				
9	80%	1.5340	0.7670	17	80%	0.0462	0.0231				
	90%	1.4892	0.7446		90%	0.0465	0.0233				
	100%	1.4906	0.7453		100%	0.0504	0.0252				

Additionally, the comparative analysis of link's travel cost is illustrated with link 1, 3, 9, 11, 17, 29 when service level $\alpha = 70\%$ and $B = 2$. The results are shown in Fig. 4 to Fig. 9. The results show that one link's expansion may bring other link's travel cost variation and some time may cause some link's travel cost increase. Travel cost decrease of this link is generally higher than the variation of other links, and the link with a maximal decrease is the restricted link 9.

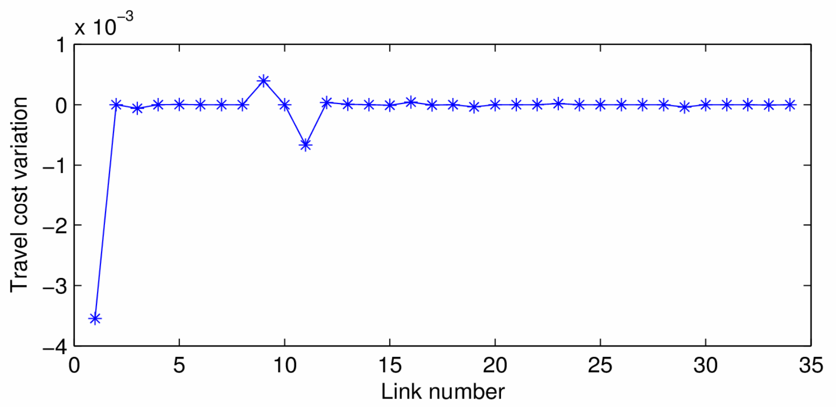


Fig. 4. Travel cost variation of links when Link 1 is expanded

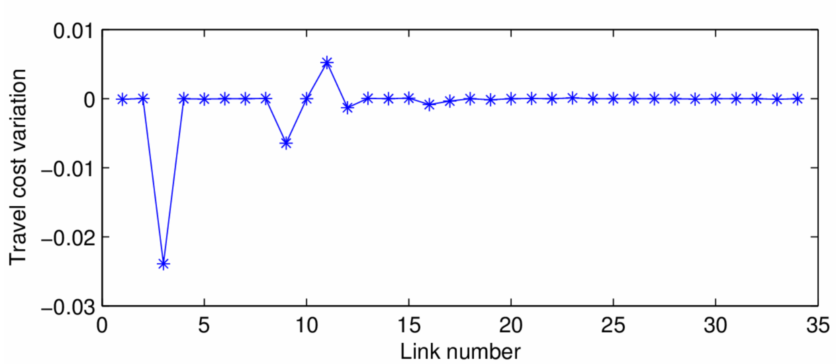


Fig. 5. Travel cost variation of links when Link 3 is expanded

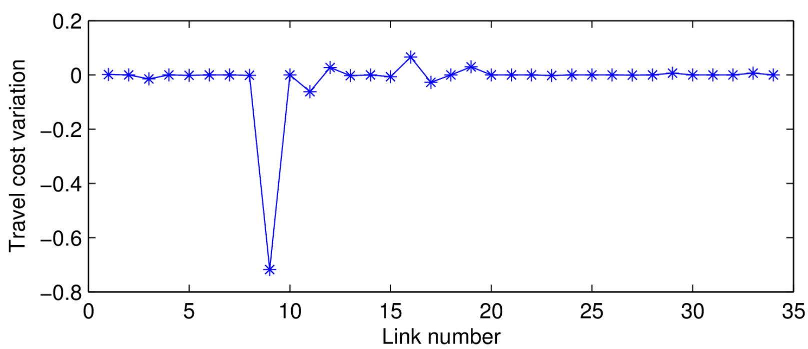


Fig. 6. Travel cost variation of links when Link 9 is expanded

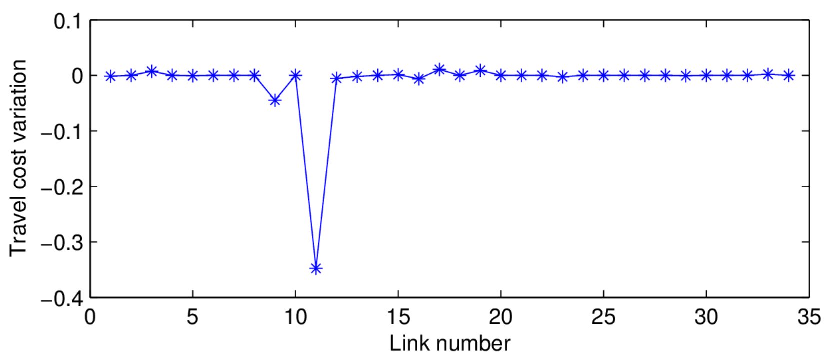


Fig. 7. Travel cost variation of links when Link 11 is expanded

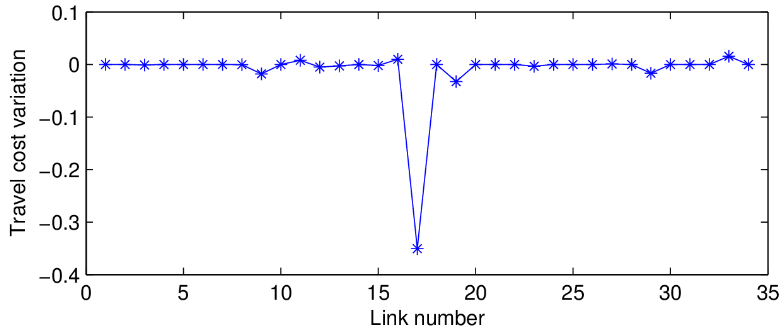


Fig. 8. Travel cost variation of links when Link 17 is expanded

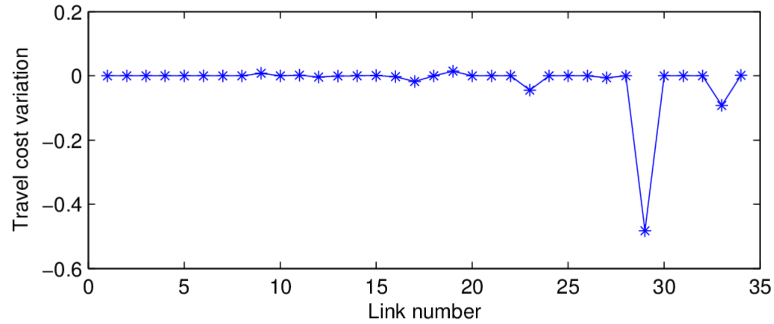


Fig. 9. Travel cost variation of links when Link 29 is expanded

5 Conclusions

Different expansion project will cause different capacity expansion of the urban traffic network. It is important to evaluate the expansion effect of links, so as to meet the demand for the expansion of traffic network capacity, and the minimum cost of expansion. This study puts forward the definition of a link's contributive degree to evaluate the expansion effect of links for traffic network capacity. According to the stochastic user equilibrium model, a bi-level programming model for determining the maximum contributive degree of link is proposed. In order to improve the computational efficiency, the impact area of a link's expansion is quantified and an efficient-paths-based stochastic traffic assignment algorithm is adopted based on the impact area. Moreover, the evaluation process for contributive degree of links is put forward.

