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Least-squares-based System Error Estimation Using ADS-B Measurements and Its Application to Three-dimensional Radar

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Abstract. Three-dimensional radar combined with an automatic dependent surveillance-broadcast (ADS-B) device is a typical multisensor data-fusion system. To estimate the system error of three-dimensional radar, a least-squares-based radar system error method (FL-LSr) using ADS-B measurements is proposed in a local rectangular radar coordinate system. In the proposed method, a unified observed model is established first, and the radar measurements and ADS-B measurements are transformed into the unified coordinated system. Then, the outliers in radar measurements are eliminated according to the 3σ rule. The corresponding sampling time of radar measurements is regarded as the reference time, and interpolation from straight-line fitting is used to reconstruct the ADS-B measurements. Additionally, the radar measurements and ADS-B are fitted to two straight-lines, and the angle between these two straight-lines is applied to compensate the azimuth components of radar measurements. Finally, the least squares estimation algorithm is utilized to estimate the system error of the three-dimensional radar. Experimental results with real data illustrate that the proposed method can estimate radar system error effectively and accurately compared with traditional radar system error estimation methods.

Keywords: ADS-B, least squares estimation, multi-sensor data fusion, multi-target tracking, time registration

1 Introduction

The tracking system established by radar and an automatic dependent surveillance-broadcast (ADS-B) device is a typical multisensor data system. Estimating the system error of three-dimensional radar is a key problem in multisensor data fusion [1-2]. There exist two types of errors in observation processes, namely, radar random error and radar system error [3-5]. Radar random error is generated by random observation noise and target random motion, which can be eliminated by various filters [6-7]. The radar system error is caused by measurements, servo systems, antennas and other factors, and it cannot be eliminated by filtering. The radar system error is generally assumed to be a determinate error, and its

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influence needs to be eliminated. Thus, one needs to estimate radar system error and apply the estimation results to compensate radar measurements. The existence of system error not only directly reduces the tracking accuracy of sensors but also causes serious failures of track associations between measurements from the same target among observed multiple targets, even generating several trajectories of the same targets. In this situation, it is difficult to form a unified and accurate tracking situation. Therefore, estimating the system error of three-dimensional radar is the primary problem of multisensor data fusion for a multisensor tracking platform.

An aerial surveillance system is established by ADS-B devices, various radars and other sensors. This system utilizes the technologies of data fusion to process time-sequential measurements from multiple sensors; it then obtains more accurate and wide-coverage surveillance information than it could from a single sensor. Consequently, it can realize the joint surveillance of cooperative and noncooperative targets and obtain a consistent and global estimate of various targets in the observed field. ADS-B is an advanced aerial target surveillance technology. It utilizes high-precision airborne GPS and information generated by other airborne equipment as data sources and uses ground-to-air and air-to-air chains for communication. Moreover, ADS-B receives broadcast information to other aircraft while it transmits its own position, speed and other status parameters to other equipment in the air or on the ground. Thus, use of the ADS-B technologies can achieve the global perception of flight situations in a surrounding surveillance area. Compared with radar, ADS-B is low-cost and high-precision, and it can continuously monitor aerial targets. Therefore, an ADS-B aided radar system error estimate can improve the tracking accuracy effectively for noncooperative targets.

The existing error-estimation methods generally assume that the radar system error is fixed; they include the real-time accuracy control quality (RTQC) method, the least-squares (LS) method, the generalized least-squares (GLS) method, the earth-centered earth-fixed-based GLS (ECEF-GLS) method, the maximum likelihood (ML) method, the expansion dimension (ED) method, and other improved related methods [8-11]. General characteristics of radar networking systems are as follows: (1) the system errors of various radars are different; (2) the farther the radar detection distance, the greater the observation error; (3) because different radars possess sampling times or sampling intervals, it is easy to introduce errors when one carries out time registration for radar measurements with target situations of high motion speed and uncertain motion models. Based on these facts, and noting that the estimation methods based on a least-squares model mostly ignore the effects of second- and higher-order terms, their estimation accuracy is also reduced to some degree.

Because the system error estimation is mutually related to track association, some scholars jointly estimate system error by combining it with track association. A multiradar anti-bias track association based on reference topology features is proposed in [12]; it can solve the multiradar track association in situations with system bias. An anti-bias track association method based on distance detection is developed in [13] for an anti-bias error problem in which sensor timing errors and high target densities exist in complex tracking scenarios. Based on the statistical characteristics of a Gaussian random vector, a multiradar antibias track association method is presented for complex situations in [14]. A bias-error track association method based on topology distances is proposed in [15] that introduces the topology information of targets. In addition, the incorporation of neural network algorithms into sensor system error estimation is a good solution. Unfortunately, these types of estimation methods are complex, and they are restricted in realistic situations [16].

In practice, if the real position of a cooperative target is known, the estimation of radar system error becomes relatively simple [17]. An ADS-B device is based on the technologies of GPS. It can provide accurate positions of a moving cooperative target, which are very close to the real positions of the target. Additionally, compared with radar measurements, ADS-B measurements are denser and more accurate [18]. Exploiting the advantages of ADS-B, a straight-line-fitting-based system error estimation method (FL) is proposed in [19] from a graphics perspective. Although the FL method is relatively simple and practical, its estimation accuracy should be further improved. The joint system error estimation method using ADS-B measurements is presented for the joint platform of radar and ADS-B in [20]. However, the calculation complexity of this estimation method is too large.

For the joint-tracking platform of three-dimensional radar and ADS-B, a least-squares-based system error estimation method using ADS-B measurements is proposed on basis of [21]. In the proposed method, the radar measurements and ADS-B measurements are preprocessed, the ADS-B measurements are regarded as the real positions of a moving target, and the least-squares algorithm is utilized to estimate the radar system error. Finally, an experiment of real test data was applied to illustrate the feasibility and effectiveness of the proposed method.

