Journal of Computers Vol. 30 No. 4, 2019, pp. 233-239
doi:10.3966/199115992019083004022

Regularization Parameter Adaptive Selection for Blurred Image Restoration

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Received 7 March 2019; Revised 7 May 2019; Accepted 29 May 2019

Abstract. It is very important for inverse problem to choose regularization parameter properly. With too little regularization parameter λ, reconstructions are too smooth. Conversely, with too much regularization parameter, the reconstructions have highly oscillatory artifacts owing to noise amplification. Currently, there are several regularization parameter selection methods for Tikhonov regularization problems such as the discrepancy principle and the generalized cross-validation method, but it mainly is for linear problem. Our main contributions are as follows. Firstly, we prove the principle of determining regularized parameters for nonlinear total variation problems. Secondly, a new adaptive parameter selection method for the nonlinear total variation regularization is proposed. In the proposed method, we use a fast total variation method for image restoration, then employ the restored image to estimate the regularization parameter. Experimental results show the proposed algorithm is very efficient and the quality of recovered images by our proposed method is competitive with other image restoration methods.

Keywords: adaptive selection, blurred image, image restoration, regularization parameter

1 Introduction

Image restoration is a process of image reconstructing by means of the degraded images observed. It is a foundation of image understanding, pattern recognition and machine vision and so on. Generally speaking, the model of blur image may be expressed

$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \tag{1}$$

where $g(x,y)$ is the blur image, $h(x,y)$ is the PSF, $f(x,y)$ is the original clear image, $n(x,y)$ is the noise, $*$ expresses the two-dimensional convolution.

Among the various image restoration techniques in both image processing and computer vision, total variation (TV) minimizing-based scheme [1-3] has been employed successfully due to its good property, such as robustness and preserving the edge [4-5]. This algorithm seeks an equilibrium state (minimal energy) of an energy functional comprised of the TV norm of the image I, Ω denotes a rectangular domain in R^2 and the fidelity of this image to the noise blur input I_0:

$$E_{TV} = \int_\Omega (|\nabla I| + \frac{1}{2}\lambda(h * I - I_0)^2)dx, \tag{2}$$

where λ, h, $*$ and ∇ denote a positive regularization parameter, a point spread function, a convolution operator and a gradient operator, respectively. It is well-known that it is very important for image deblurring to select the regularization parameter λ properly. With too little regularization parameter λ, reconstructions are too smooth. Conversely, with too much regularization parameter λ, the reconstructions have highly oscillatory artifacts owing to noise amplification [6]. Therefore, our

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motivation is to investigate an adaptive method of selecting regularization parameters to solve the ill-conditioned problem caused by noise.

It is a challenging problem to select the regularization parameter λ adaptively, but scholars have proposed a series of methods for selecting regularization parameters. For example, the unbiased predictive risk estimator (UPRE) method [7], the generalized cross validation (GCV) method [8], the discrepancy principle [9-10], L-curve method [11], and so on. These regularization parameter selection methods are currently developed for linear Tikhonov regularization or the truncated singular value decomposition (TSVD) regularization. However, the formulation in (1) for TV image restoration is a nonlinear, therefore these evaluation formulas based on the methods above is difficult to be derived. Although [12] and [13] have presented some good regularization methods for TV, those methods are only suitable for image denoising. For blurred images, which are degraded by motion, defocus or atmospheric turbulence, those methods in [12] and [13] are not applicable because of the effect of point spread function (PSF). In the [18], an effective blind image deblurring method based on a data-driven discriminative prior is proposed, but it is a linear Tikhonov regularization and its regularization is a constant.

Regularization is a technique which makes slight modifications to the learning algorithm such that the model generalizes better. This in turn improves the model's performance on the unseen data as well. L1 and L2 are the most common types of regularization. Currently, Regularization methods have been widely used in the fields of deep learning, machine learning and image deblurring for its ability to solve the ill-posed problem caused by noise. However, the performance of deep CNNs on image deblurring still falls behind conventional optimization-based approaches on handling large blur kernels [18-20]. Therefore, we still adopt the strategy based on optimization to study the image deblurring algorithm.

In order to attack the problem, according to Euler-Lagrange equation of (2) and supposed noise model, we examine the relationship between the regularization parameter λ and the divergence of image I. It is shown that the relationship can be expressed by a function. On this basis, a novel approach for adaptively selecting the regularization parameter image restoration is proposed. Our experimental results show that this algorithm is very efficient and the quality of restored images by the proposed method is competitive with those restored by the existing restoration methods.

The contributions of this work are as follows:

Prove the principle of determining regularized parameters for nonlinear total variation problems.

Present a new adaptive parameter selection method for the nonlinear total variation regularization.

