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Thickness Optimization of Multi-layer High Temperature Clothing

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Abstract. The design problem of multi-layer high-temperature clothing is based on the numerical solution of partial differential equation. Based on the solution of the problem, it is obtained that the temperature satisfies the law of heat conduction equation in the same layer transfer, the temperature and thermal density continuity conditions are considered at the critical point of the layer and layer. In this paper, the mathematical model of partial differential equation of multi-layer heat conduction is established for simulation experiment. The mathematical model of dual objective optimization is established to solve the optimum thickness of a certain layer, and the difference algorithm of partial differential equation is designed. The boundary condition is obtained by the constraint condition in the solution of the optimal thickness. The optimal solution is obtained by traversing the thickness of the simulation experiment. In addition, the temperature distribution of each layer is simulated in detail. In order to find out whether a certain thickness satisfies the fact law of temperature change, this paper takes the average difference of layer II in data simulation experiment as the criterion to judge whether it conforms to the actual situation. Thus the index of this judgment can be quantified. In the traversal search, the average difference of each thickness is calculated, and the optimum thickness is selected. In the dual objective mathematical model, the control variable method is used to solve the average difference between the thickness of the fourth layer and the thickness of the second layer, from which the Pareto solution set is found and the optimal solution is selected from the Pareto solution set.

Keywords: multi-layer high temperature heat conduction, thermal density continuity, partial differential equation, difference algorithm, multi-objective optimization

1 Introduction

With the demand of industrial manufacture and life, some industries need to operate in high temperature environment, which is undoubtedly a great challenge for human. For example, during the process of steelmaking or fire, the relevant staff should work in a high temperature environment. In order to reduce the harm caused by high temperature to the staff, it is indispensable to prevent the relevant heat insulation during the work. Protective clothing is one of the most widely used insulation precautions. The common protective clothing includes firefighting clothing, which is one of the important equipment to protect the personal safety of firefighters active in the front line of fire fighting. It is not only an indispensable part of the fire rescue scene. It is also an anti-fire device for firefighters to protect their bodies from harm [1]. Therefore, it is particularly important to adapt to the fire-scene rescue activities. Firefighting clothing contains many different material layers such as waterproof and breathable layer and heat insulation layer. In order to protect the life safety of the related high-temperature workers the state has issued the design standards of the related high-temperature clothing. In the production of high-temperature working clothing, it is particularly important to design clothing suitable for the application environment. Under the condition of satisfying the working conditions, according to the function of different material layers, the design of optimal clothing with thickness and cost is of great significance in production [2], which

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can increase the production efficiency. This has attracted the attention of scholars in related fields. In this paper, taking the A title of the 2018 National University Student Mathematical Model Competition as an example, the optimal high temperature clothing is designed by establishing the relevant mathematical model.

The so-called heat conduction refers to the physical phenomenon of heat transfer from the high temperature part to the low temperature part when the temperature of each part in the medium is not uniform [3]. Microscopically, the heat conduction depends on the movement of internal microscopic particles in the medium, but does not happen in the macroscopic relative displacement between the various parts of the matter, and the heat is transferred from the high temperature to the low temperature [4]. In response to this phenomenon, Fourier's law was put forward by French famous scientist Fourier in 1822. The law states that in the process of heat conduction, the amount of heat conduction passes through a given section in a given time. It is proportional to the rate of temperature change and the cross section area perpendicular to the cross section, while the direction of heat transfer is related to the temperature. The direction of degree rise is opposite [5]. The heat conduction equation [6-8] can be derived from the law of conservation of energy. Heat conduction equation is one of the classical partial differential equations, which is not only beautiful in form, but also a partial differential equation with numerical solutions. Heat conduction equation plays an important role in related engineering field and guides the production of relevant social industry.

Partial differential equations mainly have initial value problem and boundary problem, or both have fusion as initial boundary value problem. The partial differential equation of heat conduction is a typical initial-boundary value problem. There are generally two methods for solving partial differential equations. One is to solve accurately and obtain the multivariate function relationship, the main algorithms are Fourier transform and so on. The other is the approximate algorithm, including the difference algorithm and so on. The problems studied in this paper are discrete quantities. It is difficult to obtain the function distribution by Fourier transform. Therefore, in this paper, the differential method [9-10] is used to solve the partial differential equation (including the ordinary differential equation of interface temperature continuity). The time of different material layers is divided by the same method, according to the different actual conditions of the material layer, different thickness partition is adopted under the condition that the explicit difference scheme converges [11], and then the temperature distribution of each time is iterated out. The average difference of I and II layer temperature is used as a quantitative index of whether or not it accords with the actual temperature distribution. In the process of calculating the optimal thickness, the average temperature difference of each I and II layer thickness is calculated by traversing the difference step size, and compared with the result of simulation, the optimal solution is obtained.