The remainder of this paper is organized as follows: in Section 2, the unified observation model for three-dimensional radar and ADS-B device is established; in Section 3, the main steps of preprocessing radar measurements and ADS-B measurements are given; in Section 4, a least-squares-based radar system error estimation method is developed; Section 5 presents the experimental results and the performance comparison with other four methods; and finally, the conclusions are provided in Section 6.

2 Unified Observation Model for Three-dimensional Radar and ADS-B Device

As shown in Fig. 1, a unified observed model for a radar and ADS-B device is established to observe a cooperative target. Generally, two sets of radar measurements and ADS-B measurements from a cooperative target in common observed time are utilized to estimate the radar-system error, and then, the estimated system error is applied to compensate the radar measurements, which consist of range, azimuth and height components.

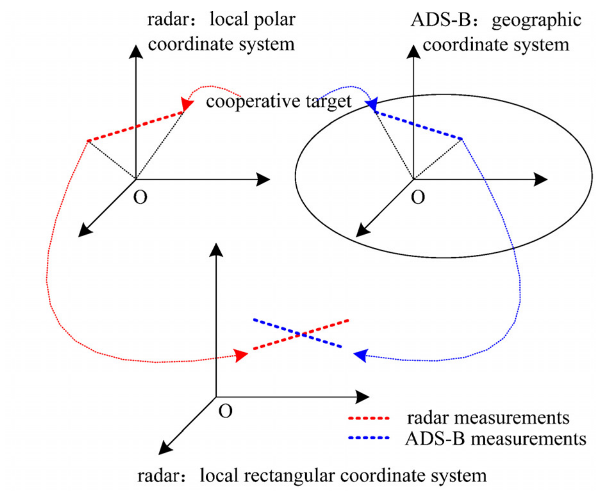


Fig. 1. Unified observed model for three-dimensional radar and ADS-B device

For ease of description, the measurements from radar and ADS-B are denoted as radar measurements and ADS-B measurements. Moreover, the radar measurements and ADS-B measurements are from a cooperative target. In other words, the radar measurements are associated with the ADS-B measurements in this paper. Theoretically, radar measurement $\boldsymbol{z}_{R,i}$ consists of a real target position $\boldsymbol{x}_{R,i}$, radar system error $\boldsymbol{b}_R$, and random error $\boldsymbol{e}_{R,i}$, and can be expressed by

$$\boldsymbol{z}_{R,i} = \boldsymbol{x}_i + \boldsymbol{b}_R + \boldsymbol{e}_{R,i} \tag{1}$$

Here, $\boldsymbol{z}_{R,i} = [r_{R,i}, \theta_{R,i}, h_{R,i}]^{\mathrm{T}}$, $\boldsymbol{x}_i = [r_i, \theta_i, h_i]^{\mathrm{T}}$, $\boldsymbol{b}_R = [\Delta r_R, \Delta\theta_R, \Delta h_R]^{\mathrm{T}}$, and $\boldsymbol{e}_{R,i} = [v^r_{R,i}, v^\theta_{R,i}, v^h_{R,i}]^{\mathrm{T}}$; $r_{R,i}$, $\theta_{R,i}$, and $h_{R,i}$ denote the corresponding range, azimuth and height components of radar measurement $\boldsymbol{z}_{R,i}$; r_i, θ_i, and h_i, Δr_R, $\Delta\theta_R$, and Δh_R, and $v^r_{R,i}$, $v^\theta_{R,i}$, and $v^h_{R,i}$ denote the corresponding ranges, azimuths and height components of real target position $\boldsymbol{x}_{R,i}$, system error $\boldsymbol{b}_R$, and random error $\boldsymbol{e}_{R,i}$, respectively, and they are assumed to be subject to zero-mean Gaussian noise, namely,

$$\begin{bmatrix} v^r_{R,i} \\ v^\theta_{R,i} \\ v^h_{R,i} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\delta^r_R)^2 & 0 & 0 \\ 0 & (\delta^\theta_R)^2 & 0 \\ 0 & 0 & (\delta^h_R)^2 \end{bmatrix} \right) \tag{2}$$

where δ^r_R, δ^θ_R, and δ^h_R denote constants.

Because ADS-B measurement $z_{A,i}$ has high accuracy, it can be regarded as the real position of a target.

Then, Eq. (1) can be replaced by

$$\boldsymbol{z}_{R,i}=\boldsymbol{z}_{A,i}+\boldsymbol{b}_R+\boldsymbol{e}_{R,i} \tag{3}$$

Rearranging, we have

$$\boldsymbol{b}_R=\boldsymbol{z}_{R,i}-\boldsymbol{z}_{A,i}-\boldsymbol{e}_{R,i} \tag{4}$$

3 Preprocessing Radar Measurements and ADS-B Measurements

3.1 Coordinate Transformation

Because ADS-B measurements utilize the geographical coordinates, one needs to transform ADS-B measurements from the geographical coordinate system to the local rectangular coordinate system for uniform processing of radar measurements and ADS-B measurements. Assume $(\lambda_{A,i},\phi_{A,i},h_{A,i})$ are the geographical coordinates of an ADS-B measurement; its corresponding local rectangular coordinates can be calculated by

$$[x_{A,i},y_{A,i},z_{A,i}]^{\mathrm{T}}=D(\lambda_R,\phi_R)\left[P(\lambda_{A,i},\phi_{A,i},h_{A,i})-P(\lambda_R,\phi_R,h_R)\right] \tag{5}$$

