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Abstract. Describing spatial relations is a challenging task in image understanding and contentbased image retrieval. There are only few works focused on describing complex spatial relations, and they usually use mathematical system which is not self-adapted, i.e. one model is only suitable for certain group of datasets. To address these problems, we proposed a self-adapted method for describing complex spatial relations. With our method, the complex spatial relation model can be quickly and accurately generated from very few labeled samples without prioriknowledge. The proposed method is tested on several benchmark datasets, and the experiment results demonstrate the superior performance and the robustness of our method.

Keywords: classification, complex spatial relations, feature selection, qualitative spatial reasoning

# 1 Introduction

Describing the relative position of objects arranged in complex spatial configurations is a challenging task in image understanding [1-5] and content-based image retrieval (CBIR) [6-9]. Spatial relations are pervasive and form an important part of the information process. Spatial relationships have been widely studied for more than two decades. Lots of spatial relation models were proposed, such as Interval Algebra [10], Directed Interval Algebra [11], Rectangle Algebra [12], Region Connection Calculus (RCC) [13], 4-Intersection Model [14], Disk Algebra [15], Cardinal Direction Calculus (CDC) [16].

Spatial relations in the real world may be complex. Obviously simple spatial relation models are not suitable for dealing with complex spatial scenes. Recently, there have been few works focused on complex spatial relations. Models like 9-intersection model and its evolutionary versions were used to represent complex topological relation between regions, lines or points etc. [15, 17-18]. Li et al. studied a string based method to capture the detailed topological relation between convex regions [19]. Wang et al. extended their works to simple regions and applied their method to image retrieval [20].

Traditional approaches use mathematical system to distinguish complex spatial relations. It requires a complex spatial relation model which is a manual work. A pre-designed complex spatial relation model is only suitable for certain group of datasets. They are not self-adapted models. For example, CDC [16] is not suitable for topological relations and [19-20] are not suitable for directional relations. To address these problems, we proposed a new method for describing complex spatial relations. It can be self-adapted to different types of spatial relations, such as topology, direction, distance or more complex situations. It automatically generates complex spatial relation models with machine learning methods (MLSR).

To summarize, the main contribution of our work is that we proposed a new method for describing complex spatial relations. This method can be self-adapted to different scenario and it has better performance in the configurations with complex spatial relations. With little extension, our method can be applied in sketch-based image retrieval and other similar applications.

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## 2 Spatial Relation Model

Before the main method of our work, some related preliminaries of spatial relation model are given in this section.

#### **Definition 1.** (Spatial Object Domain)

Spatial Object Domain M is all the spatial object pairs which are considered for a particular spatial relation model, normally, M is an infinite set. Spatial Relation Sample Set D is a finite subset of M, D includes all the instances recorded in the system, and the *i*th sample is  $(A_i, B_i) \in D$ .

The Spatial Objects mentioned in this paper are all region objects in two-dimensional space, and only binary spatial relations are considered.

**Definition 2.** (Single-Label Spatial Relation Model)

Single-Label Spatial Relation Model *R* on a Spatial Relation Sample Set *D* is a function:

$$R(A_i, B_i) = r, r \in B_R = \{r_1, \dots, r_n\},$$
(1)

where  $A_i$ ,  $B_i$  are two spatial objects in a binary spatial object domain M and r belongs to  $B_R$ , which is the basic relation set of R. For any sample  $(A_i, B_i) \in D$ , a label  $R(A_i, B_i)$  is attached to it.

**Definition 3.** (Multi-Label Spatial Relation Model)

Multi-Label Spatial Relation Model on a Spatial Relation Sample Set D is a function:

$$R(A_i, B_i, r) = t \in \{0, 1\}, r \in B_R = \{r_1, \dots, r_n\},$$
(2)

where  $A_i$ ,  $B_i$  are two spatial objects in a binary spatial object domain M and r belongs to  $B_R$ . For any sample  $(A_i, B_i) \in D$ , labels  $\{r \mid R(A_i, B_i, r) = 1\}$  are attached to it.

In this paper, the complexities of spatial relations come from two aspects: the complexity of spatial objects and the complexity of relations. Other than simple objects which only have shape, complex spatial objects have structures like multi-bodies, holes or nesting. The complexity of relations means that the number of the basic relations is very huge or even infinite. Spatial relations can be classified into some major categories such as topology, direction or distance.