2 Analysis of high Temperature Garment Thickness Optimization and Its Solution

2.1 Optimization of High Temperature Clothing Thickness

Thermal-proof clothing is usually composed of three layers of fabric material, which are recorded as I, II and III layers. Among them, I layer contacts with the outside environment, and there is still a gap between III layer and skin. The gap is marked as IV layer, as shown in Fig. 1. The density, specific heat and thermal conductivity of each layer are ρ_i , C_i and λ_i , i=1, 2, 3, 4. The thicknesses of I and III layers are x_1, x_3, and the thicknesses of II and IV layers are in $\left[x'_2, x''_2\right]$ and $\left[x'_4, x''_4\right]$. When the temperature of the environment is U_0, the initial skin temperature is u_0, the thickness of II layers is x_2, the thickness of IV layers is x_4 and the working time is T_0, the temperature of the outer skin changes with time. When the temperature of the environment is U_0', the initial skin temperature is u_0, determine the optimum thickness of layer II and IV, and when working for T_1, ensure that the working time beyond u_1 does not exceed ΔT and the skin temperature does not exceed u_2.

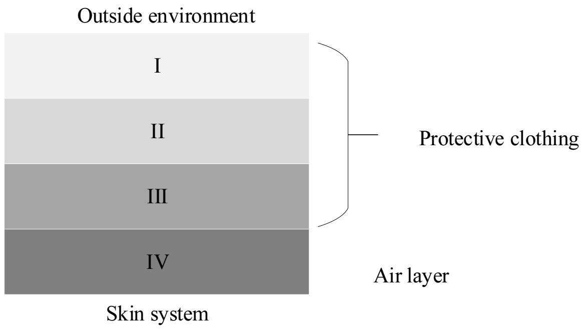

Fig. 1. Heat proof clothing - air layer - skin surface system

2.2 Analysis of the Solution to the Optimization Problem of High Temperature Clothing Thickness

Thickness optimization of high temperature garment is a dual objective optimization problem. It is difficult to find out the law of heat conduction directly by establishing an optimization model to solve the problem. In this paper, the temperature distribution law of multi-layer high-temperature heat conduction is obtained through simulation experiments of skin layer temperature changes in specific environment through known heat-proof clothing. First, the general single-objective optimization model is solved by using this law, and then the dual objective optimization is transformed into single-objective optimization by using control variable method. The solution is solved step by step, as shown in Fig. 2.

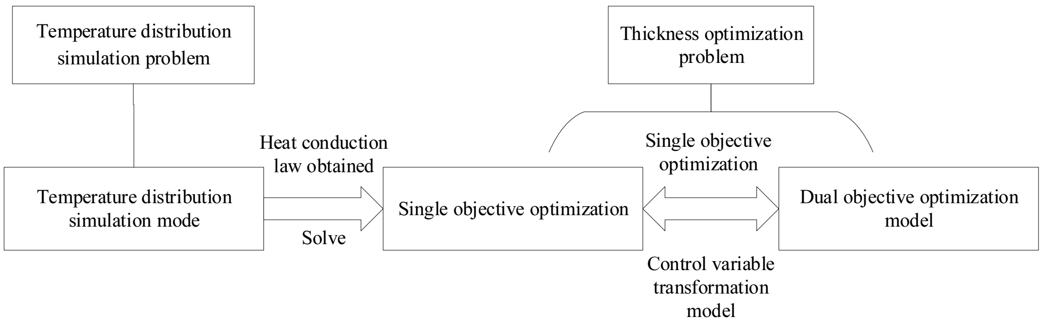

Fig. 2. Analysis of solution

According to the above analysis, the two step method will be used to solve the problem of high temperature clothing thickness optimization.

The first step is the simulation of temperature distribution. In this paper, the mathematical model of temperature distribution simulation is established by using the relationship of skin layer temperature variation under the condition of known initial parameters, and the difference algorithm is used to solve it. According to the temperature distribution of each material layer, the law of heat conduction in multi-layer high temperature clothing is obtained. The temperature distribution model is also established for other optimization problems.

The second step is the problem of thickness optimization. After obtaining the heat conduction law, the span of solving the dual objective problem directly is too long. In this paper, we first solve a single objective optimization problem and give a transition. When solving the dual objective problem, we only need to control one variable unchanged to transform it into a single objective optimization problem, and then traverse the controlled variables to get the solution set.