Here, (λ_R,ϕ_R,h_R) is the geographic coordinate of the radar position; $D(\lambda_R,\phi_R)$ and $P(\lambda_R,\phi_R,h_R)$ can be respectively calculated by

$$D(\lambda_R,\phi_R)=\begin{bmatrix}-\sin\lambda_R & \cos\lambda_R & 0\\ -\sin\phi_R\cos\lambda_R & -\sin\phi_R\sin\lambda_R & \cos\phi_R\\ \cos\phi_R\cos\lambda_R & \cos\phi_R\sin\lambda_R & \sin\phi_R\end{bmatrix} \tag{6}$$

$$P(\lambda,\phi,h)=\begin{bmatrix}(N+h)\cos\phi\cos\lambda\\ (N+h)\cos\phi\sin\lambda\\ N[(1-\rho^2)+h]\sin\phi\end{bmatrix} \tag{7}$$

where $N=R_e\big/\sqrt{1-\rho^2\sin^2\varphi}$, $\rho=\sqrt{1-r_e^2\big/R_e^2}$ is the eccentricity of the earth, $r_e=6356.752$ km is its short-axis radius, and $R_e=6378.137$ km is its long-axis radius.

Furthermore, ADS-B measurements should be transformed from the local rectangular coordinate system to a local polar coordinate system. Polar coordinates $(r_{A,i},\theta_{A,i},h_{A,i})$ corresponding to ADS-B measurement $(x_{A,i},y_{A,i},z_{A,i})$ can be expressed by:

$$\begin{cases} r_{A,i}=\sqrt{x_{A,i}^2+y_{A,i}^2+z_{A,i}^2}\\ \theta_{A,i}=\arctan(x_{A,i}\big/y_{A,i})\\ h_{A,i}=z_{A,i}\end{cases} \tag{8}$$

Here, $r_{A,i}$, $\theta_{A,i}$, and $h_{A,i}$ correspond to the range, azimuth and height components of the ADS-B measurement.

For a three-dimensional radar, the range and azimuth components of a radar measurement are more accurate than its height component. To avoid the incorporation of height error into the other two components and to reduce the complexity of estimation algorithms, the local rectangular coordinate system can be approximately described by

$$\begin{cases} x_i' = r_i \sin\theta_i \\ y_i' = r_i \cos\theta_i \\ z_i' = h_i \end{cases} \tag{9}$$

Here, r_i, θ_i and h_i denote the range, azimuth and height components of a radar measurement in the polar coordinate system; x_i', y_i' and z_i' denote the corresponding components on the x-, y- and z-axes, respectively.

For ease of discussion, the radar measurements and ADS-B measurements mentioned in following paragraph are calculated by Eq. (9). They are assumed to have been obtained from a cooperative target with a straight-line motion model. The assumption also satisfies the actual situations. Although the motion mode of a moving target possesses uncertainties and its trajectory generally approximates a curved line, one can obtain ADS-B measurements in its straight section. Noting the motion inertia of aerial targets, the trajectory of a moving target can be easily divided into straight sections for relatively short times [19]. In addition, one can utilize the ADS-B measurements of a civilian airliner as the cooperative target; there exist many straight sections of this target. Therefore, the above assumption is reasonable in actual situations.

3.2 Time Registration

In actual situations, the sampling times of radar measurements and ADS-B measurements are approximately periodic. Generally, the sampling times of ADS-B measurements are denser than those of radar measurements. Fig. 2 displays the sampling times of radar measurements and ADS-B measurements in a real-time experiment. From Fig. 2, the sampling interval of ADS-B measurements is shorter than that of radar measurements, and it approximates 1 s. For ease in the following processing of measurements and accurate estimation of radar system error, one utilizes the sampling time sequence $\{t_{R,i}\}_{i=1}^{M}$ of radar measurements as reference times and interpolates values of ADS-B measurements. Then, these interpolation values are used to reconstruct new ADS-B measurements that are applied to estimate the system error of three-dimensional radars.

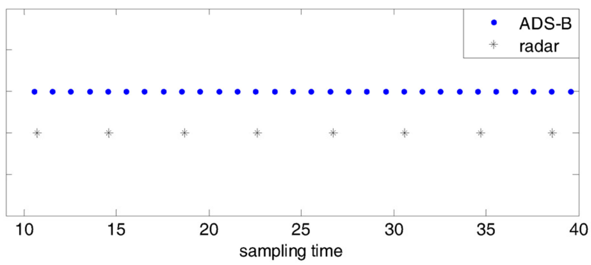


Fig. 2. Sampling time of radar and ADS-B in actual surveillance situations

As mentioned above, the sampling interval is relatively short, approximately 1 s per sample. Consequently, the motion model of a moving target can be assumed as a straight line in each sampling interval. Thus, one can utilize two neighboring ADS-B measurements ($\mathbf{z}_{A,j}$ and $\mathbf{z}_{A,j+1}$, $t_{A,j} < t_{R,i} < t_{A,j+1}$) close to the sampling time $t_{R,i}$ of the radar measurement to calculate a new interpolation value.

$$\begin{cases} \hat{x}_{A,i} = x_{A,j} + \hat{v}_{x,i}(t_{A,j+1} - t_{R,i}) \\ \hat{y}_{A,i} = y_{A,j} + \hat{v}_{y,i}(t_{A,j+1} - t_{R,i}) \\ \hat{z}_{A,i} = z_{A,j} + \hat{v}_{z,i}(t_{A,j+1} - t_{R,i}) \end{cases} \tag{10}$$

Here,

$$\begin{cases} \hat{v}_{x,i} = (x_{A,j+1} - x_{A,j}) / (t_{A,j+1} - t_{A,j}) \\ \hat{v}_{y,i} = (y_{A,j+1} - y_{A,j}) / (t_{A,j+1} - t_{A,j}) \\ \hat{v}_{z,i} = (z_{A,j+1} - z_{A,j}) / (t_{A,j+1} - t_{A,j}) \end{cases} \tag{11}$$

where $t_{A,j}$ and $t_{A,j+1}$ correspond to sampling times j and $j+1$ of the ADS-B measurements $z_{A,j}$ and $z_{A,j+1}$, respectively.

