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Variational Bayesian-based Multiple-model Algorithm Combined with Auxiliary Knowledge

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Abstract. The theoretical framework of the multiple-model algorithm combined with auxiliary knowledge is first proposed in this paper. In order to improve the tracking accuracy of tracking algorithms that is in situation with unknown measurement noise, where Automatic Dependent Surveillance-Broadcast (ADS-B) equipment is employed to keep track of aircraft, a variational Bayesian-based multiple-model algorithm combined auxiliary knowledge (KAVBMM) is proposed. To solve the state and noise distribution, the variational Bayesian approximation is adopted for performing multiple known distribution approximation and estimate the measurement noise variance. Meanwhile, the KAVBMM algorithm utilizes a multiple-model method to adapt the maneuvering change of the target and adjusts the measurement noise value according to Navigational Accuracy Category (NAC) for position information. The results of simulation experiment and read-data experiment shows that the proposed KAVBMM algorithm is can improve the tracking performance.

Keywords: ADS-B, multiple-model, target tracking, variational Bayesian

1 Introduction

The Automatic Dependent Surveillance-Broadcast (ADS-B) technology, a sort of aircrafts running surveillance technology based on global navigation satellite system, has been extensively applied in the Air Traffic Control (ATC) systems nowadays [1-3]. The ADS-B measurement link includes both the air-to-air data and the ground-to-air data. The aircrafts loaded with the ADS-B device are capable of sending out messages automatically and periodically, including altitude, heading, velocity, and other states. Therefore, this information is usually referenced to keep track of air traffic in ATC systems. The ADS-B out service predictions are based on the expected GNSS satellite geometry and performance. The ADS-B equipment sends off the Navigational Accuracy Category for Position (NACp) code along with position reports [4]. Not only the NACp in the ADS-B measurement is limited to providing a range of measurement noise accuracy, but its true measurement noise variance is also unknown.

As for the general aviation aircraft ATC system, the interactive multiple-model (IMM) algorithm is usually applied to track the general aviation aircraft with an excellent mobility [5]. For example, in literature [6] an IMM algorithm intended to track the aircraft is proposed. When the statistical feature of measurement noise is determined, the aircraft can achieve a higher accuracy of state estimation under

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maneuvering conditions. In multiple-model algorithms, the drawbacks of traditional filters, such the Kalman filter and its various improved versions, are manifested in that accurate state models and statistical feature of measurement noise, which requires to be known [7]. Measurement noise has impact on the contribution degree of measurement in state estimation, which is the evaluation quality of external measurements in a target tracking system. In general, the measurement model is determined by the sensor properties. In practical applications, both the state model and the statistical feature of measurement noise are unknown or partially known, and such uncertainty will cause the performance of the multiple-model algorithm deteriorated and even bring divergence [8]. Many studies have been performed on the unknown measurement noise variance in target tracking process. For example, Osborne suggests that an algorithm is capable to estimate the variance of measurement noise, which is assumed to be constant [9]. This algorithm is based on the idea of the IMM algorithm to address the uncertainty caused by sensor measurement noise. Nevertheless, the calculation complexity is relatively big. When the preset value of equivalent measurement noise fails to match the real value, the tracking accuracy become low. Sage and Husa proposes an algorithm based on the suboptimal unbiased maximum posterior estimator [10]. This algorithm is capable to estimate the first and second moments of measurement noise and system noise simultaneously. The mentioned methods above have exerted effect to some extent by estimating the mean and variance of measurement noise in real-time. The variational Bayesian method takes advantage of a posterior distribution where pluralities of known distributions are difficult to solve, and the calculated amount is reduced significantly compared with the abovementioned methods [11]. For the variational Bayesian estimation, Smidl and Quinn proposed a theoretical framework with the emphasis depends on the application of iterative Bayesian inference in signal processing [12]. In the reference [13], Sarkka puts forward a variational Bayesian-based adaptive Kalman filter (VB-AKF) algorithm for signal filtering with a variational Bayesian approximation method to estimate the measurement noise variance at the current moment through iteration. As a result, it involves a small workload of calculation to achieve an excellent filtering performance.

In order to obtain an accurate position estimation of a target, the noise needs to be filtered in ADS-B measurements. Since the unknown measurement noise variance lies in an interval for NACp, the traditional filters can't be directly applied. Under the situations with unknown measurement noise variance, the VB-AKF relies on the variational Bayesian approximation method to estimate the current noise variances. It requires a small calculation load and can estimate the unknown noise variance. The VB-AKF has an excellent target tracking performance for the uniform motion model. Nevertheless, it fails to work well when the target maneuvers. This is because the maneuvering motion model causes uncertainty, which makes it difficult for any single motion model to describe the actual target motion. For maneuvering target tracking, the IMM algorithm is usually applied in ATC systems [14-15]. However, it does not work with the unknown measurement noise. Hence, we present a variational Bayesian-based IMM (VB-IMM) algorithm for ADS-B data at a conference for communication [16]. Here, we further propose a modified variational Bayesian-based multiple-model algorithm combined auxiliary knowledge (KAVBMM) method based on the spirit of the VB-AKF. The major contributions are given as follows: (1) in situation with unknown measurement noise covariance, we propose a theoretical framework of the multiple-model algorithm combined with auxiliary knowledge; (2) on the basis of this theoretical framework, a variational Bayesian-based multiple-model algorithm combined auxiliary knowledge (KAVBMM); (3) furthermore, the variational Bayesian approximation is adopted for performing multiple known distribution approximation and estimate the measurement noise variance; (4)finally, two experiments with simulated data and real data are designed to illustrate the validity of the proposed KAVBMM method.

