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A Type Ultrasonic Testing Algorithm Based on Finite Rate of Innovation

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Abstract. There exists no proper sub-Nyquist sampling method that can be directly applied to ultrasound imaging. In this paper, on the basis of previous research, combining FRI theory with phased array ultrasound imaging features, proposed a sub-Nyquist sampling algorithm that can be applied to ultrasound imaging, which is referred to as multi-channel sampling of mixing Fourier series coefficients. The paper analyzes the reason for hybrid integral method firstly; then gives the sampling frameworks for limited FRI signal and the infinite long FRI signal separately; at last applies the schema to actual data of A type ultrasonic to do relevant simulation. The experimental results show that compared with the traditional Shannon sampling theorem multi-channel solution sampling frequency is only 0.024% (1/T/fs) of the traditional sampling frequency.

Keywords: FRI (Finite Rate of Innovation), multi channel, sub-Nyquist sampling, ultrasonic testing

1 Introduction

With the development and application of ultrasonic testing technology, the required number of sensors is increasing, which leads to extraordinary growth of processing data volume. Since the traditional Nyquist sampling needs a large amount of data which contains lots of redundant information, in addition, this data obtaining pattern conducts sampling first and compressing later which requires large amounts of time and space for data access. The compression sensing (CS) [1] theory, developed in recent years, points out that the accurate reconstruction of signals can be got via sampling the sparse signals that is lower than the lowest frequency of Shannon theorem. The method that samples signals which are lower than the Nyquist frequency is called Sub Nyquist sampling method. Hence applying this method in ultrasonic testing, which reduces the sampling frequency, becomes a direction of the ultrasonic testing technology development.

Although certain progress was made on the practicality of Sub Nyquist sampling, research on applying the CS theory in the field of ultrasonic testing is rather rare. French scientists Friboulet [2] introduced CS theory to the field of ultrasonic testing for the first time in 2010; certain progress was made. Then Israeliwhich realized the single channel sampling framework to reduce the ultrasonic imaging data. Then the team researched on ultrasonic imaging of multi-channel sampling and sparse sampling in frequency domain, etc. There are also some scholars from domestic colleges and universities, such as Chinese Academy of Sciences, Tsinghua University, etc, who research on application of CS in the field of ultrasonic imaging. Accordingly, Sub Nyquist sampling method was proposed in the paper on the basis of FRI theory, predecessors' research, and combining with the characteristics of ultrasonic, which is suitable for ultrasonic detection, namely the multi-channel mixed carrier integral sampling plan: multiplying the original signal with carrier to generate new analog signal in each channel, then integral sampling the new signal to get a set of linear transformation about original signal's Fourier coefficient, afterwards Fourier coefficient of original signal can be got through linear transformation, finally signal is reconstructed with spectral method. This schema not only has the advantages of high stability and strong ability to resist noise, but also it apples to A type ultrasonic testing as well as B type ultrasonic testing after deformation. Rarely prior knowledge is needed even for the infinite aperiodic FRI signal. Moreover, the circuit structure is simple and the sampling frequency is much lower than the traditional way of sampling method.

2 Algorithm Design

2.1 Finite-length FRI Signal Sampling Framework

Multi-channel direct sampling framework of Fourier coefficient. The FRI signal can be rebuilt according to the spectrum analysis as long as *K* Fourier coefficients are known. So how to obtain the Fourier coefficient of FRI signal directly is the key to the problem.

The multi-channel direct sampling framework of Fourier coefficient is as shown in Fig. 1 where T is the sampling period, as well as the time duration of finite-length FRI signal; K is an odd number, and in each channel, multiply the original signal $x(t)$ by a complex exponential function, then integrate and divide by T, finally get the Fourier coefficient vector x. However, there is a defect that the frequency of the carrier functions in each channel is different, so it requires a variety of different-frequency crystal oscillator, causing problems to the circuit design.