3.3 Eliminating Outliers [19]

Radar data generally contain some measurement outliers that seriously deviate from real positions of the target due to synthetic effects of various incidental factors. These outliners may degrade the of radar system errors. Moreover, the estimation parameters of the least squares model are sensitive to these outliners. Therefore, one needs to utilize the 3σ rule to eliminate the outliers. The concrete processing steps are as follows:

Step 1. With the components $\{(x_{R,i}, y_{R,i})\}_{i=1}^{M}$ of radar measurements, one can utilize the least squares fitting algorithm to estimate the equation of the straight-line $\hat{l}_R$ as in [21]:

$$\hat{l}_R : y_{R,i} = \hat{a}_R x_{R,i} + \hat{b}_R \tag{12}$$

$$\hat{a}_R = \frac{\sum_{i=1}^{M} x_{R,i} y_{R,i} - \frac{1}{M}\sum_{i=1}^{M} x_{R,i} \sum_{i=1}^{M} y_{R,i}}{\sum_{i=1}^{M} x_{R,i}^2 - \frac{1}{M}\left(\sum_{i=1}^{M} x_{R,i}\right)^2} \tag{13}$$

$$\hat{b}_R = \frac{1}{M}\left[\sum_{i=1}^{M} y_{R,i} - \hat{a}\sum_{i=1}^{M} y_{R,i}\right] \tag{14}$$

where M is the amount of measurement components, $\hat{a}_R$ and $\hat{b}_R$ are two parameters of the straight-line $\hat{l}_R$.

Step 2. Calculate the distance $d_{R,i}$ between each radar measurement $z_{R,i}$ and the straight line $\hat{l}_R$:

$$d_{R,i} = \frac{|\, \hat{a}_R x_{R,i} - y_{R,i} + \hat{b}_R \,|}{\sqrt{\hat{a}_R^2 + 1}} \tag{15}$$

Step 3. Calculate the standard deviation:

$$\sigma_{R,l} = \sqrt{\frac{\sum_{i=1}^{M} (d_{R,i} - \bar{d}_R)^2}{M-1}} \tag{16}$$

Here, $\bar{d}_R = \frac{1}{M}\sum_{i=1}^{M} d_{R,i}$.

Step 4. According to the 3σ rule, if $d_{R,i} > 3\sigma_{R,l}$, $z_{R,i}$ is an outlier, and one needs to eliminate it from the set of radar measurements. Otherwise, one needs to keep the corresponding radar measurement.

Step 5. Use the updated set of radar measurements to refit the equation of the straight line.

Step 6. Repeat step (2) to step (5) until all of the radar measurements satisfy the 3σ rule.

3.4 Rough Estimation of Radar Azimuth

Considering the sensitivity of the system error estimation algorithm to observation errors, here one needs to calculate rough values of the system error and then apply them to compensate radar measurements. For this purpose, assuming that a target moves in a straight line, one can use the least squares fitting algorithm to calculate the equations of radar measurements and ADS-B measurements as follows:

$$\begin{cases} \hat{l}_R : y_{R,i} = \hat{a}_R x_{R,i} + \hat{b}_R \\ \hat{l}_A : y_{A,i} = \hat{a}_A x_{A,i} + \hat{b}_A \end{cases} \tag{17}$$

Then, calculate the angle between these two straight lines to utilize as the rough value of the azimuth error of radar measurements:

$$\Delta\hat{\theta}_{R,0} = \arctan\left|\frac{\hat{a}_R - \hat{a}_A}{1 + \hat{a}_R \hat{a}_A}\right| \tag{18}$$

Finally, apply the rough value to compensate the azimuth component of radar measurements:

$$\theta'_{R,i} = \theta_{R,i} + \Delta\theta_{R,0} \tag{19}$$

4 Least-squares-based Radar System Error Estimation

After the preprocessing of radar measurements and ADS-B measurements, $(r_{R,i}, \theta_{R,i}, h_{R,i})$ and $(r_{A,i}, \theta_{A,i}, h_{A,i})$ denote the radar measurement and ADS-B measurement, respectively, of target T in a polar coordinate system at time t_i; $(x'_{R,i}, y'_{R,i}, z'_{R,i})$ and $(x'_{A,i}, y'_{A,i}, z'_{A,i})$ denote its radar measurement and ADS-B measurements in a rectangular coordinate system; Δr_R, $\Delta\theta_R$ and Δh_R denote the range error, azimuth error and height error of a three-dimensional radar. If one ignores the influence of random error on the estimation process, one can obtain the following equation using Eq. (9):

$$\begin{cases} x'_{R,i} = (r_{R,i} - \Delta r_R)\sin(\theta_{R,i} - \Delta\theta_R) \\ y'_{R,i} = (r_{R,i} - \Delta r_R)\cos(\theta_{R,i} - \Delta\theta_R) \\ z'_{R,i} = h_{R,i} - \Delta h_R \end{cases} \tag{20}$$

Because Δr_R and $\Delta\theta_R$ are very small, one can ignore the influences of their second-order terms. Then, Eq. (20) can be further modified by

$$\begin{cases} x'_{R,i} = r_{R,i}\sin\theta_{R,i} - \Delta r_R \sin\theta_{R,i} - \Delta\theta_R r_{R,i}\cos\theta_{R,i} \\ y'_{R,i} = r_{R,i}\cos\theta_{R,i} - \Delta r_R \cos\theta_{R,i} + \Delta\theta_R r_{R,i}\sin\theta_{R,i} \\ z'_{R,i} = h_{R,i} - \Delta h_R \end{cases} \tag{21}$$