The rest of this paper is organized as follows. In Section 2, a brief review of the NACp in ADS-B measurement is introduced. Section 3 gives an implementation process of the KAVBMM algorithm, and the experiment results on the simulation and the real ADS-B measurements are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2 NACp in ADS-B Measurement

The ADS-B error stems from GPS positioning. A majority of the existing methods rely on representing the actual measurement error distribution by Gaussian model [17]. In ADS-B measurements, the NACp is classified into 12 categories [4], as listed in Table 1. Here, Estimated Position Uncertainty (EPU) is defined as the estimation accuracy error range of measurements in the horizontal position. The actual positions fall within the circle with a 95% probability and a 5% probability outside the range of the circle, which is centered on the aircraft reporting position. The EPU value of the NACp corresponding the level is used as the radius. This error representation is known as Circular Error Probability (CEP), which is widely utilized in the field of navigation positioning [18]. When the probability is 95%, the circular probability error can be expressed as CEP_{95}. For example, when the NACp level is 9, the theoretical containment radius is (10m, 30m]. When the NACp is ≥ 9, it consists of EPU and Vertical Estimated Position Uncertainty (VEPU). VEPU defines the accuracy error range of accuracy error of measurements in the vertical direction. Assuming σ is the given error and H is the height position reported, the actual position falls within [H-σ, H+σ] with a probability of 95%, and falls outside the range with a probability of 5%, as shown in Fig. 1. The corresponding values of the EPU and the VEPU are indicated in Table 1. Therefore, after obtaining the NACp in the ADS-B measurements, the CEP range of measurements can be ascertained. It can be seen from the range of the VEPU and EPU, which is fairly large. They can be calculated directly by using the variable-point Bayes, and the number of iterations is too large. Hence, their influences must be considered in the designed.

Table 1. Position accuracy corresponding to the NACp

NACp	VEPU (CEP_{95})	EPU(CEP_{95})
0	N/A	>18.52 km
1	N/A	(7.408km, 18.52km]
2	N/A	(3.704km, 7.408km]
3	N/A	(1.852km, 3.704km]
4	N/A	(926m,1852m]
5	N/A	(555.6m, 926m]
6	N/A	(185.2m, 555.6m]
7	N/A	(92.6m, 185.2m]
8	N/A	(30m, 92.6m]
9	(15m, 45m]	(10m, 30m]
10	(4m, 15m]	(3m, 10m]
11	<4m	<3m

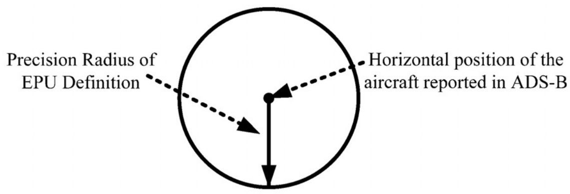


Fig. 1. Corresponding diagram between the EPU and the ADS-B report position

The measurement noise variance is adopted in the filtering, and the EPU or VEPU needs to be converted into the corresponding Root Mean Square Error (RMSE). The relationship between the EPU or VEPU with RMSE is usually applied in target tracking, and then RMSE is defined by

$$\text{RMSE}=(\sigma_\varphi^2+\sigma_\lambda^2)^{1/2} \quad \textbf{(1)}$$

where σ_φ indicates the longitude error of the target position and σ_λ denotes its latitude error. According to the corresponding reference [18], one can further obtain:

$$\text{CEP}_{95}=1.2272\times(\sigma_\varphi+\sigma_\lambda)\,. \quad \textbf{(2)}$$

Then, Eq. (1) can be further modified as

$$\text{RMSE}=1.1\times\text{CEP}_{95}\,. \quad \textbf{(3)}$$

After obtaining the EPU (CEP_{95}) corresponding to the NACp of the target, the range of the standard deviation or variance of the target position can be determined.

3 KAVBMM Algorithm Implementation Process

In the multiple-model algorithm, the measurement model can be expressed by:

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k(i) \tag{4}$$

where $\boldsymbol{v}_k(i)$ represents the different measurement noise models, $i \in \{1,2,\cdots,r\}$. The real measurement noise variance is screened by the NACp aided.

In the process of screening, let the real measurement noise variance model be **S**. The measurement noise variance model with the NACp aided as **A** in the current filter, which is obtained by screening the total measurement noise variance model **B**. Namely, **A** contains **S**. **C**=**B**-**A** when $\mathbf{C} \cap \mathbf{S} = \Phi$. The property of model **A**'s state estimation is better than that of model **B**, and the former one is closer to the optimal model **S**'s state estimation. Meanwhile, it needs to utilize the conclusion drawn in Literature [15]. It applies the NACp to improve the tracking performance of multiple model algorithms. Based on the above analysis, the KAVBMM algorithm is further proposed in this paper.