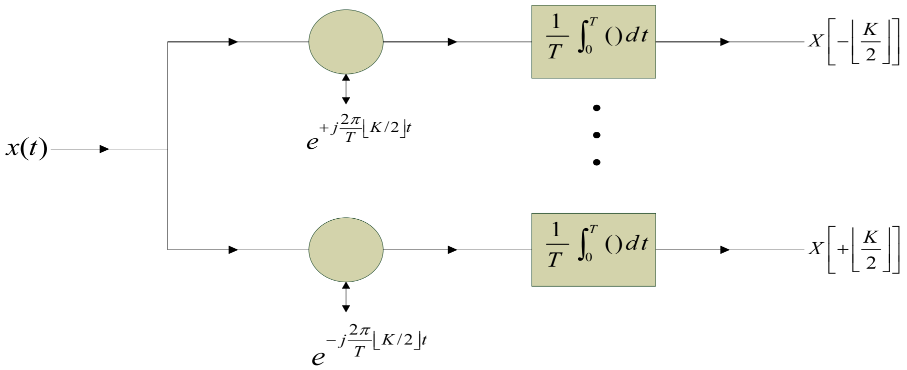


Fig. 1. Multi-channel direct sampling framework of Fourier coefficient

Multi-channel indirect sampling framework of Fourier coefficient. The improved sampling framework in Fig. 1 is as shown in Fig. 2, which is more practical than the direct sampling framework: firstly, if there is one channel out of order in the direct sampling framework as shown in Fig. 1, the accuracy of signal reconstruction will be greatly affected; but in the indirect sampling framework as shown in Fig. 2, the sampling result in each channel is the sum of several Fourier coefficients, so it has better robustness; secondly, the carrier frequency composition of each channel is the same, and only the coefficient is different, so simplifying the carrier signal generating circuit is possible. The mathematical model of sampling framework as shown in Fig. 2 is derived as follows:

Suppose that there are P channels, it can be known from Fig. 2 that the carrier signal of the i channel is:

$$s_i(t) = \sum_{k\in\kappa} s_{ik} e^{-j\frac{2\pi}{T}kt} \tag{1}$$

The coefficient of each channel is not the same; set the sampling vector, then the sampling point of the i channel is:

$$c_i = \frac{1}{T}\int_0^T x(t)\sum_{k\in\kappa} s_{ik} e^{-j\frac{2\pi}{T}kt} dt = \sum_{k\in\kappa} s_{ik} x[k] \tag{2}$$

Set S as the matrix, the (i, k) element is s_{ik}, then the formula (2) can be modified into matrix form:

$$\boldsymbol{c=Sx} \tag{3}$$

x is the Fourier coefficient vector; if and only if S column has full rank, namely P, S is left invertible, and:

$$x = S + c \tag{4}$$

The hybrid matrix S is determined by the carrier signal $s_i(t)$ we choose. It can be known that the direct sampling method of Fourier coefficient is a special case of the indirect sampling method of Fourier coefficient; P = K, and S = I.

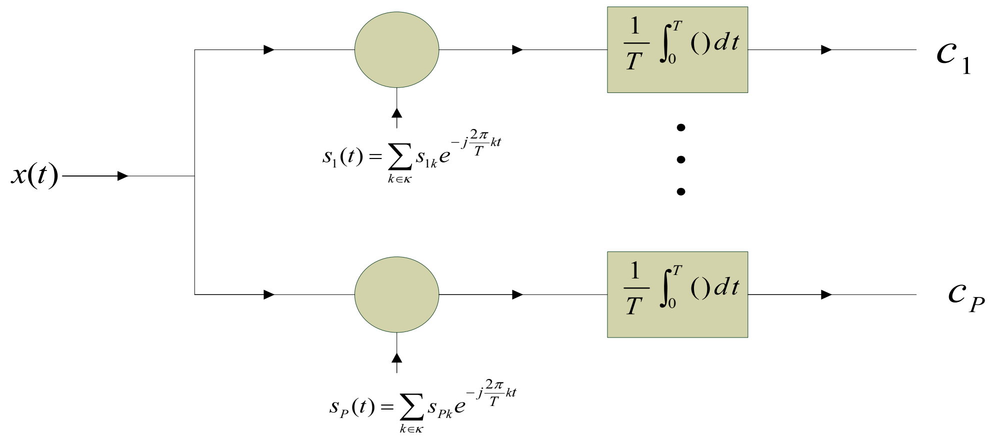


Fig. 2. Multi-channel indirect sampling framework of Fourier coefficient for finite-length FRI signal