The case study performed for evaluating the contributive degree of links illustrates the effectiveness of the optimization model and algorithm, and some results have been got as follows:

(1) The contributive degrees of one link with different investment are different. Generally, the higher the value of service level, the bigger the contributive degree will be.

(2) The contributive degrees of some links may be negative and expansion of these links may actually reduce the capacity of the urban traffic network which is consistent with Yang and Bell [4].

(3) Expansion effect of some links with small service level value is not obvious.

(4) The contribution degrees of links with different service level values are different when expanded. Generally, the contributive degree is a little bigger when the link with higher value of service level and the link with maximal contributive degree is the restricted link which has the biggest value of the service level. Therefore, the restricted link should be priority expanded.

(5) One link's expansion may bring other link's travel cost variation and some time may cause some link's travel cost increase. Travel cost decrease of this link is generally higher than the variation of other links, and the link with a maximal decrease is the restricted link.

When determining the maximal contributive degree of link in this work, the green-to-cycle-length ratio g/C of intersection is set to be fixed. Considering a general road network, most of travel time delay strongly depends on the signal setting at intersections. In future work, the authors will analysis the influence of signal timing setting of intersection on link's contributive degree. Moreover, the development of Intelligent Transport Systems (ITS) is a trend for requiring more timely, accurate and reliable transport related information. The impact of ITS on route choice behavior is becoming more and more important. We also plan to analysis link's contributive degree with ITS applications.