3.1 Variational Bayesian Iterative Method

The optimal Bayesian filter needs to obtain the posterior distribution $p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k-1})$ when the measurement noise variance is unknown. The predictive distribution of the system state $\boldsymbol{x}_k$ and the measurement noise variance $\boldsymbol{R}_k$ is obtained by the Chapman-Kolmogrov equation:

$$p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k-1}) = \int p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}) p(\boldsymbol{R}_k \mid \boldsymbol{R}_{k-1}) p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k-1}) \times p(\boldsymbol{x}_k, \boldsymbol{R}_{k-1} \mid z_{1:k-1}) d\boldsymbol{x}_{k-1} d\boldsymbol{R}_{k-1}\,. \tag{5}$$

Given the next measurement z_k, the predictive distribution is updated to a posterior distribution by applying the Bayes' rule:

$$p(\boldsymbol{x}_k, \boldsymbol{R}_{k-1} \mid z_{1:k-1}) \propto p(z_k \mid \boldsymbol{x}_k, \boldsymbol{R}_k) p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k-1})\,. \tag{6}$$

The variational Bayesian approximation assumes that $p(\boldsymbol{x}_k, \boldsymbol{R}_{k-1} \mid z_{1:k-1})$ for $\boldsymbol{x}_k$ and $\boldsymbol{R}_{k-1}$ given the measurements $z_{1:k-1}$ conform to Gaussian and independent Inverse-Gamma distributions [19].

$$p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k-1}) \approx N(\boldsymbol{x}_k \mid \overline{m}_k, \overline{P}_k) \times \prod_{i=1}^{d} inv-Gamma(\sigma_{k,i}^2 \mid \overline{\alpha}_{k,i}, \overline{\beta}_{k,i})\,. \tag{7}$$

The inverse gamma distribution is a distribution function performed by the inverse of a gamma distribution, two of which are shape parameters and scale parameters. The inverse gamma distribution is primarily applied to Bayesian statistics, to estimate the marginal posterior distribution of a normal distribution with unknown variance. The distribution function can be written as:

$$f(x \mid \alpha, \beta) = \frac{\beta^{\alpha 11}}{\Gamma(\alpha)} x^{-\alpha-1} \exp(-\frac{\beta}{x})\,. \tag{8}$$

where α indicates shape parameter, β denotes scale parameter, $\Gamma(\cdot)$ is Gamma Distribution and $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}$.

According to the variational Bayesian method, it is assumed that $\boldsymbol{x}_k$ and $\boldsymbol{R}_k$ are independent of each other. The predictive distribution of $\boldsymbol{x}_k$ and $\boldsymbol{R}_k$ is given by the Chapman-Kolmogrov equation:

$$p(\boldsymbol{x}_k, \boldsymbol{R}_k \mid z_{1:k}) \approx Q_k(\boldsymbol{x}_k) Q(\boldsymbol{R}_k) \tag{9}$$

where $Q_k(\boldsymbol{x}_k)$ conforms to a normal distribution and $Q(\boldsymbol{R}_k)$ is consistent with an inverse gamma distribution. By calculating the relative entropy of the true value of the posterior probability density and the approximation, which is also known as the Kullback-Leibler divergence, variational Bayesian approximation of the following is performed:

$$\mathrm{KL}[Q_k(\boldsymbol{x}_k)Q(\boldsymbol{R}_k)\,|\,p(\boldsymbol{x}_k,\boldsymbol{R}_k \mid z_{1:k})] = \int Q_k(\boldsymbol{x}_k)Q(\boldsymbol{R}_k)\times\log\frac{p(\boldsymbol{x}_k,\boldsymbol{R}_k \mid z_{1:k})}{Q_k(\boldsymbol{x}_k)Q(\boldsymbol{R}_k)}\mathrm{d}\boldsymbol{x}_k\mathrm{d}\boldsymbol{R}_k\,. \quad \textbf{(10)}$$

To minimize the KL divergence, the iterative calculations are performed on the $Q_k(\boldsymbol{x}_k)$ and $Q(\boldsymbol{R}_k)$ using the variable integration method.

$$Q_k(\boldsymbol{x}_k) \propto \int \ln p(\boldsymbol{z}_k,\boldsymbol{x}_k,\boldsymbol{R}_k \mid z_{1:k})\mathrm{d}\boldsymbol{R}_k \quad \textbf{(11)}$$

$$Q_k(\boldsymbol{R}_k) \propto \int \ln p(\boldsymbol{z}_k,\boldsymbol{x}_k,\boldsymbol{R}_k \mid z_{1:k})\mathrm{d}\boldsymbol{x}_k\,. \quad \textbf{(12)}$$

The dynamical model $p(\boldsymbol{R}_k \mid \boldsymbol{R}_{k\text{-}1})$ for the measurement noise variance is unknown. To reflect the changes in the measurement noise variance, variational Bayesian approximation takes a heuristic approach in the calculation process, which predicts the posterior distribution parameter by the means of first-order approximation [13].

$$\bar{\alpha}_{k,i} = \rho_i \alpha_{k-1,i}, \bar{\beta}_{k,i} = \rho_i \bar{\beta}_{k-1,i}, i = 1,2,\cdots,d \quad \textbf{(13)}$$

where $\rho_i \in [0,1]$ is the prediction weighted attenuation coefficient in the interval, to indicate the correlation between the noise at the previous moment and the current moment. When the measurement noise variance difference between time k-1 and time k is small, ρ_i shows a larger value; when the difference is larger, ρ_i shows a smaller value.