3 Mathematical Model for Optimization of Clothing Thickness at High Temperature

3.1 Mathematical Model of Temperature Distribution Simulation

This paper mainly introduces the mathematical model of multi-layer heat conduction. In this model, the high-temperature clothing is simplified as a multi-layer flat wall, thus the heat transfer problem in space is simplified to one-dimensional heat transfer problem. The one-dimensional coordinate system is established from the outside to the skin layer, which is shown in Fig. 3.

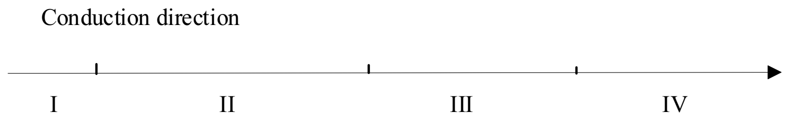

Fig. 3. One-dimensional conduction coordinate system

Heat conduction equation can be derived from Fourier law and energy conservation equation.

$$\frac{\partial u}{\partial t} - a\frac{\partial^2 u}{\partial x^2} = 0(a > 0) \quad \textbf{(1)}$$

Formula (1) is a one-dimensional heat conduction equation. The coefficient a only depends on the constant of the material itself, approach $a_i = \frac{\lambda_i}{C_i \rho_i}$. It is called the thermal diffusion rate (also known as the thermal conductivity), which is a measure of the temperature variation ability of the material. u is a binary function of t and x, describing the relationship between temperature and time and space.

For the convenience of solving the optimization problem of high temperature clothing thickness, we continue to define the thickness of each layer as x_i. The change of skin temperature over time is $U(t)$. Each interface layer is l_j, we define $l_0 = 0$, $l_j = \sum_{i=1}^{j} x_i$, i=1, 2, 3, 4.

The temperature conduction at the interface no longer satisfies the above partial differential equation because the temperature conductivity can't be solved. From the point of view of energy transfer, energy is always transferred from high temperature to the part of low temperature with time, and follows the law of conservation of energy. From the point of view of temperature transfer, temperature has continuity and no abrupt change. At this point, temperature should satisfy the continuous condition of temperature and thermal density.

$$\begin{cases} u^- = u^+ \\ \left(\lambda_j \dfrac{\partial u}{\partial x}\right)^- = \left(\lambda_{j+1} \dfrac{\partial u}{\partial x}\right)^+ \end{cases} \quad \textbf{(2)}$$

$x = l_j$, $j = 1,2,3$, which represents the three interface layers in the temperature transfer. Above the equation describes the law of heat conduction at the interface.

In this paper, the mathematical model of temperature distribution for data simulation is set up as follows.

$$\begin{cases} \dfrac{\partial u}{\partial t} - a_i \dfrac{\partial^2 u}{\partial x^2} = 0 & x \in (l_{i-1}, l_i) i = 1,2,3,4 \\ u^- = u^+ & x \in \{l_1, l_2, l_3\} \\ \left(\lambda_j \dfrac{\partial u}{\partial x}\right)^- = \left(\lambda_{j+1} \dfrac{\partial u}{\partial x}\right)^+ & x \in \{l_1, l_2, l_3\} \\ u(x,0) = u_0 & 0 < x \le l_4 \\ u(0,t) = U_0 & 0 \le t \le T_0 \\ u(l_4,t) = U(t) & 0 \le t \le T_0 \end{cases} \quad \textbf{(3)}$$

3.2 Mathematical Model of Thickness Optimization Problem

To determine the optimal thickness of II layer and IV layer, a dual objective optimization model is established.

Objective function: $\begin{matrix} \min x_2 \\ \min x_4 \end{matrix}$

Constraints:

$$\begin{cases} \frac{\partial u}{\partial t} - a_i \frac{\partial^2 u}{\partial x^2} = 0 & x \in (l_{i-1}, l_i) i = 1,2,3,4 \\ u^- = u^+ & x \in \{l_1, l_2, l_3\} \\ \left(\lambda_j \frac{\partial u}{\partial x}\right)^- = \left(\lambda_{j+1} \frac{\partial u}{\partial x}\right)^+ & x \in \{l_1, l_2, l_3\} \\ u(x,0) = u_0 & 0 < x \le l_4 \\ u(0,t) = U_0 & 0 \le t \le T_0 \\ u(l_4, T_1) \le u_1 \\ u(l_4, T_1 - \Delta T) \le u_2 \\ T_1 \le T_0 \end{cases} \tag{4}$$

4 An algorithm for Optimizing the Thickness of High-temperature Clothing

4.1 Difference Algorithm for Temperature Distribution Simulation Model

In the same layer material, the partial differential equation of heat conduction is transformed into a difference scheme in order to solve the numerical solution of the partial differential equation of heat conduction.