2 The Fast Adaptive Image Restoration Method

In order to implement the fast adaptive image restoration, we first arbitrarily take the regularization parameter λ and apply the method in [14] to restore the image. Then a new regularization parameter model is used to confirm the parameter λ. By repeated iteration, proper regularization parameter is selected and the approximate ideal image is obtain.

2.1 The Regularization Parameter Selection Model

Image deblurring is an inverse problem, that can be formulated as the minimizing the energy functional [15] comprised of the norm of the image I and the fidelity of this image to the blurred-noisy input image I_0:

$$\hat{I} = \arg\min_I E_{TV} = \arg\min_I \int_\Omega \phi(|\nabla I|) + \frac{1}{2}\lambda(h * I - I_0)^2)dx, \tag{3}$$

where Ω denotes a rectangular domain in R^2, on which the image intensity function $I:\Omega \to [0,1]$ is defined.

The Euler-Lagrange equation is

$$\lambda(h * I - I_0) * h^* - div(\phi' \frac{\nabla I}{|\nabla I|}) = 0, \tag{4}$$

where h^* denotes conjugation of h, div denotes divergence operator.

In this paper, we choose $\phi(|\nabla I|)=\sqrt{\beta+|\nabla I|^2}$ and $\beta=10^{-8}$ in our experiment. Therefore,

$$div(\phi'\frac{\nabla I}{|\nabla I|})=\frac{(\beta+I_y{}^2)I_{xx}+(\beta+I_x{}^2)I_{yy}-2I_xI_yI_{xy}}{(\beta+|\nabla I|^2)^{3/2}} \tag{5}$$

when the noise is approximated by an additive white Gaussian process of standard deviation σ, the equation (6) can be obtained, where $|\Omega|$ represents the image area. In general case, this noise model is also rational.

$$\frac{1}{|\Omega|}\int_\Omega (I*h-I_0)^2 dxdy=\sigma^2 . \tag{6}$$

Let G, DIV, H^* denote the Fourier transform of $(h*I-I_0)$, $div(\phi'\frac{\nabla I}{|\nabla I|})$, h^* respectively. Then according to equation (4), we get

$$\lambda G=\frac{DIV}{H^*} . \tag{7}$$

Use the inverse Fourier transform for this formulation, then

$$\lambda(h*I-I_0)=F^{-1}(\frac{DIV}{H^*}) , \tag{8}$$

where F^{-1} represents the inverse Fourier transform operator.

In order to computer the value λ, we multiply the equation (7) by $(h*I-I_0)$ and integrate over the domain Ω. we obtain

$$\lambda\int_\Omega (h*I-I_0)^2 dxdy=\int_\Omega (h*I-I_0)F^{-1}(DIV)dxdy . \tag{9}$$

According to the equation (6) and (9),

$$\lambda=\frac{\int_\Omega (h*I-I_0)F^{-1}(DIV)dxdy}{|\Omega|\sigma^2} \tag{10}$$

Thereby, regularization parameter λ may be obtained, if the true image I is known. In our experiment, we replace the standard deviation σ with the estimate value $\hat{\sigma}$. When the estimate value $\hat{\sigma}$ lies in the interval [0.5 σ, 2 σ], the regularization parameter may be selected properly.

2.2 The Fast Total Variation Image Restoration Algorithm

However, in practice, the true image I is not possible to be known. In order to get the estimate $\hat{I}$ of the true image I, a minimization algorithm [14] is used. This algorithm is derived from the well-known variable-splitting and penalty techniques in optimization. An auxiliary variable is introduced in [14] to conveniently use fast Fourier transform and alternative iteration to solve the minimizer of (1).

2.3 Regularization Parameter Adaptive Selection for Blurred Image Restoration Method

In the equation (7), if $|H^*|\to 0$, it is to result in the non-solution of the regularization parameter λ, so we modify (7) into

$$\lambda G=\frac{DIV}{H^*+\alpha} , \tag{11}$$

where parameter α is a constant and $\alpha>0$. In our experiment, let $\alpha=10^{-8}$.

In view of analysis above and discussion, the description of the proposed algorithm is given as follows:

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Step 1. Initialize the image I = I_0 , the tolerance tol , regularization
        parameter λ > 0 , the last iteration image Iold = 0 .
Step 2. While ( mean(|I − Iold|) > tol ) do
Step 3. Blurred image restoration use the method of [14].
Step 4. Computer div(φ' ∇I/|∇I|) with the equation (5).
Step 5. Computer λ(h*I − I_0) with the equation (8).
Step 6. Obtain the regularization parameter λ by the equation (10).End
        while
Setp 7. Output restoration image I .
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3 Experimental Results

In this section, numerical results are presented to demonstrate the performance of our proposed algorithm for blurred image restoration involving a little Gaussian noise. The results are compared with those obtained by Richard-Lucy filter (RL) [16-17]. Peak signal-to-noise ratio (PSNR) is used to evaluate the performance of the two methods. It is defined as

$$PSNR = 10\log_{10}\frac{255^2}{\frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j}^{n-1}(I(i,j)-I_{true}(i,j))^2} \quad \textbf{(12)}$$

where I_{true} and I are the original image and the restored image, respectively.