Then, the measurement noise variance estimation $\hat{\boldsymbol{R}}_k$ can be estimated as:

$$\hat{\boldsymbol{R}}_k = \mathrm{diag}\left(\frac{\beta_{k,1}}{\alpha_{k,1}},\frac{\beta_{k,2}}{\alpha_{k,2}},\cdots,\frac{\beta_{k,d}}{\alpha_{k,d}}\right) = \mathrm{diag}(\sigma_{k,1}^2,\sigma_{k,2}^2,\cdots,\sigma_{k,d}^2) \quad \textbf{(14)}$$

where $\alpha_{k,i}$ represents shape parameter, $\beta_{k,i}$ indicates scale parameter, and $1,2,\cdots,d$. $\mathrm{diag}(\sigma_{k,1}^2,\sigma_{k,2}^2,\cdots,\sigma_{k,d}^2)$ denotes the measurement noise variance.

The $\hat{\boldsymbol{R}}_k$ and state estimation m_k^{n+1} [20] will be obtained after N steps of iteration.

$$\hat{\boldsymbol{R}}_k^{(n)} = \mathrm{diag}(\beta_{k,1}^{(n)}/\alpha_{k,1}^{(n)},\cdots,\beta_{k,d}^{(n)}/\alpha_{k,d}^{(n)}) \quad \textbf{(15)}$$

$$m_k^{(n+1)} = m_k^- + \boldsymbol{P}_k^-\boldsymbol{H}_k^T(\boldsymbol{H}_k\boldsymbol{P}_k^-\boldsymbol{H}_k^T + \hat{\boldsymbol{R}}_k^{(n)})^{-1}(\boldsymbol{z}_k - \boldsymbol{H}_k m_k^-) \quad \textbf{(16)}$$

$$\boldsymbol{P}_k^{(n+1)} = \boldsymbol{P}_k^- - \boldsymbol{P}_k^-\boldsymbol{H}_k^T(\boldsymbol{H}_k\boldsymbol{P}_k^-\boldsymbol{H}_k^T + \hat{\boldsymbol{R}}_k^{(n)})^{-1}\boldsymbol{H}_k\boldsymbol{P}_k^- \quad \textbf{(17)}$$

$$\beta_{k,i}^{(n+1)} = \beta_{k,i}^- + \frac{1}{2}\left(\boldsymbol{z}_k - \boldsymbol{H}_k m_k^{(n+1)}\right)_i^2 + \frac{1}{2}\left(\boldsymbol{H}_k\boldsymbol{P}_k^{(n+1)}\boldsymbol{H}_k^T\right)_i. \quad \textbf{(18)}$$

Finally, set $m_k = m_k^N$, $\boldsymbol{P}_k = \boldsymbol{P}_k^N$, $\beta_{k,i} = \beta_{k,i}^N$. As shown in Table 1. It is difficult to meet the requirements of the application due to the overlarge noise span of the ADS-B measurement. If the number of iterations is excessively large, the real-time performance of the algorithm cannot be ensured. While, if the iteration setting interval is overly large, the accuracy cannot be ensured.

3.2 The KAVBMM Algorithm Implementation Process

The multiple-model algorithm relies on two or more filters which run in parallel, with each filter using a different model for one mode of target motion. It can obtain the state estimate by a weighted sum of the estimates from all filters with different motion models. The weights, depending on the measurement and the models, change as the motion model changes, thus keeping the one that corresponds to the true mode dominant and the rest negligible [6]. The variational Bayesian approximation is applied to the maneuvering target tracking with unknown measurement noise variance. The NACp in the ADS-B measurement only provides a range of measurement noise accuracy, and its true measurement noise variance is unknown. When filtering, the wrong measurement noise variance will reduce the estimated performance of the multiple-model algorithm significantly, and even cause divergence [21]. In this section, the KAVBMM algorithm is proposed to address this problem. The equation of state and the measurement equation can be described as

$$\boldsymbol{x}_k = \boldsymbol{F}_k(m_k^i)\boldsymbol{x}_{k-1} + \boldsymbol{w}_k \tag{19}$$

$$\boldsymbol{z}_k = \boldsymbol{H}_k(m_k^i)\boldsymbol{x}_{k-1} + \boldsymbol{v}_k \tag{20}$$

where the time is indexed by k, $\boldsymbol{x}_k$ denotes an n-dimensional state vector on the target at time k, $\boldsymbol{z}_k$ indicates a d-dimensional measurement vector, $\boldsymbol{F}_k$ refers to an $n \times n$ state transition matrix, $\boldsymbol{H}_k$ stands for an $m \times n$ measurement matrix, and m_k^i mean the motion model. The initial state shows a Gaussian prior distribution $\boldsymbol{x}_0 \sim N(m_0, \boldsymbol{P}_0)$, and m_0, $\boldsymbol{P}_0$ are assumed to be known. The process noise $\boldsymbol{w}_k$ and the measurement noise $\boldsymbol{v}_k$ are mutually independent zero-mean Gaussian white noise with covariance $\mathrm{cov}(\boldsymbol{w}_k^i) = \boldsymbol{Q}_k^i$ and $\mathrm{cov}(\boldsymbol{v}_k^i) = \boldsymbol{R}_k^i$.