Simple spatial relations focus on single category while complex spatial relations may combine multicategory. There are some previous works on combined spatial relations. Gerevini and Renz studied the combination of RCC and qualitative size [21]. [22-23] investigated the combination of RCC and direction. Wang et al. provided a general method for combined spatial relation model [20].

For complex spatial relations, it is very difficult to build a mathematical model. Therefore, this paper extracted features from geometrical information of spatial objects, and used features to represent complex spatial relations. A set of features  $F = (f_1, f_2, ..., f_k)$  are extracted from a Spatial Relation Sample Set *D*. **Definition 4.** (Sufficient Feature Set)

A feature set  $F = (f_1, f_2, ..., f_k)$  is sufficient for a Spatial Relation Model M with regard to a Spatial Relation Sample Set D if and only if any two samples in D with the same feature values have the same labels of M.

If a feature set satisfies Definition 4, it can be used to classify spatial relations of M in D. But in real applications, only a small amount of samples in D are labeled, i.e., it is difficult to verify Definition 4. This paper presents a general feature set, which is suitable for most known complex spatial relations between regions in 2D space. And the general feature set can be easily extended to adapt new applications.

## 3 Framework of Our Method

Aiming at distinguishing complex spatial relations, this section proposes a general machine learning method. In order to clearly expound the whole idea of this work, a flow chart in Fig. 1 shows the framework of the method.



Fig. 1. Framework of the general machine learning method

The first step of the general machine learning method is to get the general feature set. Each spatial relation sample has two spatial objects A and B. The method separates the sub-regions of each object, and then extracts the extended regions for each sub-region. The general feature set is just the prime spatial relations between each pair of regions.

The general feature set proposed in this paper is of great versatility, and can express most of the complex spatial relations in the applications at present. Furthermore, the users can also extend it or design new feature sets according to actual demands.

As we expected, the original general feature set could be extremely huge and sparse, so Multi-levels Feature Selection strategy is proposed to deal with it. State-of-the-art feature selection technologies such as minimal-Redundancy-Maximal-Relevance (mRMR) [24] and Relief Features (ReliefF) [25] are used here. According to the unique properties of our problem, the features are grouped into several levels of groups. The number of features grows exponentially when the level increases. The strategy tries the levels of feature group one by one, until the classification accuracy reaches the desired threshold.

In order to get the most accurate result, several classification algorithms are considered and tested such as Support Vector Machine (SVM) and k-Nearest Neighbor (kNN).

# 4 General Features for Complex Spatial Relation

This section proposed a general feature set for two-dimensional region objects with complex spatial relations. A feature for complex spatial relations contains three parts, as shown in Table 1.

Components of a Feature	Definition	
Sub-region	The individual parts of a complex spatial object.	
Extended region	Sensitive areas in a certain spatial relation, such as inside, outside and boundary	
Feature value (for Crisp Spatial Relation)	Intersect or not	

Table 1. Components of a feature

#### 4.1 Sub-regions

Complex region objects can be divided into several individual parts, and the method of division depends on the inside structure of the object. Some commonly used sub-regions are defined here, which can be further extended.

## Definition 5. (Whole Region)

Whole Region is the sum of all parts of an object, defined as SELF in this paper.

**Definition 6.** (Fully Connected Sub-regions)

For a multi-bodies region, its Fully Connected Sub-regions are represented by  $E_1$ ,  $E_2$ , etc. For a fully connected region, it has only one Fully Connected Sub-region, named by  $E_1$ .

#### **Definition 7.** (Hole Sub-regions)

A hole of a spatial object A is a connected finite space outside of A, the Hole Sub-regions of a spatial object are represented by  $H_1$ ,  $H_2$ , etc.

## Definition 8. (Concave Sub-regions)

If a fully connected spatial object A is not convex, then the Concave Sub-regions of object A are the areas within the convex hull of A, except for A and the holes in A. The Concave Sub-regions are represented by  $C_1$ ,  $C_2$ , et al. The indexes of Concave Sub-regions are organized in ascending order based on the X-coordinate of the top left corner of the Minimum Bounding Rectangle of each Concave Sub-region.