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References

[1] H. Yang, H.-P. An, The management of current traffic congestion status during the urbanization development in Guiyang, Studies in Sociology of Science 2(2)(2011) 23-28.

[2] Z. Cong, B.D. Schutter, R. Babuška, Co-design of traffic network topology and control measures, Transportation Research Part C: Emerging Technologies 54(7)(2015) 56-73.

[3] R.Z. Farahani, E. Miandoabchi, W.Y. Szeto, H. Rashidi, A review of urban transportation network design problems, European Journal of Operational Research 229(2)(2013) 281-302.

[4] H. Yang, M.G. Bell, Models and algorithms for road network design: a review and some new developments, Transport Reviews 18(3)(1998) 257-278.

[5] S.-W. Chiou, Bilevel programming for the continuous transport network design problem, Transportation Research Part B 39(4)(2005) 361-383.

[6] G.B. Dantzig, R.P. Harvey, Z.F. Lansdowne, D.W. Robinson, S.F. Maier, Formulating and solving the network design problem by decomposition, Transportation Research Part B: Methodological 13(1)(1979) 5-17.

[7] Q. Meng, H. Yang, Benefit distribution and equity in road network design, Transportation Research Part B: Methodological 36(1)(2002) 19-35.

[8] C. Yang, C. Hao, J. Zhang, On the optimum capacity of capacity expansion problems, Mathematical Methods of Operations Research 66(2)(2007) 225-233.

[9] F. Ordóñez, J. Zhao, Robust capacity expansion of network flows, Networks 50(2)(2007) 136-145.

[10] T.V. Mathew, S. Sharma, Capacity expansion problem for large urban transportation networks, Journal of Transportation Engineering 135(7)(2009) 406-415.

[11] E. Miandoabchi, F. Daneshzand, R.Z. Farahani, W.Y. Szeto, Multi-objective discrete urban road network design, Computers & Operations Research 40(10)(2013) 2429-2449.

[12] O. Baskan, Harmony search algorithm for continuous network design problem with link capacity expansions, KSCE Journal of Civil Engineering 18(1)(2014) 273-283.

[13] R.E. Msigwa, Y. Lu, Y. Ge, L. Zhang, A smoothing approach for solving transportation problem with road toll pricing and capacity expansions, Journal of Inequalities and Applications 2015(1)(2015) 1-17.

[14] D.J. Graham, E.J. Mccoy, D.A. Stephens, Quantifying causal effects of road network capacity expansions on traffic volume and density via a mixed model propensity score estimator, Journal of the American Statistical Association 109(508)(2014)1440-1449.

[15] L. Zhang, M. Zhang, W.-C. Yang, D. Dong, Golden ratio genetic algorithm based approach for modelling and analysis of the capacity expansion of urban road traffic network, Computational Intelligence & Neuroscience 2015(2015) 1-9.

[16] P. Marcotte, Network optimization with continuous control parameters, Transportation Science 17(2)(1983) 181-197.

[17] Z.-Y. Gao, Y.-F. Song, A reserve capacity model of optimal signal control with user-equilibrium route choice, Transportation Research Part B: Methodological 36(4)(2002) 313-323.

[18] S.-W. Chiou, A hybrid approach for optimal design of signalized road network, Applied Mathematical Modelling 32(2)(2008) 195-207.

[19] A. Karoonsoontawong, S.T. Waller, Integrated network capacity expansion and traffic signal optimization problem: robust bi-level dynamic formulation, Networks and Spatial Economics 10(4)(2010) 525-550.

[20] O. Baskan, C. Ozan, Combined solution of capacity expansion and signal setting problems for signalized road networks, Transportation Research Procedia 10(2015) 61-70.

[21] S.-W. Chiou, A robust urban traffic network design with signal settings, Information Sciences 334-335(2016)144-160.

[22] H. Aissi, D. Vanderpooten, Robust capacity expansion of a network under demand uncertainty: a bi-objective approach, Networks 68(3)(2016)185-199.

[23] H. Yang, M.G. Bell, Q. Meng, Modeling the capacity and level of service of urban transportation networks, Transportation Research Part B: Methodological 34(4)(2000) 255-275.

[24] H. Sun, Z. Gao, J. Wu. A bi-level programming model and solution algorithm for the location of logistics distribution centers, Applied Mathematical Modelling 32(4)(2008) 610-616.

[25] X.-F. Yang, L.-F. Liu, Y.-Z. Li, R.-C. He, Determining the efficient paths based on effect degree, Journal of Transportation Systems Engineering and Information Technology 11(6)(2011) 104-110.