2.2 Infinite-length FRI Signal Sampling Framework

The sampling framework is as shown in Fig. 3 with considering a finite-length FRI signal:

$$x(t)=\sum_{k\in\mathbb{Z}} a_l h(t-t_l), t_l\in\mathbb{R}, a_l\in\mathbb{C} \tag{5}$$

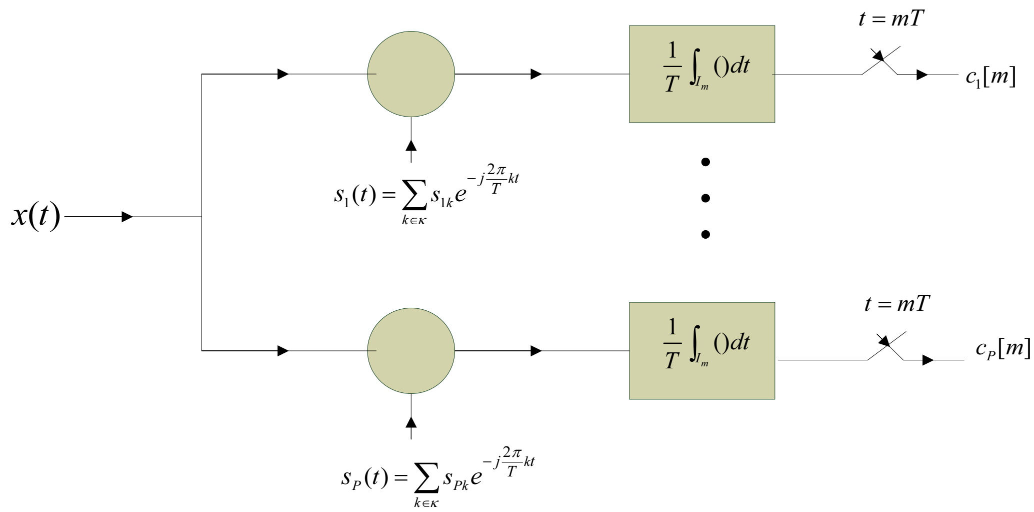


Fig. 3. Multi-channel indirect sampling framework of Fourier coefficient for infinite-length FRI signal

Assuming that as long as the number of short pulse does not exceed L at any time, then ROI of the signal is not greater than 2L/T, so the sampling period of the signal can be set as T (there are P2L sampling channels, so the relative sampling frequency is P/T); through the calculation:

$$c[m]=Sx[m] \tag{6}$$

Where $\boldsymbol{x}[m]$ is the Fourier coefficient vector during the I_m; if and only if S is left invertible, $\boldsymbol{x}[m]=\boldsymbol{S}^{+}\boldsymbol{c}[m]$. Extend the formula (4) to the infinite FRI signal, it can be obtained:

$$y[m]=H^{-1}x[m]=V(t[m])a[m] \tag{7}$$

Where $\boldsymbol{a}[m]$ and $\boldsymbol{t}[m]$ are the delay vector and amplitude vector during I_m.

2.3 Carrier Wave

In the actual circuit, the square wave $p_i(t)$ is often used to generate the carrier wave $s_i(t)$, because the generating circuit of square wave is simpler. Next is the way to obtain the actual carrier signal $s_i(t)$ from square wave $p_i(t)$ in the form of smoothing; and deduce the calculation formula of the hybrid matrix S.