Note that ADS-B measurements have high accuracy; namely, the system error of ADS-B is very small compared with that of radar. Then, one can ignore the system error of ADS-B and regard the ADS-B measurements as the real positions of a target. Under this assumption, one can obtain the equations

$$\begin{cases} x'_{A,i} = r_{A,i}\sin\theta_{A,i} \\ y'_{A,i} = r_{A,i}\cos\theta_{A,i} \\ z'_{R,i} = h_{R,i} - \Delta h_R \end{cases} \tag{22}$$

and further deduce the following from the geometric spatial relationship between $(x'_{R,i}, y'_{R,i}, z'_{R,i})$ and $(x'_{A,i}, y'_{A,i}, z'_{A,i})$:

$$\begin{cases} r_{R,i}\sin\theta_{R,i} - \Delta r_R \sin\theta_{R,i} - \Delta\theta_R r_{R,i}\cos\theta_{R,i} = r_{A,i}\sin\theta_{A,i} \\ r_{R,i}\cos\theta_{R,i} - \Delta r_R \cos\theta_{R,i} + \Delta\theta_R r_{R,i}\sin\theta_{R,i} = r_{A,i}\cos\theta_{A,i} \\ z'_{R,i} = h_{R,i} - \Delta h_R \end{cases} \tag{23}$$

Eqs. (23) can be replaced by the following equations by constant deformation:

$$\begin{cases} \Delta r_R \sin\theta_{R,i} + \Delta\theta_R r_{R,i} \cos\theta_{R,i} = r_{R,i} \sin\theta_{R,i} - r_{A,i} \sin\theta_{A,i} \\ \Delta r_R \cos\theta_{R,i} - \Delta\theta_R r_{R,i} \sin\theta_{R,i} = r_{R,i} \cos\theta_{R,i} - r_{A,i} \cos\theta_{A,i} \\ z'_{R,i} = h_{R,i} - \Delta h_R \end{cases} \quad \textbf{(24)}$$

Let $H_i = \begin{bmatrix} \sin\theta_{R,i} & r_{R,i}\cos\theta_{R,i} & 0 \\ \cos\theta_{R,i} & -r_{R,i}\sin\theta_{R,i} & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\beta = [\Delta r_R, \Delta\theta_R, \Delta h_R,]^{\mathrm{T}}$ and $Z_i = \begin{bmatrix} r_{R,i}\sin\theta_{R,i} - r_{A,i}\sin\theta_{A,i} \\ r_{R,i}\cos\theta_{R,i} - r_{A,i}\cos\theta_{A,i} \\ h_{R,i} - z'_{R,i} \end{bmatrix}$. Then, Eq. (20) can be further modified as

$$Z_i = H_i \beta \quad \textbf{(25)}$$

For M radar measurements, one can obtain the following from to Eq. (25)

$$\left[Z_1, Z_2, \cdots H_M\right]^{\mathrm{T}} = \left[H_1, H_2, \cdots H_M\right]^{\mathrm{T}} \beta \quad \textbf{(26)}$$

If we let $H = \left[H_1, H_2, \cdots H_M\right]^{\mathrm{T}}$ and $Z = \left[Z_1, Z_2, \cdots Z_M\right]^{\mathrm{T}}$, then Eq. (26) can be expressed by

$$Z = H\beta \quad \textbf{(27)}$$

Because Eq. (23) is overdetermined, one can utilize the least squares estimation algorithm to solve it, as in [21]:

$$\beta = (H^{\mathrm{T}} H)^{-1} H^{\mathrm{T}} Z \quad \textbf{(28)}$$

5 Experimental Results and Analysis

An experiment with real data has been conducted to evaluate the performance of the proposed method in comparison with the other four system-error estimation methods: the straight-line fitting-based method (FL), the least squares-based method in rectangular coordinates (LSr), the least squares-based method in polar coordinates (LSp), the straight-line-fitting-based and least-squares-based method in polar coordinates (FL-LSp), and the straight-line fitting-based and least squares-based method in rectangular coordinates (FL-LSr).

5.1 Real Data Scenarios

The real data are radar measurements (30 points) and ADS-B measurements (115 points) from a cooperative target. The trajectories for radar measurements and ADS-B measurements are shown in Fig. 3. The cooperative target moves with a straight-line model in Fig. 4. To estimate the system error, ADS-B measurements were transformed from a geographic coordinate system to a rectangular coordinate system, and radar measurements were transformed from a polar coordinate system to a rectangular coordinate system. As observed in Fig. 3 and Fig. 4, the sampling times of ADS-B measurements are denser than those of radar measurements. To estimate the radar system error, the ADS-B measurements are aligned with radar measurements, where the sampling time of radar measurements in a common sampling interval is used as the reference time. Then, the interpolation values of ADS-B measurements corresponding to the reference time are calculated, and these values then are utilized to reconstruct the ADS-B measurements. Moreover, they are regarded as the real positions of the moving target. After the transformation and interpolation processing, the new ADS-B measurements (interpolation values) are given in Fig. 5. Thus, the radar measurements and new ADS-B measurements have the same sampling time and number of samples, which consist of 22 points. For ease of reference, all of the ADS-B measurements mentioned in the following context denote these new ADS-B measurements.