#### Definition 9. (Outline Sub-regions)

For a fully connected spatial object A, its Outline Sub-regions are the inside area of the convex hull and the maximum inscribed circle of A, named as  $OL_1$  and  $OL_2$  respectively. And for a multi-bodies object A, it contains only one Outline Sub-region, which is the convex hull and the area inside of it, defined as  $OL_1$ . Fig. 2 shows two examples of Outline Sub-regions. The red ring and the area inside of it in Fig. 2(a) represent  $OL_2$ , and the blue closed curves together with the area surrounded by them illustrated in Fig. 2(a) and Fig. 2(b) are symbolized as  $OL_1$ .



Fig. 2. Two examples of Outline Sub-regions

#### 4.2 Extended Regions

Three categories of spatial relation, topology, direction and distance, which cover most of the known spatial relation models, are considered in this paper to define sensitive areas. After referring to previous works, this paper provides some possible sensitive areas for determining the spatial relations. Since sensitive areas are defined by extending the sub-regions of an object, they are named as Extended Regions.

## **Definition 10.** (Extended Regions for Topology)

For a sub-region S, its Extended Regions for Topology are I, B and O. As is shown in Fig. 3, I is the inside area of S, i.e., the closure of S minus the boundary of S, B is the boundary of S. And O is the outside area of S, i.e., the whole 2D space minus the closure of S.



Fig. 3. An example of extended regions for topology

## **Definition 11.** (Extended Regions for Distance)

For a sub-region S, supposing  $(x_0, y_0)$  is the center of its Minimum Circumcircle of S, its Extended Regions for Distance can be defined in Table 2.

Table 2. Extended regions for distance

Extended Region	Definition
DR1	$\{(x, y)   (x - x_0)^* (x - x_0) + (y - y_0)^* (y - y_0) \le R^*R \}$
DR2	$\{(x, y)   R * R < (x - x_0) * (x - x_0) + (y - y_0) * (y - y_0) \le (9/4) * R * R \}$
DR3	$\{(x, y)   (9/4) R^* R < (x - x_0) (x - x_0) + (y - y_0) (y - y_0) \le 4 R^* R \}$
DR4	$\{(x, y)   4*R*R < (x - x_0)*(x - x_0)+(y - y_0)*(y - y_0)\}$

Definition 12. (Extended Regions for Inner Distance)

For a sub-region *S*, it has four Extended Regions for Inner Distance, which are successively defined as IR1, IR2, IR3 and IR4 from the periphery to the center of *S*. Supposing *R* is the radius of the maximum inscribed circle of *S* and the coordinate of the center of the circle is  $(x_0, y_0)$ , then Extended Regions for Inner Distance can be defined in Table 3.

Table 3. Extended regions for inner distance

Extended Region	Definition
IR1	$\{(x, y)   R^*R < (x - x_0)^* (x - x_0) + (y - y_0)^* (y - y_0), (x, y) \in S\}$
IR2	$\{(x, y)   (4/9) R^* R < (x - x_0) (x - x_0) + (y - y_0) (y - y_0) \le R^* R \}$
IR3	$\{(x, y)  (1/9) R^R < (x - x_0) (x - x_0) + (y - y_0) \le (4/9) R^R \}$
IR4	$\{(x, y)   (x - x_0)^* (x - x_0) + (y - y_0)^* (y - y_0) \le (1/9)^* R^* R\}$

## Definition 13. (Extended Regions for Cardinal Direction)

For a sub-region S,  $(x_1, y_1) - (x_2, y_2)$  is its Minimum Bounding Rectangle, which is also defined as OC, i.e., the center part of sub-region S. The definitions of the Extended Regions for Cardinal Direction are shown in Table 4.

Table 4. Extended regions for cardinal direction

Extended Region	Definition	Extended Region	Definition
OC	$\{(x, y)   x_1 \leq x \leq x_2, y_1 \leq y \leq y_2\}$	OE	$\{(x, y) x_2 < x, y_1 \le y \le y_2\}$
ONE	$\{(x, y) x_2 < x, y_2 < y\}$	ON	$\{(x, y) x_1 \le x \le x_2, y_2 < y\}$
ONW	$\{(x, y) x < x_1, y_2 < y\}$	OW	$\{(x, y) x < x_1, y_1 \leq y \leq y_2\}$
OSW	$\{(x, y) x < x_1, y < y_1\}$	OS	$\{(x, y) x_1 \le x \le x_2, y < y_1\}$
OSE	$\{(x, y) x_2 < x, y < y_1\}$		

## Definition 14. (Extended Regions for Inner Direction)

For a sub-region *S*, the Minimum Bounding Rectangle is divided into nine equal parts, three on each row. The definitions of each part are shown in Table 5.