The mathematical model of square wave is as shown below:

$$p_i(t) = \sum_{m\in\mathbb{Z}} \sum_{n=0}^{N-1} a_i[n]p(t - nT/N - mT) \tag{8}$$

Where $\boldsymbol{a}_i[n]$ is the coefficient vector with the length of N, and the element can be set as 1; set $P=N=K$, $p(t)$ is:

$$p(t) = \begin{cases} 1 & t \in [0, T/N] \\ 0 & t \notin [0, T/N] \end{cases} \tag{9}$$

The continuous Fourier transform is:

$$p(\omega) = \frac{T}{N} e^{-j\frac{T}{2N}\omega} \bullet \sin c(\frac{T}{2\pi N}\omega) \tag{10}$$

Generate the carrier signal si(t) from the square signal pi(t). It can be seen from the formula (8) that $p_i(t)$ is a function with the cycle of T. The periodic function can be represented by Fourier coefficient as follows:

$$p_i(t) = \sum_{k\in\mathbb{Z}} d_i[k] e^{j\frac{2\pi}{T}kt} \tag{11}$$

The Fourier coefficient $d_i[k]$ can be calculated by the following formula:

$$\mathrm{d_i}[k] = \frac{1}{T}\int_0^T p_i(t) e^{-j\frac{2\pi}{T}kt} dt \tag{12}$$

It can be seen that compared to the formula (1), the formula (11) is the infinite sum; therefore, in order to generate $s_i(t)$, an extra filter $g(t)$ is needed to filter $p_i(t)$:

$$s_i(t) = p_i(t) * \mathrm{g(t)} \tag{13}$$

However, in order to achieve this function, $g(t)$ must be restricted as follows:

$$G(\omega) = \left\{ \begin{matrix} nonzero & \omega = \frac{2\pi}{T}k, & k \in \kappa \\ 0 & \omega \neq \frac{2\pi}{T}k, & k \notin \kappa \\ arbitrary & elsewhere & \end{matrix} \right\} \tag{14}$$

Where $G(\omega)$ is the Continuous Fourier Transform of $g(t)$; according to this condition, choose $g(t)$ as the ideal low-pass filter. It can be obtained that:

$$s_i(t) = \sum_{k\in\mathbb{Z}} d_i[k] G(\frac{2\pi}{T}k) e^{j\frac{2\pi}{T}kt} = \sum_{k\in\kappa} d_i[k] G(\frac{2\pi}{T}k) e^{j\frac{2\pi}{T}kt} \tag{15}$$

It can be known through the formula (1) and formula (15) that:

$$s_{ik} = d_i[-(k - \lfloor K/2 \rfloor].G(\frac{2\pi}{T}(-(k - \lfloor K/2 \rfloor))) \tag{16}$$

The result will be used in the hybrid matrix S. The actual multi-channel indirect sampling circuit of Fourier coefficient is as shown in Fig. 4.

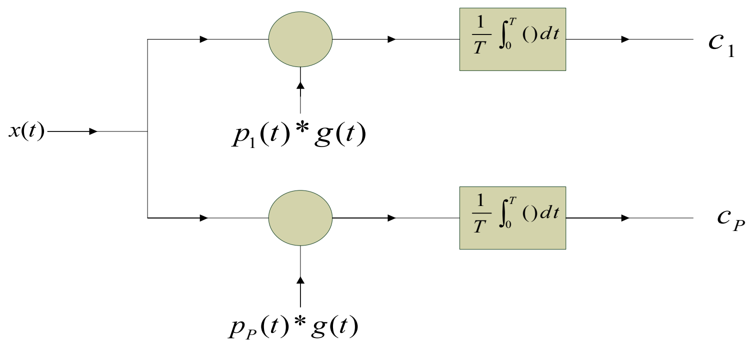


Fig. 4. Actual multi-channel indirect sampling circuit of Fourier coefficient

Calculation formula of the hybrid matrix S. Calculation formula of the hybrid matrix S is derived as follows:

Through the formulas (8) and (12), it can be obtained that:

$$\begin{aligned} d_i[k] &= \frac{1}{T}\sum_{m\in Z}\sum_{n=0}^{N} a_i[n]\int_0^T p(t-nT/N-mT)e^{-j\frac{2\pi}{T}kt}dt \\ &= \frac{1}{T}\sum_{n=0}^{N} a_i[n]\sum_{m\in Z}\int_{-mT}^{-(m-1)T} p(t-nT/N)e^{-j\frac{2\pi}{T}kt}dt \\ &= \frac{1}{T}\sum_{n=0}^{N} a_i[n]\sum_{m\in Z}\int_{-\infty}^{\infty} p(t-nT/N)e^{-j\frac{2\pi}{T}kt}dt \\ &= \frac{1}{T}\sum_{n=0}^{N} a_i[n]P(\frac{2\pi}{T}k)e^{-j\frac{2\pi}{N}kn} \end{aligned} \tag{17}$$

It can be obtained through combining with the formulas (16) and (17) that:

$$S_{ik} = \frac{1}{T}\sum_{n=0}^{N-1} a_i[n]P(\frac{2\pi}{T}k^{'})G(\frac{2\pi}{T}k^{'})e^{-j\frac{2\pi}{N}k^{'}n} \tag{18}$$

The hybrid matrix S can be expressed as:

$$S = AW\Phi \tag{19}$$

Where A is the matrix of P, the $(i,\ n)$ element $\boldsymbol{A_{in}}=\boldsymbol{a_i}[n]$; W is the diagonal matrix $W_{nk} = e^{-j\frac{2\pi}{N}k^{'}n}$ of $N{\times}K$ matrix, and the diagonal element is:

$$\Phi_{kk} = \frac{1}{T}P(\frac{2\pi}{T}k^{'})G(\frac{2\pi}{T}k^{'}) \tag{20}$$

Substitute the formula (10) into the above formula:

$$\Phi_{kk} = \frac{1}{N}e^{-j\frac{\pi}{N}k^{'}}\sin c(\frac{k^{'}}{N})G(\frac{2\pi}{T}k^{'}) \tag{21}$$

When $g(t)$ is the ideal low-pass filter, the formula (21) can be modified into:

$$\Phi_{kk} = \frac{1}{N}e^{-j\frac{\pi}{N}k^{'}}\sin c(\frac{k^{'}}{N}) \tag{23}$$

3 Implementation and Simulation of the Algorithm

3.1 Application in the A-type Ultrasound Imaging

The experiment is conducted in the form of MATLAB simulation. The imaging data is a set of one-dimensional signal from the GE Health's Vivid-I ultrasonic imaging system [8]. The original signal is polluted by the noise, so the low-pass filtering and modulation are conducted in advance, and the square wave is used in the simulation as a carrier; the specific simulation is as follows:

(1) Use the ultrasonic industrial imaging system to collect a set of one-dimensional original signal; the center frequency of the probe f_c=3.4235MHz, the sampling frequency f_s=40MHz, the imaging depth R_{max}=0.16m, the ultrasonic wave velocity in analyte C=1540, and the signal duration can be calculated as $T=2R_{max}/C=2.0810^{-4}$sec, and the number of data points in each imaging $SamplingNo\_classic=T\times f_s$=8320; this simulation only uses half data, namely $T=T/2=1.0410^{-4}$sec, $SamplingNo\_classic$= $SamplingNo\_classic/2$=4160;

(2) According to the characteristics of ultrasonic industrial probe, it can be known that $h(t)$ is appropriate as gaussian pulse, and the gaussian parameter $\sigma = 3\times 10^{-7}\,\mathrm{sec}, h(t) = e^{-\frac{t^2}{\sigma^2}}$;

(3) Use the sampling framework as shown in Fig. 2 to conduct the integral sampling for the original signal, set L=4, $P=N=K=2L oversampling+1$; the original signal noise is large, so set $oversampling$=5, then $P=N=K$=4, $\kappa = \{-20,-19,\cdots,0,\cdots,19,20\}$;

(4) Choose the square wave signal as the carrier wave; the square wave signal conforms to the formula (8) and (9), the number of square wave is 41 with the cycle of T, width of T/N, the filter $g(t)$ is the low-pass filter with the bandwidth of $2\pi/T$, only use one square wave signal generator;