Grid selection. The plane Xt is meshed. Separating extraction h, k for X directional (direction of thickness) and step size of t directional (direction of time), using two families of parallel lines $(h = 0,1,2\cdots)$, $t = t_j = jk(k = 0,1,2\cdots)$, to bring the Xt plane is divided into rectangular meshes, the point is (x_i, t_j). For simplicity, let $(i, j) = (x_i, t_j)$, $u(i, j) = u(x_i, t_j)$, where $i = 0,1,2,\cdots, j = 0,1,2,\cdots$.

List difference scheme. Point (i, j) in the grid department, using backward difference quotient formula to $\frac{\partial u}{\partial t}$, using the first order central difference quotient formula to $\frac{\partial^2 u}{\partial x^2}$. Express (1) as:

$$\frac{u(i+1, j) - u(i, j)}{k} - a\frac{u(i+1, j) - 2u(i, j) + u(i-1, j)}{h^2} = O(k + h^2) \tag{5}$$

The difference approximation of the one-dimensional heat conduction equation obtained is as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{k} - a\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} = 0 \tag{6}$$

In order to solve the definite solution of partial differential equation of heat conduction by difference equation, it is necessary to discretize the definite solution condition.

For initial conditions and boundary conditions of one kind, when time is equal to 0, the temperature of the thickness boundary value is all u_0, when the thickness is equal to 0, the time boundary temperature is all U_0, the temperature of the skin changes to $U(t)$, so we can obtain as follows:

$$\begin{cases} u_{i,0} = u(x_i, 0) = u_0 (i = 0, 1, \cdots, n) \\ u_{0,j} = u(0, t_j) = U_0 \left(j = 0, 1, \cdots, m\right) \\ u_{n,j} = u(x_n, t_j) = U(t_j)\left(j = 0, 1, \cdots, m\right) \end{cases} \tag{7}$$

where $n = \frac{x}{h}, m = \frac{T_0}{k}$.

For the convenience of calculation, we define $r = \frac{ak}{h^2}$, where r is called difference coefficient. If we want to reach the difference effect, we must satisfy the difference necessary and sufficient conditions for partial convergence $1 - 2r > 0$. To reformulate formula (6) in the following form.

$$u_{i,j+1} = ru_{i+1,j} + \left(1 - 2r\right)u_{i,j} + ru_{i-1,j}. \tag{8}$$

By combining formula (7) with formula (8), we obtain a difference scheme for solving the problem expression (1) as follows.

$$\begin{cases} u_{i,j+1} = ru_{i+1,j} + \left(1 - 2r\right)u_{i,j} + ru_{i-1,j} (i = 0, 1, 2, \cdots, n, j = 0, 1, \cdots, m) \\ u_{i,0} = u(x_i, 0) = u_0 (i = 0, 1, \cdots, n) \\ u_{0,j} = u(0, t_j) = U_0 \left(j = 0, 1, \cdots, m\right) \\ u_{n,j} = u(x_n, t_j) = U(t_j)\left(j = 0, 1, \cdots, m\right) \end{cases} \tag{9}$$

Due to Heat proof suit-Air layer-skin gauge system is composed of four layers of different media that lead to partial differential equation coefficient is different. If using the same mesh ensure each corresponding differential coefficient of convergence is difficult. At the same time if the grid us the thickness are also unable to get the fabric of each layer boundary layer temperature. According to the above analysis, we can first estimate the partition of each grid, namely the corresponding *k*, *h*.

We can see from the formula (8) in the process of iteration the temperature of every grid is decided by the temperature of the upper part, top left and top right, by the moment before the same location, namely the previous position, a position after three common decision. While the first row, the first column, and the last column of the grid are all known data, iterating from top to bottom, to ensure that the difference method is feasible, and that the time equalization must be equal. It is only necessary to discuss the reasonable *h* value for each material layer according to the actual temperature conductivity under the condition that the difference coefficient is known to converge.