The KAVBMM algorithm assumes that $\boldsymbol{w}_k^i$ corresponding to $m_k^i (i = 1, 2, \cdots, S)$, and it is assumed that m_k^i takes a value in the finite set with a Markov transition probability matrix.

The variational Bayesian approximation in the KAVBMM is presented as follows:

(1) Assume the model-conditioned $\hat{\boldsymbol{R}}_k$.

(2) Use the IMM method to predict the system state $\hat{\boldsymbol{x}}_k$ with $\hat{\boldsymbol{R}}_k$ given in the previous step.

(3) Update $\hat{\boldsymbol{R}}_k$ by using $\hat{\boldsymbol{x}}_k$.

(4) Repeat steps (2) and (3) above for N times of iteration.

(5) The final iterative output of $\hat{\boldsymbol{x}}_k$ and $\hat{\boldsymbol{R}}_k$ are obtained by N iterations.

The specific process of the KAVBMM algorithm is shown below. The process is split into two parts: time update and measurement update, as shown in Fig. 2.

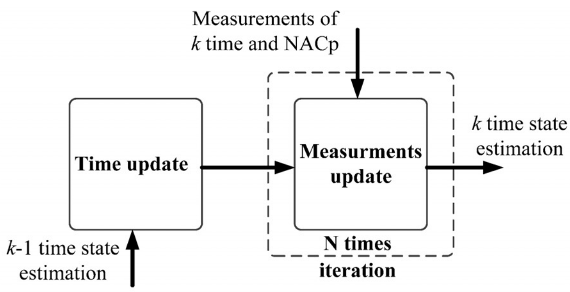


Fig. 2. Flowchart of the variational Bayesian iterative method

3.2.1 Time Update

Parameters prediction.

$$\alpha_{k|k-1,i} = \rho_i \alpha_{k-1,i} \tag{21}$$

$$\beta_{k|k-1,i} = \rho_i \beta_{k-1,i} \tag{22}$$

where d indicates the dimension of the measurement vector. The KAVBMM algorithm uses a fixed set of ρ_i, and the specific value is set according to the scene.

Input interaction.

It is assumed that the probability of model m_{k-1}^{i} at time k-1 is μ_{k-1}^{i}, and the matching model is m_{k}^{j} at time k, so the model predictive probability is:

$$\mu_{k-1|k-1}^{i/j} = \frac{1}{\bar{c}_j} p_{ij} \mu_{k-1}^{i} \tag{23}$$

$$\bar{c}_j = \sum_i p_{ij} \mu_{k-1}^{i}, \ i, j = 1, 2, ..., S \tag{23}$$

where s is the number of models. State estimate is updated to

$$\hat{\boldsymbol{x}}_{k-1|k-1}^{(0)j} = \sum_{i=1}^{r} \boldsymbol{x}_{k-1|k-1}^{i} \mu_{k-1|k-1}^{i/j} . \tag{24}$$

The covariance estimate is updated to

$$\boldsymbol{P}_{k-1|k-1}^{(0)j} = \sum_{i=1}^{r} \mu_{k-1|k-1}^{i/j} \left\{ \boldsymbol{P}_{k-1|k-1}^{i} + [\boldsymbol{x}_{k-1|k-1}^{i} - \hat{\boldsymbol{x}}_{k-1|k-1}^{0j}][\boldsymbol{x}_{k-1|k-1}^{i} - \hat{\boldsymbol{x}}_{k-1|k-1}^{0j}]^{\mathrm{T}} \right\} . \tag{25}$$

Predicted state and covariance.

Finally, $\hat{\boldsymbol{x}}_{k|k}^{(0)j}$ and $\boldsymbol{P}_{k|k}^{(0)j}$ are estimated by a weighted sum of estimates from all filters.

$$\hat{\boldsymbol{x}}_{k|k}^{(0)j} = \boldsymbol{F}_{k,k-1}^{j} \hat{\boldsymbol{x}}_{k-1|k-1}^{(0)j} \tag{26}$$

$$\boldsymbol{P}_{k|k}^{(0)j} = \boldsymbol{F}_{k,k-1}^{j} \boldsymbol{P}_{k-1|k-1}^{(0)j} (\boldsymbol{F}_{k,k-1}^{j})^{T} + \boldsymbol{Q}_k \tag{27}$$

where $j = 1, 2, \cdots, s$.

3.2.2 Measurement Update (N iterations)

Set the initial value of the iteration.