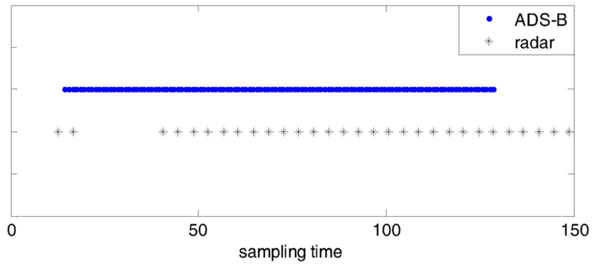


Fig. 3. Sampling times of radar and ADS-B

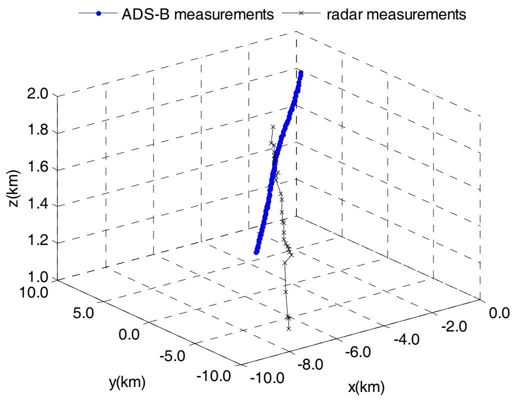


Fig. 4. Radar and ADS-B measurements

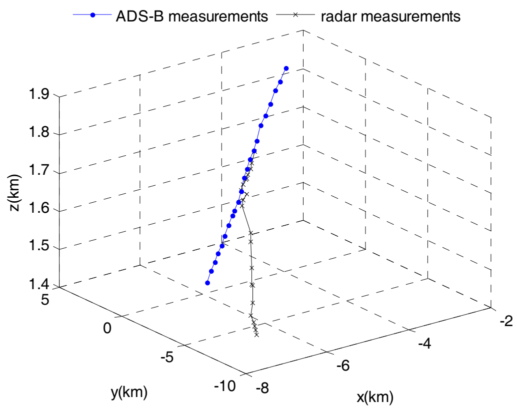


Fig. 5. Preprocessed radar and ADS-B measurements

Fig. 6 shows the lines fitted to radar measurements and ADS-B measurements. As known from Fig. 6, there are system errors in the radar measurements and ADS-B measurements that are approximately distributed on both sides of the corresponding straight-lines, particularly for ADS-B measurements. This demonstrates that ADS-B measurements, which are based on GPS, have high accuracy. Consequently, this verifies that the strategy of regarding ADS-B measurements as the real positions of a target to correct radar measurements is feasible. Therefore, one can calculate the angles of two straight-lines corresponding to radar measurements and ADS-B measurements and then utilize the angle between the lines to compensate the azimuthal components of radar measurements. Table 1 presents the estimation results for five estimation methods; the estimation of radar system errors by the FL-LSr, FL-LSp, and LSp methods are approximately consistent. Thus, the registration performance of these three estimation methods is also consistent.

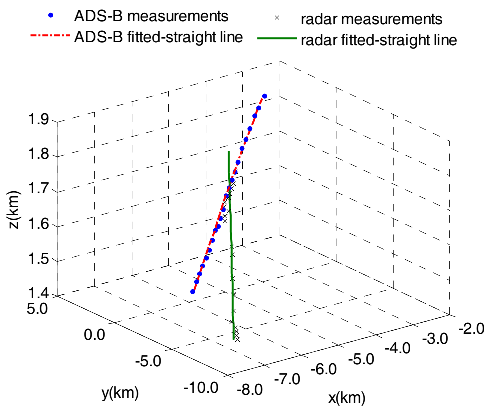
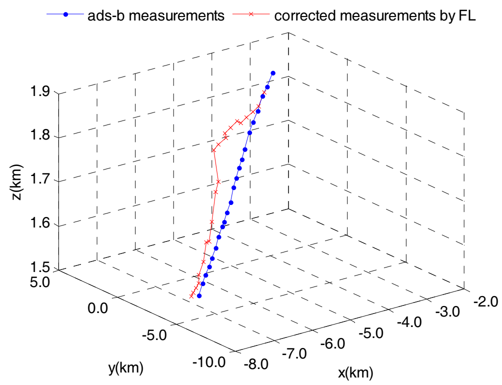


Fig. 6. Fitting straight-lines by radar and ADS-B measurements

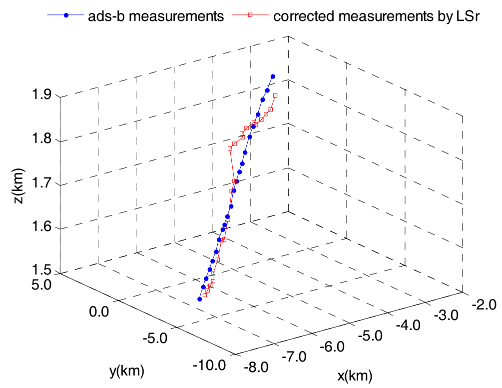
Table 1. Estimation results of radar system errors

methods	FL	LSr	LSp	FL-LSr	FL-LSp
range (m)	——	418.0	322.1	332.4	332.1
azimuth (mrad)	170.8	159.9	169.0	172.5	172.6
height (m)	——	145.5	145.5	145.5	145.5

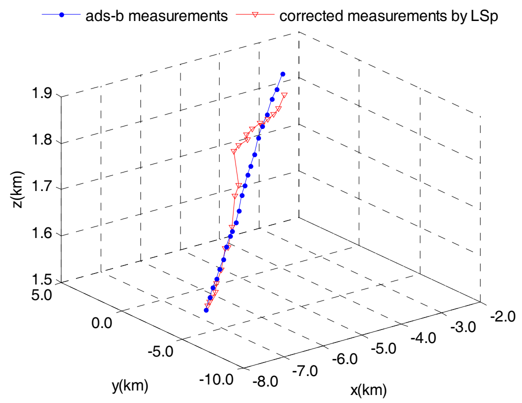
Fig. 7 provides the radar measurements after registering by five methods. For clear expression, the registration results for each estimation method are shown in a figure. As observed in Fig. 7, the radar measurements after registering by five methods approach the ADS-B measurements. This fact illustrates that all five estimation methods can improve the accuracy of radar measurements, and demonstrates that they are valid. Moreover, the improvements of the FL-LSr, FL-LSp, and FL-LSp methods are better than those of the LSr method and the improved effects of the FL method.