The physical law of temperature conduction at the interface does not follow the partial differential equation of heat conduction, but obeys the continuity equation of temperature and thermal density. In this equation, only the first order partial derivative of *x* exists, and the first order backward difference quotient is used to replace the partial derivative:

$$\lambda^- \frac{u_{i-1,j} - u_{i,j}}{h^-} = \lambda^+ \frac{u_{i,j} - u_{i+1,j}}{h^+} \tag{10}$$

where *j* refers to any point in time, and *i* is located, only if it is at the junction of the material layer place where l_1, l_2, l_3, h^-, λ^- is the thickness grid and thermal conductivity of the left side of the interface; h^+, λ^+ is the thickness grid and thermal conductivity on the right side of the interface. To simplify:

$$\begin{cases} r^* = \frac{\lambda^- h^+}{h^- \lambda^+} \\ u_{i,j} \frac{r^* u_{i-1,j} + u_{i+1,j}}{1 + r^*} \end{cases} \tag{11}$$

From the formula, we can see that the temperature in the heat transfer at the interface is determined by the temperature values of the left and right sides of the grid, that is, the former position at the same time and the latter position together. In the iterative process, different material layers correspond to different difference coefficients, and the temperature distribution at the non-interface of the next time layer is iterated through the heat conduction difference equation of the same layer material. Then the temperature distribution at the interface is iterated according to the thermal density continuous difference scheme of the same time layer, and the temperature distribution of the grid is iterated out step by layer.

The specific steps of the temperature distribution algorithm are as follows:

Step 1. The division of time and thickness of each layer is determined to ensure that each layer is equally divided, and the difference coefficient of each material layer is calculated and converged. The total fraction of calculation time is *m*, and the total number of thickness is *n*.

Step 2. Initialize initial boundary value, the first line of the grid (all positions at time 0) except the first element set to u_0, and the first column of the grid (all points of time when thickness is 0) set to U_0. The iteration starts at the second time layer, let $j=1$.

Step 3. By traversing the total thickness layer number, if the position *i* is not at the interface, the corresponding difference coefficient *r* is obtained, and the temperature is calculated according to the difference formula of the partial derivative equation. When $i=n$, that can get directly from the $U(t)$.

Step 4. After the calculation of the temperature at the non-interface of each time layer, the temperature at each interface of the time layer can be iterated out according to the continuous difference scheme of the thermal density of the time layer.

Step 5. When all thermometers in the *j* time layer are completed, let $j=j+1$. If the calculation of $j> m$ temperature distribution is completed otherwise it is transferred to Step 3.

4.2 An Algorithm for Solving Optimal Models

The constraints given in the model cannot be substituted into the temperature distribution algorithm. First of all, according to the temperature distribution $U(t)$, we set an undetermined parameter $U^{'}(t)$ using the undetermined coefficient method. The following constraints are used to solve $U^{'}(t)$ specific parameters. The specific parameters can be adapted to the above temperature distribution algorithm in the case of specified thickness.

$$\begin{cases} u(l_4,0)=u_0 \\ u(l_4,T_1)\le u_1 \\ u(l_4,T_1-\Delta T)\le u_2 \\ u(0,t)=U_0 \end{cases} \tag{12}$$

According to the temperature distribution of the difference equation, an average difference of the layer I and II can be obtained, which can be used as an index to judge whether or not it accords with the actual temperature.

$$\Delta u=\frac{1}{T_0}\sum_{t=0}^{T_0}\left[u(t,l_0)-u(t,l_1)\right] \tag{13}$$

In the analysis of the problem, it is mentioned that this paper studies the problem of single objective optimization before optimizing the single objective model. The dual objective problem of the problem can be transformed into a single objective model by means of the control variable method. First, the traversal search algorithm of the single objective optimization model is presented. The steps are as follows:

Step 1. Solve the function $U^{'}(t)$ by constraint condition. According to the mesh of the II layer in the temperature distribution that has been solved, the grid is divided into *h*, which is used as the step size for the traversal calculation, let $y=x_2^{''}$. If $x^{'}_2$ is not an *h* common multiple, then in the interval $\left[x^{'}_2,x_2^{''}\right]$, the minimum common multiple of *h* is assigned to *y*.

Step 2. The mesh generation of other material layers is consistent with that of the temperature solution algorithm. According to the calculation time of *y*, the total fraction is *m* and the total thickness is *n*.

Step 3. The thickness y of the second layer and other quantities are brought into the temperature distribution algorithm, and the average difference between three and four layers Δu_y is calculated. $y = y + h$, if $y > x_2^{''}$, we go to the next step. Otherwise execute Step 2.

Step 4. Output the corresponding value Δu_y of each y value, and compare with Δu and select the best value.