Set the initial value $\alpha_{k|k,l}^{(0)}$ of the iteration:

$$\alpha_{k|k,l}^{(0)} = 0.5 + \alpha_{k|k-1,l} . \tag{28}$$

Set the initial value $\beta_{k|k,l}^{(0)}$ of the iteration:

$$\beta_{k|k,l}^{(0)} = \beta_{k|k-1,l} . \tag{29}$$

Suppose the NACp be level r at time k, the corresponding accuracy interval is $[L_r + 1, L_r]$, $L_0 > L_1 > L_2 > \cdots > L_{12} = 0$. When the initial value of the l-th diagonal element in $\hat{\boldsymbol{R}}_k$ exceeds the scope of current accuracy category, $(\hat{\boldsymbol{R}}_k < L_{r+1}) \cup (\hat{\boldsymbol{R}}_k < L_{r+1})$. $\beta_{k|k,l}^{(0)}$ will be reset to $\beta_{k|k,l}^{(0)} = L_{r+1} \times \alpha_{k|k,l}^{(0)}$. This step realizes the introduction addition of the NACp information into the KAVBMM algorithm.

Iterative calculation.

By applying the equation (30), after N iterations, state estimation and measurement noise covariance estimation are obtained:

$$\hat{\boldsymbol{x}}_{k|k}^{(n+1)j} = \hat{\boldsymbol{x}}_{k|k}^{(0)j} + \boldsymbol{P}_{k|k}^{(0)j} \boldsymbol{H}_k^{T} (\boldsymbol{H}_k \boldsymbol{P}_{k|k}^{(0)j} \boldsymbol{H}_k^{T} + \hat{\boldsymbol{R}}_{k|k}^{(n)})^{-1} (\boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k}^{(0)j}) \tag{30}$$

$$\boldsymbol{P}_{k|k}^{(n+1)j} = \boldsymbol{P}_{k|k}^{(0)j} - \boldsymbol{P}_{k|k}^{(0)j} \boldsymbol{H}_k^{T} (\boldsymbol{H}_k \boldsymbol{P}_{k|k}^{(0)j} \boldsymbol{H}_k^{T} + \hat{\boldsymbol{R}}_{k|k}^{(n)})^{-1} \boldsymbol{H}_k \boldsymbol{P}_{k|k}^{(0)j} . \tag{31}$$

The likelihood function $\Lambda_k^{(n+1)j}$ is calculated by using the measurement residual $\upsilon_k^{(n+1)j}$ and the residual covariance $S_k^{(n+1)j}$ is obtained after Kalman filtering.

$$\Lambda_k^{(n+1)j} = \frac{1}{(2\pi)^{d/2} \left| S_k^{(n+1)j} \right|^{1/2}} \exp\left\{ -\frac{1}{2} (\upsilon_k^{(n+1)j})^T (S_k^{(n+1)j})^{-1} \upsilon_k^{(n+1)j} \right\}. \tag{32}$$

The model probability $\mu_k^{(n+1)j}$ of the model m_k^j is obtained.

$$\mu_k^{(n+1)j} = \frac{1}{c} \overline{c}_k^{(n+1)j} \Lambda_k^{(n+1)j} \tag{33}$$

$$c = \sum_{j=1}^{r} \overline{c}_k^{(n+1)j} \Lambda_k^{(n+1)j} . \tag{34}$$

Get the overall estimate of the state and covariance:

$$\hat{\boldsymbol{x}}_{k|k}^{(n+1)} = \sum_{j=1}^{r} \boldsymbol{x}_{k|k}^{(n+1)j} \mu_k^{(n+1)j} \tag{35}$$

$$\boldsymbol{P}_{k|k}^{(n+1)} = \sum_{j=1}^{r} \mu_k^{(n+1)j} \left\{ \boldsymbol{P}_{k|k}^{(n+1)j} + \left[\hat{\boldsymbol{x}}_{k|k}^{(n+1)} - \hat{x}_{k|k}^{(n+1)j} \right] \left[\hat{\boldsymbol{x}}_{k|k}^{(n+1)} - \hat{\boldsymbol{x}}_{k|k}^{(n+1)j} \right]^T \right\}. \tag{36}$$

Parameters update.

Finally update shape parameters $\alpha_{k|k,l}$ and scale parameter $\beta_{k|k,l}^{(n+1)}$.

$$\alpha_{k|k,l} = 0.5 + \alpha_{k|k,l}^{(0)} \tag{37}$$

$$\beta_{k|k,l}^{(n+1)} = \beta_{k|k,l}^{(0)} + \frac{1}{2} (\boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k}^{(n+1)})^2 + \frac{1}{2} (\boldsymbol{H}_k \boldsymbol{P}_{k|k}^{(n+1)} H_k^{\mathrm{T}}) . \tag{38}$$

Here, $l = 1, 2, \cdots, d$; $j = 1, 2, \cdots, s$; $n = 1, 2, \cdots, N-1$. The measurement noise covariance parameter is updated where *N* indicates the iteration times for measurement updating. When the *N* times iteration is completed, set

$$\beta_{k|k,i} = \beta_{k|k,i}^{(N)} \tag{39}$$

$$\mu_k^j = \mu_k^{(N)j} \tag{40}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k}^{(N)} \tag{41}$$

$$P_{k|k} = P_{k|k}^{(N)} . \tag{42}$$

4 Experimental Results and Analysis

4.1 Simulation Experiment and Discussion

In this simulation, the motion of the maneuvering target includes two types of model, namely, constant velocity (CV) and coordinated turn (CT). In order to simulate the characteristics of maneuvering measurement, the zero-mean white Gaussian noise was added to the simulation data. The simulation parameters are set as follows: sampling interval T=1s, the NACp is set to (level 8, 5, 7, and 6), corresponding to the true value of the measurement noise variance, (3m, 10m], (10m, 30m], (30m, 92.6m], and (92.6m, 185.2m], respectively. The trajectory is assumed as follows: The initial position is located at [1000*m*, 1000*m*, 1000*m*], and the initial velocity is [200*m*/s, 200*m*/s, 200*m*/s]. The motion process of the target is split into five different periods as shown in Table 2.