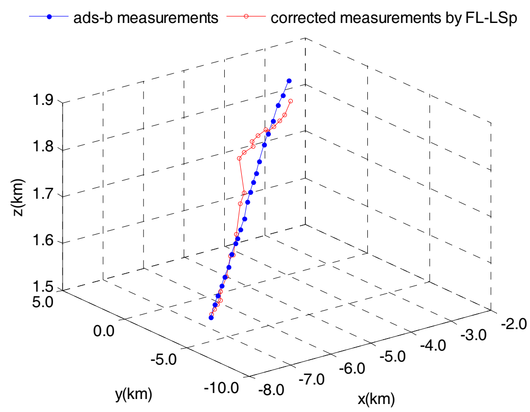
(a) FL method



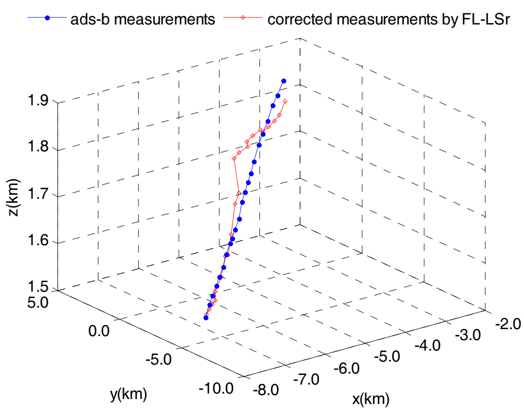
(b) LSr method



(c) LSp method



(d) FL-LSp method



(e) FL-LSr method

Fig. 7. Registration results for radar measurements

To compare the performance of the FL-LSr, FL-LSp, and LSp methods, Fig. 7 displays the registration mean-square errors (MSE) in the range, azimuth, and height components of the radar measurements. From Fig. 7, for the estimation of radar range errors, the FL-LSr, FL-LSp, and LSp methods have the same estimation performance, which is better than that of the LSr method. Similarly, for the estimation of radar azimuth errors, the performances of the FL-LSr, FL-LSp and LSp methods are consistent. Moreover, their performance in estimating range error is higher than that of the FL method, and the performance of the FL method is better than that of the LSr method. For the estimation of radar height errors, these estimation methods are consistent because they utilize the least-squares algorithm to estimate the radar-height errors individually. Therefore, the FL-LSr method, FL-LSp method and LSp method possess the same estimation performance; their overall performance is better than those of the LSr and FL methods. In particular, the estimation performance of the FL method is approximately the same as those of the FL-LSr, FL-LSp, and LSp methods. This fact illustrates that the FL method is a simple but valid estimation from a graphics perspective, and it provides a good strategy for further solving the system-error estimation problem.

In addition, Table 2 provides the averaged MSEs for the range, azimuth, and height components of radar measurements. Here, it assumes that the ADS-B measurements are the real positions of the target. The results are similar to the analyzed results. All five estimation methods can largely improve the accuracy of the radar azimuth. For quantitative analysis of the improved effects of radar measurements after registration, one can define the index of range, azimuth, and height components, and they can concretely be expressed by

$$\begin{cases} \eta_r = \dfrac{\varepsilon_{r0} - \varepsilon_r}{\varepsilon_{r0}} \times 100\% \\ \eta_\theta = \dfrac{\varepsilon_{\theta 0} - \varepsilon_\theta}{\varepsilon_{\theta 0}} \times 100\% \\ \eta_h = \dfrac{\varepsilon_{h0} - \varepsilon_h}{\varepsilon_{h0}} \times 100\% \end{cases} \tag{29}$$

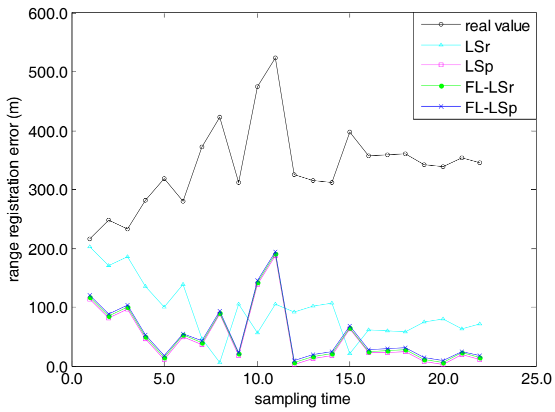
Table 2. Mean square error of registration results

methods	original error	FL	LSr	LSp	FL-LSr	FL-LSp
range (m)	347.6	347.6	104.9	71.0	71.0	71.0
azimuth (mrad)	168.7	9.8	12.8	9.5	9.5	9.5
height (m)	140.1	140.1	45.1	45.1	45.1	45.1