(5) After the integral sampling, obtain the vector c;

(6) According to the square wave signal, write the A matrix;

(7) According to the formula (19) and (22), get the S matrix;

(8) Use the formula (4) to calculate the Fourier coefficient vector x;

(9) Use the formula $y = H^{-1}x$, obtain the vector y, and get the equation $y_k = \sum_{l=1}^{L} a_l e^{-j2\pi k t_l/\tau}, k \in \kappa$;

(10) Adopt the Annihilating Filter Method and the least square method to solve the equation set, and get the parameter $\{a_l, t_l\}_{l=1}^{4}$; the reconstruction effect is as shown in Fig. 5 where the signal size and time return to 1:

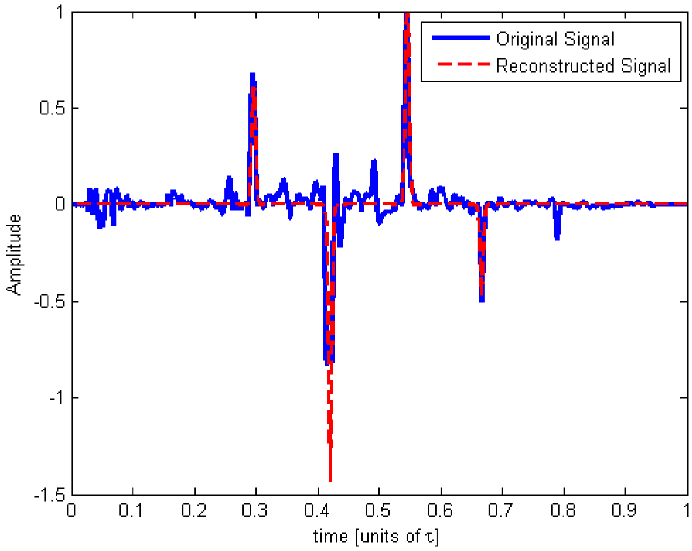


Fig. 5. signal reconstruction, P=41

It can be seen that compared to the traditional Shannon's theorems sampling, the sampling frequency of multi-channel method is only 0.024% of the traditional one ($1/T/fs$), also 0.8% lower than the SoS filter.

3.2 Compare the Anti-noise Performance with Other Similar Methods

Two carrier waves are used here, one is the square wave, and another one is SoS (Sum of Sincs) filter; at the same time, other similar methods refer to the Integrators filter and Exponential filter, two famous multi-channel Nyquist sampling methods in FRI system. The simulation process is as follows:

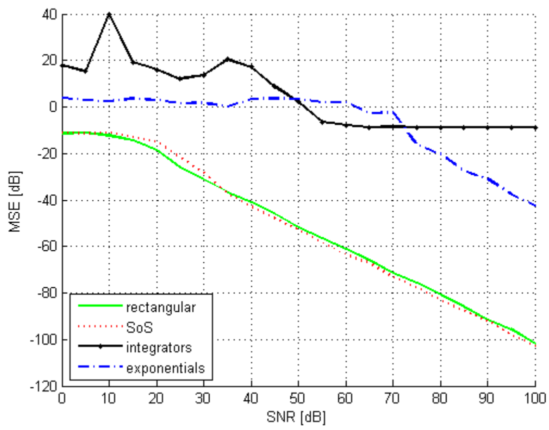
(1) The input signal is the ideal pulse signal polluted by the white noise with the signal length T=1, divided into three conditions: when the pulse number L=2, delay vector $\boldsymbol{t}$=$[0.256,\ 0.38]^T$, amplitude vector $\boldsymbol{a}$=$[1,0.8]^T$; when the pulse number L=4, delay vector $\boldsymbol{t}$=$[0.213\ 0.452\ 0.664\ 0.7453]^T$, amplitude vector $\boldsymbol{a}$=$[1\ \ 0.9\ \ 0.7\ \ 0.6]^T$; when the pulse number L=10, delay vector t conforms to the normal distribution, and the amplitudes are all set to 1;