The dual objective optimization model is transformed into a single objective optimization model by using the control variable method to find the Pareto solution set. As the same, according to the constraint conditions, $U^{'}(t)$ is constructed. To control the thickness of the fourth layer for a custom, the algorithm of calculating the single objective optimization model is used to calculate Δu_y. And then we traverse the fourth layer thickness to get all the Δu_y solution set, and the Pareto solution set is found by the comparison of Δu_y in the solution set with Δu, and the optimal solution is found from it. The steps for solving the dual objective optimization model are as follows.

Step 1. Solve the function $U^{'}(t)$ by constraint condition. The mesh of the IV layer in the temperature distribution that has been solved is divided into h_4, let $y_4 = x_4^{'}$.

Step 2. Let $h_4 = y_4$, order the sum h_4 y_4 that has already obtained in the algorithm of single objective computation, then the set of all solutions is outputted.

Step 3. $y_4 = y_4 + \Delta y_4$, if $y_4 > x_4^{''}$, we go to the next step. Otherwise, execute Step 2.

Step 4. Output all corresponding Δu_y, and compare with Δu, then the Pareto solution set is select the best value by comparison.

5 Data simulation Experiment and Analysis

5.1 Calculation of Temperature Distribution

According to the above model and algorithm, this paper will solve and calculate the data given by the 2018 National College Students Mathematical Modeling Competition A, in which the parameter value of special garment materials is Table 1.

Table 1. Parameter values for special garment materials

Hierarchy	Density (kg/m^3)	Specific heat ($J/(kg{\cdot}^{\circ}C)$)	Thermal conductivity ($W/(m{\cdot}^{\circ}C)$)	Thickness (mm)
I stratum	300	1377	0.082	0.6
II stratum	862	2100	0.37	0.6-25
III stratum	74.2	1726	0.045	3.6
IV stratum	1.18	1005	0.028	0.6-6.4

By changing the change of temperature and time on the outside of the skin, the correlation law of heat conduction was obtained when the ambient temperature was 75 ºC, the thickness of Layer II was 6 *mm*, the thickness of IV layer was 5 *mm* and the working time was 90 minutes. When the ambient temperature is 80 ºC, determine the optimum thickness of layers II and IV to ensure that at 30 minutes of operation, the lateral temperature of the dummy's skin does not exceed 47 ºC, and the time over 44 ºC does not exceed 5 minutes.

The operating environment of this paper is AMD A10-9600P RADEON R5, 10COMPUTE CORES 4C+6G 2.8GHz processor, 4.00GB memory, Windows 1064-bit operating system, using Matlab2016a programming.

Firstly, the mesh of temperature distribution is divided, so that it is convenient to consider the uniform division in the time direction for iteration, and when divide the time into k=1s, the conditions for estimating the convergence of each thickness of the travel partition coefficient are estimated, as shown in Table 2.

Table 2. Convergence conditions for each layer

I	II	III	IV
$h > 0.6mm$	$h > 0.6mm$	$h > 0.8mm$	$h > 6.9mm$

Therefore, the fourth layer is only 5 *mm*, to meet the fourth layer of convergence is obviously impossible, so give up k= 1*s*. When time is equal to k=0.5*s*, the conditions for estimating the convergence of travel sub-coefficients are estimated, as shown in Table 3.

Table 3. Convergence conditions for each layer

I	II	III	IV
$h > 0.4mm$	$h > 0.5mm$	$h > 0.6mm$	$h > 4.9mm$

It was found that the fourth layer was 5 *mm*, which happened to meet the equal conditions, so that the k=0.5*s*.

In the case of time equal to 0.5*s*, for us through the four layers of different thickness to take different steps, in order to ensure that the convergence of the case at the same time to obtain the calculation of the boundary value of the layer and layer, we will divide the I layer into 1 lattice each 0.6*mm*, the II layer divided into 5 lattice per lattice length 1.2*mm*, the III layer is divided into 2 lattice 1.8*mm*, the IV layers per lattice is divided into 1 lattice length 5*mm* per lattice. The results of the differential coefficients of the partial differential equations of each layer are calculated as shown in Table 4.

Table 4. Iterative difference coefficient r

r_1	r_2	r_3	r_4
0.2757	0.0710	0.0542	0.4722

The difference coefficient of a continuous differential equation of thermal density is shown in Table 5.

Table 5. Continuous differential coefficient of thermal density r^*

r_{12}^*	r_{23}^*	r_{34}^*
0.4432	12.333	4.4643

The temperature distribution of the grid can be obtained by iteration through the algorithm of temperature distribution. The relationship of temperature varies with time at the interface and the temperature distribution diagram are shown in Fig. 4 and Fig. 5.