Table 2. Change of target flight model

Model	Time of duration (s)	Velocity or angular velocity
CV	1s~50s	200m/s,200m/s,200m/s
CT	51s~80s	+3°/s
CV	81s~130s	200m/s,200m/s,200m/s
CT	131s~160s	-3°/s
CV	161s~200s	200m/s,200m/s,200m/s

Experimental parameters are set as follows. The scalar factor vector $\rho=[1-e^{-2},1-e^{-2},1-e^{-2}]^{\mathrm{T}}$, N=5. The initial value of the model probability is [0.3, 0.3, 0.4]. The model transition probability matrix Π of the Markov chain is described as follows:

$$\Pi=\begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.05 & 0.15 & 0.8 \end{bmatrix}. \tag{43}$$

Two experiments are designed for this simulation as follows. Moreover, these simulation experiments are conducted by using a computer with a dual-core CPU of Pentium 4 2.93 GHz, 1-GB RAM. The programs are performed by using the Matlab 2014b version software.

Experiment 1. It is assumed that the measurement noise in the IMM algorithm is unknown, it is set to a fixed standard deviation of 100 meters, and the IMM algorithm and the KAVBMM algorithm are applied to filter the measurement, respectively. Fig. 4 shows the distribution of the running model for the KAVBMM algorithm. It can be seen that the algorithm is capable to estimate the motion model of the target. Fig. 5 presents the comparison of the real noise variances and the estimation of the KAVBMM algorithm. Fig. 6 shows the ratio of the measurement noise variance to the real value of the KAVBMM algorithm, where the four sub-pictures a, b, c, and d use different N values. It can be seen that the KAVBMM algorithm can accurately estimate the standard deviation of the measurement noise, and the state estimation accuracy can be maintained at a high level. After measuring the noise change each time, the KAVBMM algorithm can converge within a short space of time (about 10 seconds or so) and estimate the variance of the measurement noise in an accurate way. In Fig. 6, the error at time 40 is merely 5m due to its small real value, while 110m at time 140, for which the ratio change is relatively inconspicuous. As for the setting of N, it is obtained from a large number of experiments. The experiment shows that 5 iterations can lead to good estimation results, and the rising number of times will improve the estimation accuracy. However, the increase is limited, and the calculation load will be increased proportionately. Therefore, N=5 iterations are adopted in this paper, which is a compromise choice between tracking accuracy and calculation load.