Extended Region	Definition	Extended Region	Definition
IC	$\{(x, y)   x_1 + (x_2 - x_1)/3 \le x \le x_2 - (x_2 - x_1)/3, y_1 + (y_2 - y_1)/3 \le y \le y_2 - (y_2 - y_1)/3\}$	IE	$\{(x, y)   x_2 - (x_2 - x_1)/3 < x \le x_2, y_1 + (y_2 - y_1)/3 \le y \le y_2 - (y_2 - y_1)/3\}$
INE	$ \{ (x, y)   x_2 - (x_2 - x_1)/3 < x \le x_2,  y_2 - (y_2 - y_1)/3 < y \le y_2 \} $	IN	$ \{(x, y)   x_1 + (x_2 - x_1)/3 \le x \le x_2 - (x_2 - x_1)/3, y_2 - (y_2 - y_1)/3 \le y \le y_2 \} $
INW	$\{(x, y)   x_1 \le x < x_1 + (x_2 - x_1)/3,  y_2 - (y_2 - y_1)/3 < y \le y_2\}$	IW	$\{(x, y)   x_1 \le x < x_1 + (x_2 - x_1)/3, y_1 + (y_2 - y_1)/3 \le y \le y_2 - (y_2 - y_1)/3 \}$
ISW	$\{(x, y)   x_1 \le x < x_1 + (x_2 - x_1)/3,  y_1 \le y < y_1 + (y_2 - y_1)/3\}$	IS	$\{(x, y)   x_1 + (x_2 - x_1)/3 \le x \le x_2 - (x_2 - x_1)/3,  y_1 \le y < y_1 + (y_2 - y_1)/3 \}$
ISE	$\{(x, y)   x_2 - (x_2 - x_1)/3 < x \le x_2,  y_1 \le y < y_1 + (y_2 - y_1)/3 \}$		

Table 5. Extended regions for inner direction

# 4.3 General Features

Regarding the spatial relations of spatial object A and B, each feature is a quadruple  $FSR_i = (SR_1, ET_1, SR_2, ET_2)$ , where  $SR_1$ (or  $SR_2$ ) is a sub-region of A(or B) and  $ET_1$ (or  $ET_2$ ) is an extended region of  $SR_1$ (or  $SR_2$ ). The feature values are defined in Table 6.

Table 6. Values of general features

Feature Value	Definition
1	A.SR <sub>1</sub> .ET <sub>1</sub> intersects with $B.SR_2.ET_2$
0	$A.SR_1.ET_1$ does not intersect with $B.SR_2.ET_2$
#	$(SR_1, ET_1, SR_2, ET_2)$ does not exist for $A, B$

The definition in Table 6 is only suitable for crisp spatial relations. As for fuzzy spatial relations, feature value is the ratio between the areas of intersection and union of two considering parts, see Eq. (3). The feature value ranges from 0 to 1. Since fuzzy and crisp spatial relations are not different much, this paper only discusses crisp spatial relations in the later parts.

$$Value(FSR_i) = \frac{area(A.SR_1.ET_1 \cap B.SR_2.ET_2)}{area(A.SR_1.ET_1 \cup B.SR_2.ET_2)}.$$
(3)

## 5 Experiments

## 5.1 Benchmark Datasets for Spatial Relation Classification

To the best of our knowledge, there has been no dataset specifically designed for classification of spatial relations. To test the proposed method empirically, five datasets are adopted in our experiments.