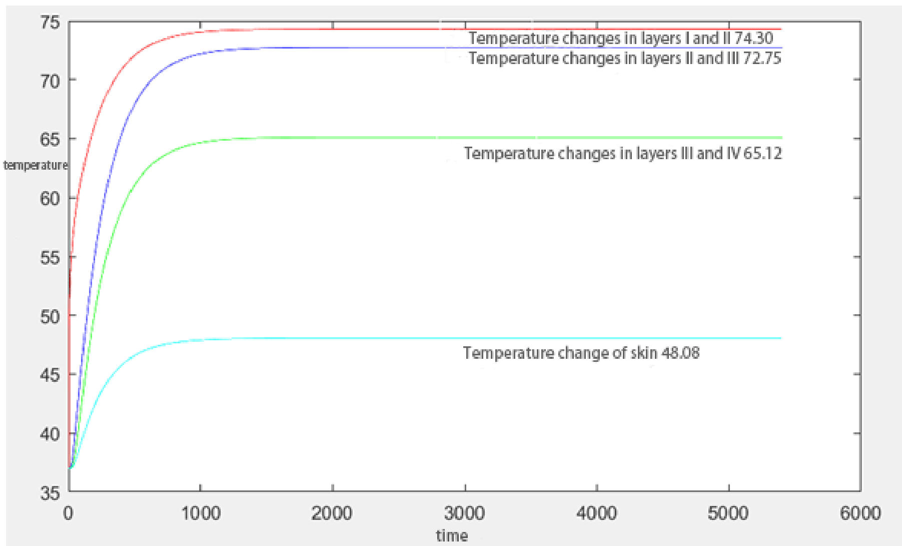

Fig. 4. Temperature changes over time at the junction

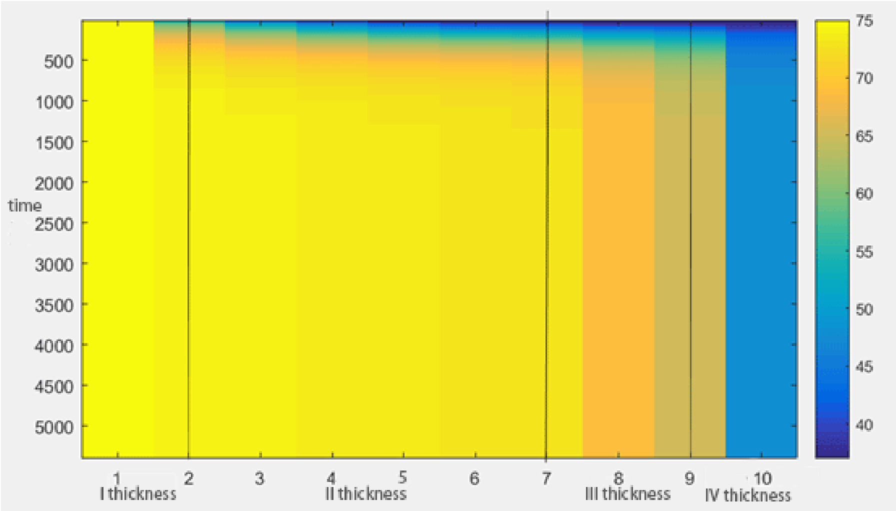

Fig. 5. Temperature changes over time at the junction

Fitting the interfacial temperature by adding two exponents to obtain the fitting test Table 6, and the fitting function is $U(t) = ae^{bt} + ce^{dt}$.

Table 6. Fitting inspection form at junction

	The I II thickness	The II III thickness	The III V thickness	The skin thickness(known)
a	74.18	72.86	65.27	48.14
b	$4.178*10^{-7}$	$-3.727*10^{-7}$	$-5.999*10^{-7}$	$-2.975*10^{-7}$
c	-21.29	-39.67	-32.27	-12.67
d	-0.004838	-0.004085	-0.003942	-0.00406
R-square	0.99	0.9989	0.9974	0.9977
RMSE	0.2861	0.1852	0.2384	0.08697

In this paper, the parameters of the temperature test with the known skin layer are in accordance with the same function form and the correlation coefficients are more than 0.99 after the fitting test is obtained. Parameter b is almost close to 0. Indicates that there is a constant of a in the function, which stabilizes with the conduction temperature of time. After the temperature is stable, the temperature difference mainly exists in the III and IV layers. The fitting test results also show that the calculated temperature changes with each layer are similar to the known temperature distribution of the skin layer, and the accuracy of the calculation results is verified.