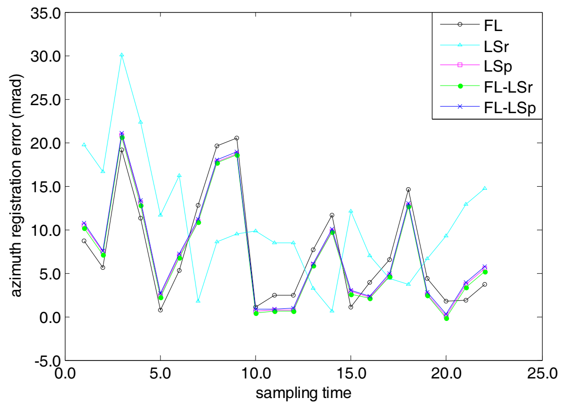
Here, ε_r and ε_{r0} respectively denote the range indexes before and after registering for average MSEs of range components; ε_θ and ε_{r0} denote the azimuth indexes before and after registering average MSEs of azimuth components; ε_h and ε_{h0} denote the azimuth indexes before and after registering average MSEs of height components; η_r , η_θ and η_h denote the range, azimuth and height indexes for registration.On the basis of the above analysis, Table 3 provides the performance indexes of the registration results for five estimation methods, and it can intuitively and quantitatively analyze the registration performance. As known from Table 3, for range registration, the FL-LSr, FL-LSp, and LSp methods can largely reduce the average MSEs 79.57%; for azimuth registration, all of the five estimation methods can largely reduce the average azimuth error of radar measurements. Here, the registration performances of the FL-LSr, FL-LSp, and LSp methods are best in the registration of azimuth error, and the registration performance of the FL method is better than that of the LSr method. Notably, the FL method can reduce the average azimuth registration error of radar measurements 94.19%, which is very close to the best registration performance of the FL-LSr, FL-LSp, and LSp methods. For height registration, all of the estimation methods have the same registration performance. On the whole, the FL-LSr, LS-LSp and LSp methods are better than other two estimation methods.

Table 3. Performance index of registration results

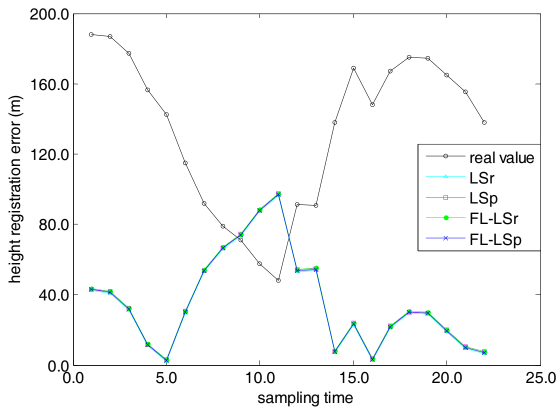
methods	FL	LSr	LSp	FL-LSr	FL-LSp
range	——	69.82	79.57	79.57	79.57
azimuth	94.19	92.41	94.37	94.37	94.37
height	——	67.81	67.81	67.81	67.81



(a) Range registration error



(b) Azimuth registration error



(c) Height registration error

Fig. 7. Root mean square error for registration results

5.2 Discussion of Experimental Results

The above experiment with real data has sufficiently analyzed the five estimation methods. The FL method possesses the advantage of simple implementation and clear physical meaning from a graphic perspective. Thus, the FL method provides a good method to estimate radar system error. Although the FL method obtains good performance in estimation of azimuth system error, it cannot estimate the range system error or height system error of a three-dimensional radar. In addition, the registration performance of the FL method depends on the assumption that its target moves as a straight-line model to some extent. The uncertainty of target motion models introduces certain constraints in real applications. Therefore, the FL method needs to further improve in its adaptability and robustness for various applications.

Similarly, the FL-LSr and FL-LSp methods are based on the FL method, and they have its advantages and weaknesses, as does the LS method. However, the FL-LSr method further utilizes the least squares algorithm to estimate the system error for the preprocessed radar and ADS-B measurements. Compared with the FL method, its estimation accuracy has been largely improved, particularly for the range system error. Although the LSr method also utilizes the least squares algorithm to estimate radar measurements, its estimation performance is not as good as that of the FL-LSr method. This is because the FL-LSr method utilizes the compensated radar measurements in the azimuth component, while the LSr method utilizes the original radar measurements. Furthermore, the original radar measurements contain larger errors than the compensated radar measurements. For the least squares model estimated in a rectangular coordinate system, it ignores the second- and higher-order items of system errors mentioned in Eqs. (21) and (22). In fact, these items also contain some system error information, and they still have a certain influence on it. In particular, when the system error becomes relatively large, the estimation accuracy of system error is seriously affected. Therefore, if one applies the least squares algorithm to estimate the system error in a rectangular coordinate system, the preprocessing and compensation of radar and ADS-B measurements are very necessary. In a polar coordinate system, the FL-LSp method and LSp method do not need to transform radar measurements to a rectangular coordinate system. This can avoid the information loss of system errors. As a result, the FL-LSp method and LSp method can also obtain good estimation accuracy.

Finally, all five estimation methods are proposed under the assumption that the system error of three-dimensional radar is constant, and the methods do not consider the influences of the uncertainty of target motion models and sensor sampling time on system error estimation. Hence, we need to further analyze the real test measurements from radar and ADS-B.

6 Conclusions

This paper analyses the strengths and weaknesses of the traditional system error estimation method. Thus, a least-squares-based system error estimation method (FL-LSr) using ADS-B measurements is proposed for three-dimensional radar in a local rectangular coordinate system. In the proposed method, the measurements from a radar and an ADS-B device are first preprocessed by unifying a coordinate system, transforming coordinates, eliminating outliers, interpolating measurements, and compensating azimuth components. Thus, observed errors in radar measurements are largely reduced by data preprocessing. Then, the least-squares estimation algorithm is applied to calculate the system error of three-dimensional radar. The experimental results using real measurements illustrate that, compared with other estimation methods, the proposed method has good accuracy performance to estimate system errors of three-dimensional radar.

To improve the accuracy and stability of system errors of three-dimensional radars in a following study, we will further consider the influence of the system errors in ADS-B measurements and the inaccuracy of sampling time to estimate the system error, and analyze the influence of the random errors of radar measurements.

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