(2) Use the multi-channel indirect sampling method of Fourier coefficient to reconstruct the signals, set P=K=N=2L+1, and simulate respectively using two carrier waves, one is the square wave, and the simulation process follows the step (3)-(10) in the simulation described above; the other one is SoS filter $g(t)=rect(\frac{t}{T})\sum_{k\in\kappa} b_k e^{j2\pi kt/T}$, set all b_k as 1, and the hybrid matrix S is calculated according to the formula $S=V(-t_s)B$;

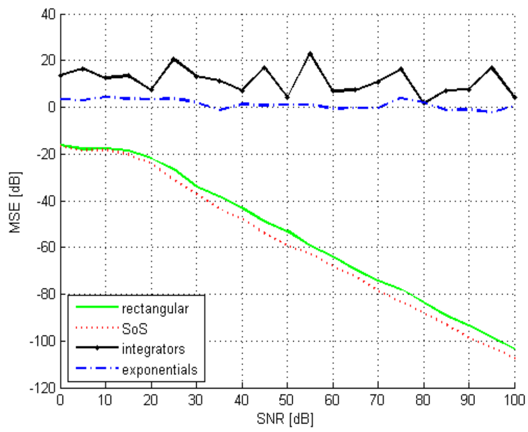
(3) Use the Integrators filter and Exponential filter to reconstruct the signal; the parameters of Exponential filter is set as α=0.2, β=0.8. Please refer to the related literature for detailed process [9-10];

(4) The simulation result is as shown in Fig. 6, the abscissa SNR is the signal-to-noise ratio defined as follows:

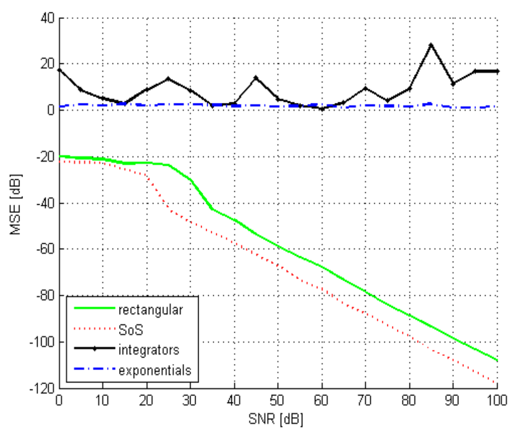
$$SNR = 10\lg\left(\frac{1}{P}\times\frac{\|c\|_2^2}{\sigma^2}\right)$$



(a) When L=2, comparison of anti-noise performance with other methods



(b) When L=4, comparison of anti-noise performance with other methods



(c) When L=10, comparison of anti-noise performance with other methods

Fig. 6. Anti-noise performance comparison of multi-channel Nyquist sampling method

C is the output vector of original signal sampling without noise pollution, ^{2} is the white noise variance. MSE is the minimum square error defined as follows:

$$MSE = 10\lg(\frac{1}{L} \times \|t = t\_est\|_2^2)$$

T is the actual delay vector, and ***t_est*** is the reconstructed delay vector;

The experiment shows that the anti-noise performance of multi-channel sampling method is much better than the other two multi-channel methods; in particular, in the case of multi-pulse signal (L=10), it has advantages of over 20db. And the anti-noise performance of SoS filter is better than that of square wave.

4 Conclusion

(1) Put forward a new kind of sampling method for FRI signal– multi-channel indirect sampling method of Fourier coefficient, introduce the property and construction methods in detail, and verify the effectiveness and stability of the method.

(2) The method can accurately and effectively reconstruct the original FRI signal, and the simulation shows that the method is feasible in industrial ultrasonic application.

(3) Compared to other commonly used FRI methods, the method has better anti-noise property; regardless of the pulse number, the method has advantage of nearly 20db in MSE with good stability.

(4) Multi-channel indirect sampling method of Fourier coefficient is a new kind of FRI method with good stability, strong anti-noise capacity, simple realizing circuit and fewer priori conditions.

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