5.2 Solving the Optimization Model

The optimization problem of a single target is solved before solving the double target model, and when the ambient temperature is 65 ºC and the thickness of IV layer is 5.5 *mm*, to ensure 60 minutes of work, the lateral temperature of the skin does not exceed 47 ºC, and the time over 44 ºC does not exceed 5 minutes, the optimal thickness of the II layer is the example, firstly, the undetermined coefficient method is used to solve the function of $U(t)$ by the constraint condition.

$$\begin{cases} u(l_4, 0) = 37 \\ u(0, t) = 65 \\ u(l_4, 3600) \le 47 \\ u(l_4, 3300) \le 44 \end{cases} \quad \textbf{(14)}$$

Given: $U(t) = a + ce^{dt}$, can be equations according to known constraints.

$$\begin{cases} U(0) = 37 \\ U(3600) \le 47 \\ U(3300) \le 44 \end{cases} \quad \textbf{(15)}$$

Since the equation is not very easy to solve, it may be useful to change the unequal sign to the equal sign condition, and the observation law of the fitting function based on the temperature distribution, make $d = \text{-}0.001$, can be solved by solving. According to the above-mentioned single objective optimization algorithm, the results table 7 can be obtained.

Table 7. Single objective optimization algorithm results

Iterations	Thickness value	Residual quantity
1	1.2	45.032722
2	2.4	0.867291
3	3.6	1.085899
4	4.8	1.300354
5	6.0	1.510826
6	7.2	1.717461
7	8.4	1.920283
⋮	⋮	⋮
18	21.6	3.422459
19	22.8	3.497310
20	24.0	3.560545

Calculated by temperature distribution $\Delta u = 1.5481$, as can be seen from the table above, the thickness difference is 45, which means that when the thickness is small, it is completely impossible, this paper selects the thickness of $x_2 = 7.2mm$ to be greater than $\Delta u = 1.5481$.

When calculating the double target model, the temperature time function of the skin surface is also calculated according to the constraint condition $U(t) = 46.01\text{-}9.01e^{\text{-}0.001t}$, the Pareto solution set is calculated by the dual objective model solution algorithm. Dual objective model solution results such as Table 8.

Table 8. Dual objective model solution results

Iterations	x_2	x_4	Residual quantity
1	1.2	5	55.42
2	2.4	5	1.81
3	3.6	5	2.49
⋮	⋮	⋮	⋮
127	8.4	5.6	5.02
128	9.6	5.6	5.57
⋮	⋮	⋮	⋮
247	8.4	6.2	5.02
248	9.6	6.2	5.57
⋮	⋮	⋮	⋮
299	22.8	6.4	7.76
300	24.0	6.4	7.80

By comparing the difference screening above to obtain a relative comparison with the real situation, some of the results filtered out of these values are shown in Table 9.

Table 9. Pareto solution set for dual target model

x_2	x_4	Residual quantity
3.6	5.1	2.49
3.6	5.2	2.49
⋮	⋮	⋮
3.6	6.2	2.49
3.6	6.3	2.49
3.6	6.4	2.49

According to the solution table and the Pareto solution set, it is found that the effect of layer IV, that is, the air layer, is not large and does not affect the calculation of the difference index. Since the ambient temperature is higher than the simulation temperature, it will be $\Delta u = 1.5481$ suitable add, choose $\Delta u = 2.49$. At this point, the optimal thickness is $x_2 = 3.6mm$, $x_2 = 5mm$.

6 Conclusion

In this paper, a partial differential equation based on heat conduction equation is established. The numerical solution of partial differential equation is obtained by using difference method, and the corresponding temperature distribution is obtained. In solving the second layer thickness optimization model, the fitting function is obtained according to the temperature distribution, and the undetermined coefficient method is used to solve the skin layer temperature function with time according to the constraint conditions of the problem. Thus, a boundary is obtained, it is convenient to use difference calculation. According to the difference of layer I and II, the index of calculation and judgement is established. After iteration, the difference index is used to judge whether it meets the requirements. The dual objective optimization model is transformed into a single objective optimization model by the control variable method, and the corresponding Pareto solution set is obtained. In the optimization model, the skin layer temperature obtained by undetermined coefficient has certain blindness, and the solution may not be accurate if the corresponding binary function is not obtained according to the partial differential equation. The experimental results show that the temperature distribution and the optimum thickness can be obtained by the calculation of the model.

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