## 5.1.1 Dataset 1

This is a single-label dataset that comes from CAD data, which describes the spatial relations between two gears. The gears are divided into 4 types according to their inner and outer shapes, i.e., toothed inside & toothed outside; toothed inside & smooth outside; smooth inside & toothed outside; smooth inside & smooth outside. This paper analyzed all types of gears and the engagement of every possible pair of gears. Although the mechanical properties of the wheel gears can be deduced from their geometric spatial relations, the existence of complex spatial relations makes it impossible to directly estimate the engagement of the gears by the existing spatial relation models. Table 7 gives the definitions of the labels in Dataset 1 and Fig. 4 shows some samples of the dataset.

Relation Label	Relative Position of Gear <i>A</i> (Red) and Gear <i>B</i> (Blue)	Engagement of the Gears: Meshed or Out of Mesh
1	B inside A	Meshed
2	<i>B</i> inside <i>A</i>	Out of Mesh
3	A inside B	Meshed
4	A inside B	Out of Mesh
5	A outside B	Meshed
6	A outside B	Out of Mesh

 Table 7. Description of labels in Dataset 1



Fig. 4. Some samples in Dataset 1

## 5.1.2 Dataset 2

Dataset 2 is a multi-label spatial relation dataset which comes from real geographical data. This dataset describes the spatial relations between two geographical entities on direction, i.e., the direction of entity B(blue) with respect to entity A(red). The spatial relation labels in this dataset are shown in Table 8, and Fig. 5 shows some examples of Dataset 2.

Direction of B	8 with	Inner D	irection	Out	er Direction	
respect to	A	Label		Label		
Center		1	l	11		
East		2	2	12		
Northeas	t	3	3	13		
North		4	1		14	
Northwes	t	4	5		15	
West	-	ť	5		16	
Southwes	t		7		17	
South		, S	2		18	
Southeas	+	(	)		10	
Southeas	ι		7		19	
1	2	3	4	4,5	5,6	
7	8	9		12	13	
<b>\$</b>	*****	\$ ar	4	8	*	
14	14,15,16	15,16	17	18	18,19	
		*			*	
11,12,13,14,15,16,17,18,19	1,6,7,8	11,12,13,18,19	13,14,15	19	11,12,18,19	
11 14 15 16	17 18 19	12.13	15 16 17		11.14.15.16	
11,14,15,16	17,18,19	12,13	15,16,17	13	11,14,15,16	

Table 8. Description of labels in Dataset 2

Fig. 5. Some samples in Dataset 2

## 5.1.3 Dataset 3

Dataset 3 is another multi-label dataset which also comes from geographical data. Dataset 3 focuses on topological relation of two geographical entities. Table 9 shows the description of the labels in Dataset 3 and Fig. 6 shows some examples of the dataset.

Label			Descriptio	n		
1	A is in the center part of B					
2	A is at the boundary of B					
3			B is in the center	part of A		
4			<i>B</i> is at the bound	arv of A		
5			A is surrounded	d by B		
6			<i>B</i> is surrounded	$\frac{1}{1}$ by A		
7			A is next to	R		
8			A is near l	R		
0	A is far away from $R$					
 7			A is lai away ii			
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4	8	9	7	1	4	

## **Table 9.** Description of labels in Dataset 3

Fig. 6. Some samples in Dataset 3

## 5.1.4 Summarization

Table 10 summarizes all the datasets in our experiments, in which there are some special requirements for the training samples here. For instance, all kinds of labels must be included. Deflective training samples may lead to a wrong model. And the training samples must be abundant enough to cover all possible

situations. So our experiments design fixed training samples for each dataset to ameliorate the problems mentioned above. This arrangement is logically suitable in real applications. If a machine wants to learn a new spatial relation model without any priori-knowledge, it must be given enough labeled data.

Dataset	Total Samples	Samples for Training	Samples for Test	Features	Classes
1	3000	2000	1000	189225	6
2	6000	4000	2000	215296	18
3	6000	4000	2000	215296	9

Table 10. Summary of Datasets in the Experiments

## 5.2 Experimental Results

#### 5.2.1 Feature Extraction and Selection

Table 11 lists the number of sub-regions for each object and the total feature number in each dataset. As shown in Table 11, the number of features for the datasets is too large, so it is necessary to use the proposed Multi-levels Feature Selection strategy to reduce the number of features. Taking Dataset 1 as an example, the feature selection algorithm mRMR and the classifier SVM are used for machine learning. For simplicity, the number of the selected features is set as the constant of 100. The classification accuracy threshold for Multi-levels Feature Selection is set as 95%, i.e., if the classification accuracy reaches the threshold, the current sub-regions group combination is suitable for the dataset. Both 100 and 95% are empirical values from the experiments.