Example 1. We restore 256×256 pixel images degraded by motion blur and a little white noise. The blurred satellite, lena, toys, barb images degraded by motion blur and a little white noise are shown in the upper of Fig. 1. The middle and bottom of Fig. 1 are corresponding restored images with RL filter and our method, respectively. Fig. 1 show the image quality of our method is more effective than that of RL filter. The comparisons of our method and RL filter are shown in Table 1. We observe the PSNRs of the our method higher than those of the RL filter. Our method is not better for computational time than RL filter, but it is still an efficient TV restoration method.

Fig. 1. Top: Blurred and noisy images by using an motion blur and corrupted by a little Gaussian noise (see Table 1). Middle: Corresponding restored images by RL filter. Bottom: Corresponding restored images by our method

Table 1. PSNR (DB), CPU time (seconds) of two methods

Image	Motion Blurred Image (angel=0)		Our Method		RL Filter	
	Blur Length	PSNR	PSNR	Time	PSNR	Time
sat	15	20.6453	35.2145	9.375	20.8470	1.6563
lena	15	20.4857	30.7589	5.4219	24.8875	1.6719
toys	11	29.0766	35.0240	3.8750	33.6423	1.6250
barb	19	21.3403	30.7018	7.1875	24.7565	1.6875

Example 2. We restore 256×256 pixel images degraded by defocus blur and a little white noise. The blurred satellite, lena, toys, barb images degraded by defocus blur and a little white noise are shown in the upper of Fig. 2. The middle and bottom of Fig. 2 are corresponding restored images with RL filter and our method, respectively. Fig. 2 show our method is more effective than that of RL filter in the presence of defocus blur and a little noise. The comparisons of our method and RL filter, shown in Table 2, also show the PSNRs of the our method are higher than those of the RL filter.

Fig. 2. Top: Blurred and noisy images by using an out of focus kernel and corrupted by a little Gaussian noise (see Table 2). Middle: Corresponding restored images by Richardson-Lucy (RL) filter. Bottom: Corresponding restored images by our method

Table 2. PSNR (DB), CPU time (seconds) of two methods

Image	Out-of-focus Blurred Image		Our Method		RL Filter	
	Blur radius	PSNR	PSNR	Time (seconds)	PSNR	Time
sat	5	21.6968	31.4347	10.7031	22.7591	1.6563
lena	7	20.8067	26.3852	10.8906	23.1517	1.7031
toys	9	25.2268	33.4541	14.6875	27.5839	1.6406
barb	11	20.2924	26.8519	5.2813	22.3417	1.6563

Example 3. We restore 256×256 pixel images degraded by Gaussian blur and a little white noise. The Corresponding restored images and the comparison results are showed in Fig. 3 and Table 3, respectively. Experimental results demonstrate our method is more efficient than the RL filter.

Fig. 3. Top: Blurred and noisy images by using an Gaussian kernel and corrupted by a little Gaussian noise (see Table 3). Bottom: Corresponding restored images by our method

Table 3. PSNR (DB), CPU time (seconds) of two methods

Image	Gaussian Blurred Image		Our Method		RL Filter	
	Blur Length	PSNR	PSNR	Time (seconds)	PSNR	Time
sat	7 (σ = 5)	22.9481	30.2639	7.8906	26.9111	1.5000
lena	9 (σ = 5)	22.6010	27.8195	18.1563	25.2002	1.5625
toys	11 (σ = 7)	27.1932	32.4592	7.6875	29.8272	1.5313
barb	13 (σ = 7)	22.3132	28.5871	7.7656	25.3751	1.5625

It is obvious from the above that our algorithm is more effective in image quality. This is attributed to the following aspects: firstly, the regularization term of total variation is adopted, which makes it advantageous in retaining details and textures; secondly, the noise estimation strategy is adopted, which ensures the rationality of adaptive parameter selection; thirdly, the adaptive parameter selection mechanism is adopted to ensure the adaptability of the algorithm under different noises, thus improving the anti-morbid problem of the algorithm.

4 Conclusions

In this letter, we proposed a fast blurred Image restoration based on total variation regularization parameter adaptive selection. It solves a problem that TV regularization parameter was usually determined by doing experiment repeatedly or experience. Our lots of experimental results show our method is very efficient and the quality of restored images by our method is quite good. However, our algorithm is based on the known PSF. Moreover, the next motivation of our research is a algorithm based on adaptive regularization parameter selection for blind image restoration.

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