Table 11. Number of Sub-regions and Features in Each Dataset

Dataset	Number of Sub-regions for Object A	Number of Sub-regions for Object <i>B</i>	Total number of Features
1	5/15	5/15	189225
2	4-16	4-16	215296
3	4-16	4-16	215296

The classification accuracies of each combination for Dataset 1 are shown in Table 12.

Combination	Total Features under the Combination	Accuracy on Training data
COM1	7569	100.0%
COM2	24389	-
COM3	21025	-
COM4	105125	-
COM5	189225	-

Table 12. Classification accuracies of each combination of the sub-regions group in Dataset 1

As is shown in Table 12, the combination COM1 is enough to get a satisfying result for Dataset 1, so the experiment did not check combinations from COM2 to COM5, and for the other four datasets, similar results were obtained, as listed in Table 13. When the experiment tried other machine learning algorithms, the same final suitable combinations and the same final accuracy on training data were achieved for all datasets, so we did not list them.

Table 13. Feature selection algorithms and classifiers for each dataset

Dataset	Feature Selection Algorithm	Classifier	Final Suitable Combination	Final Accuracy on Training data
1	mRMR	SVM	COM1	100.0%
2	MLNB	Naïve Bayes	COM3	100.0%
3	MLNB	Naïve Bayes	COM3	100.0%

All the selected features cannot be listed for the datasets, so the experiment uses two tables to analysis the semantic relationship between the selected features and the dataset. The results in each dataset using

different algorithms are similar, so only the results using the algorithms in Table 13 are presented. Table 14 and Table 15 show the proportions of the sub-regions types and extended regions types of each object (A or B) after feature selection in each dataset. The feature selection algorithms are listed in Table 13.

Dataset —	Whole Regions		Fully Connected		Hole		Outline		Concave	
	A	В	A	В	Α	В	A	В	Α	В
1	61	62	13	12	12	13	14	13	0	0
2	59	55	7	9	0	0	34	36	0	0
3	42	62	27	12	0	13	31	13	0	0

Table 14. Proportions of the sub-regions types for each object in each dataset after feature selection

**Table 15.** Proportions of the Extended Regions Types for Each Object in Each Dataset after Feature

 Selection

Dataset —	Topology		Distance		Inner Distance		Cardinal Direction		Inner Direction	
	A	В	Α	В	Α	В	A	В	A	В
1	32	33	35	34	33	33	0	0	0	0
2	0	82	0	11	0	7	54	0	46	0
3	50	50	43	43	7	7	0	0	0	0

As is seen, the sub-regions types and extended regions types in Table 14 and Table 15 are consistent with the semantics in each dataset, although the method did not provide any priori knowledge.

For example, the extended regions types for Dataset 1 in Table 15 do not contain Cardinal Direction or Inner Direction, while we only care about the engagement of two gears and direction is apparently not our concern in this spatial relation model. Furthermore, as shown in Table 14, even though the wheel gears could contain many teeth, it is unnecessary to check the Concave Sub-regions one by one to distinguish the engagement of the two gears in our general feature sets. Using the Outline Sub-regions and Hole Sub-regions together with Inner Distance Extended Regions is more efficient.

As is shown in Table 15, the extended regions of object A in Dataset 2 are all of direction types, while the ones of object B are not. This is consistent with the potential descriptions of the labels in Dataset 2, i.e. the labels are described as the direction of B with respect to A.

For Dataset 3, the sub-regions types in Table 14 can be summarized as total region types, and there are only Topology, Distance and Inner Distance types of extended regions in Table 15. It is perfectly agreed with the design of Dataset 3, because the labels in Dataset 3 are only related to topology and distance. Furthermore, the most interesting thing is that although Dataset 2 and Dataset 3 share a lot of geometric properties, their feature selection results are quite different. And classification results for both of them are 100%. This means that our general features and feature selection method can deeply look into the hidden semantic information in the labels.

From the above, it can be concluded that the machine learning method can obtain the similar conclusion of manual analysis when dealing with data without any priori-knowledge.

Furthermore, for each of the three datasets, the final feature set has been verified by Multi-levels Feature Selection with full samples (both training sample and test sample). They all satisfied Definition 4, i.e. they are sufficient for the whole dataset. It means that by using the selected features and proper learning algorithms, 100% classification accuracy on the whole dataset is possible. And the test results will confirm it in the next subsection. However, it only proves that our general features and Multi-levels Feature Selection does well in these datasets, because in real applications Definition 4 cannot be verified for unlabeled samples.

## 5.2.2 Classification

In the above steps, the final feature combination for each dataset is obtained in Table 13. Then with fixed feature selection algorithms and classification algorithms (also listed in Table 13), the experiment gets 100% accuracies of classification on the training data. Furthermore, these classifiers are tested with the test samples of each dataset, and also have 100% accuracies of classification for each dataset.

The experiment also tests several other combinations of feature selection algorithms and classification algorithms. The results show that although the feature selection and classification algorithms shift, the

final results are similar. So if using 100% of the training samples, 100% accuracies of classification can always be achieved. This shows the robustness of our general features.

Next the experiment will test what happen if using parts of the training samples. Fig. 7 shows the performance of some representative groups of methods, which are more efficient than their own types of methods. For example, RF-LP is a representative of multi-label feature selection algorithms that use the problem transformation approach to simplify the multi-label problems.



Fig. 7. Classification accuracy of the method in each dataset

The three line charts in Fig. 7 show the comparison of the performances of each group of methods used in the corresponding dataset. The horizontal axis refers to the proportion of the training data that we used, and the vertical axis represents the classification accuracy.

As is seen, when the proportion of the training data is near 100%, 100% accuracy can be achieved for all methods and all datasets. However, when the proportion of the training data is reduced, in can be found that the performance for each group of methods is slightly different. It can be concluded from Fig. 7(a) that for single-label dataset, mRMR together with SVM obtains better results than other groups of methods. For multi-label datasets, MLNB outperforms the other algorithms. Furthermore, the line chart in Fig. 7(a) has the greatest classification accuracy, and the result in Dataset 2 has the poorest classification accuracy.

## 5.2.3 Comparing with Related Works

To show the efficiency of our spatial relation features, some relevant works in the literature such as Adaptive Hierarchical Density Histogram (AHDH) [26], Directional Enlacement Histograms (DEH) [1], and Versatile Relative Position Descriptor (VRPD) [3] are compared with our proposed algorithms. Since the labeled dataset is very small, deep learning is unsuitable. The dataset uses 70% of the training samples and Fig. 8 shows that, the classification accuracy of MLSR is better than that of AHDH, DEH, and VRPD on all the test dataset. That is because MLSR is specially designed for capturing spatial relations. The results in Fig. 8 also indicate that conventional machine learning and image classification technologies cannot be directly applied to complex spatial relations classification, and our general features extracted by MLSR for complex spatial relation provide an efficient way to solve this problem.



Fig. 8. The classification accuracy of proposed algorithm with relevant works

## 6 Conclusion

A new scheme to represent complex spatial relations based on machine learning methods was proposed in this paper. More specifically, this paper first designed a set of general spatial relation features by considering different types of spatial attribute. With the general feature set, most of the potential spatial relations can be theoretically represented. Then, a number of feature selection strategies were employed to search a small-scale feature set which really provided crucial feedback to the spatial relation classification. Finally, the classification algorithm was implemented for spatial relation classification tasks.

Unlike the previous approaches which built algebra based systems, this paper mainly employs machine learning methods to classify complex spatial relations. As a distinct advantage, the complex spatial relation models can be automatically set up without any priori-knowledge. This subsequently leads to that our method is very suitable for the environment with rich data and deficient knowledge to get spatial relations. Experiments on benchmark datasets show that our method can be adapted to different types of data and achieve high accuracy.

As a future work, this work can be applied to sketch-based image retrieval. Spatial relation is an important element in sketch-based image retrieval, but the spatial relations are very complex and domain related. However, now classifier for complex spatial relation can be trained with small scale manual labeled data. Then labels for database can be obtained, and we can further query sketch image and calculate similarity for spatial relation. Combining traditional image retrieval strategies, a better sketch-based image retrieval system will be built.

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