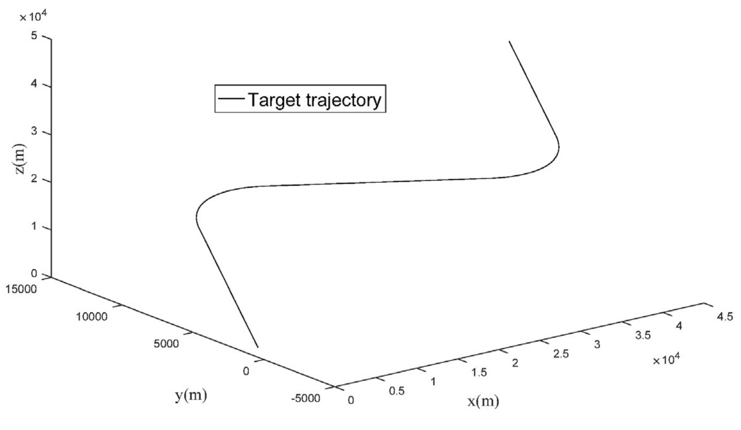


Fig. 3. Target's simulation motion trajectory

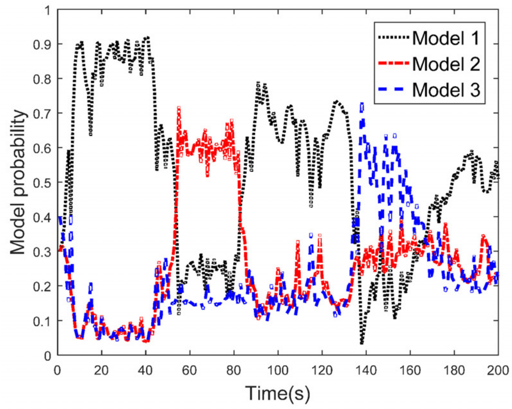


Fig. 4. Probability distribution of three motion models in the KAVBMM algorithm

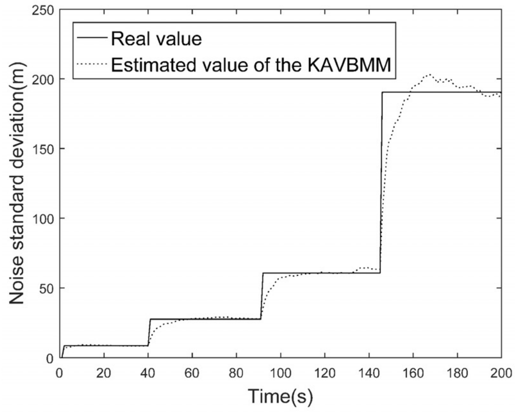
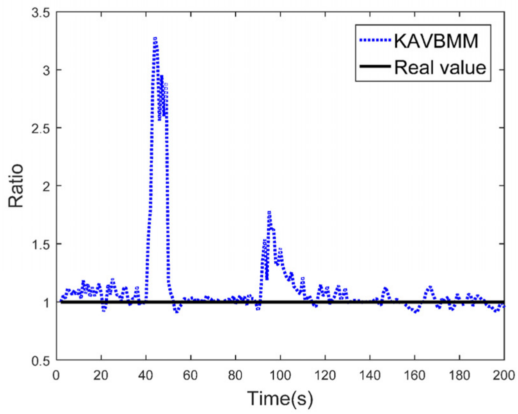
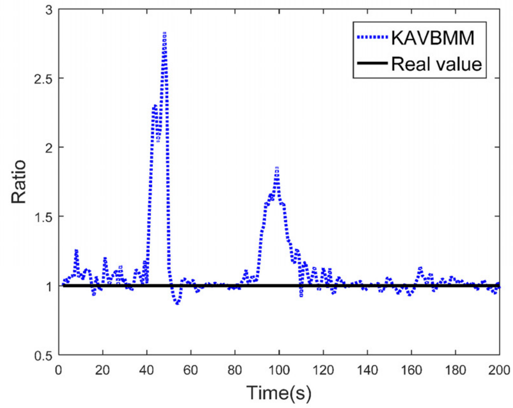


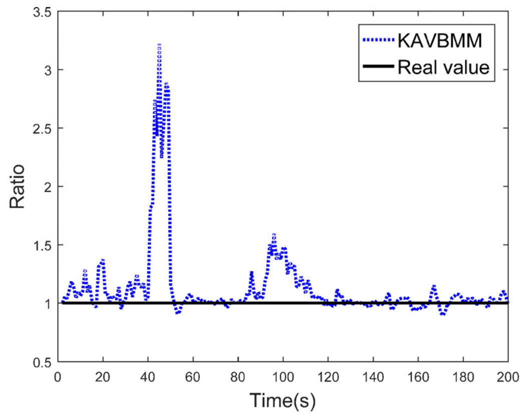
Fig. 5. Real noise variances and estimation of the KAVBMM algorithm (*N*=5)



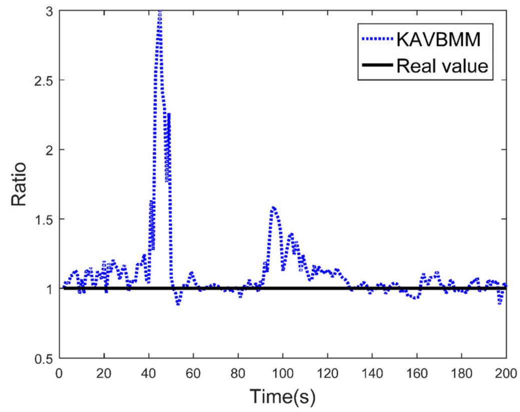
(a) iterations *N*=4



(b) iterations *N*=5



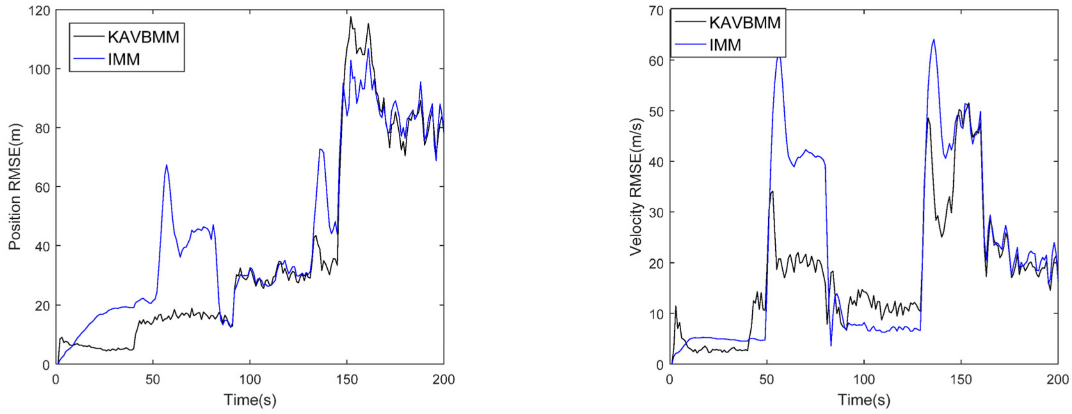
(c) iterations *N*=6



(d) iterations *N*=10

Fig. 6. Measurement noise variance ratio of the value obtained by the KAVBMM algorithm and the real value (*N* equals 4, 5, 6, 10 respectively)

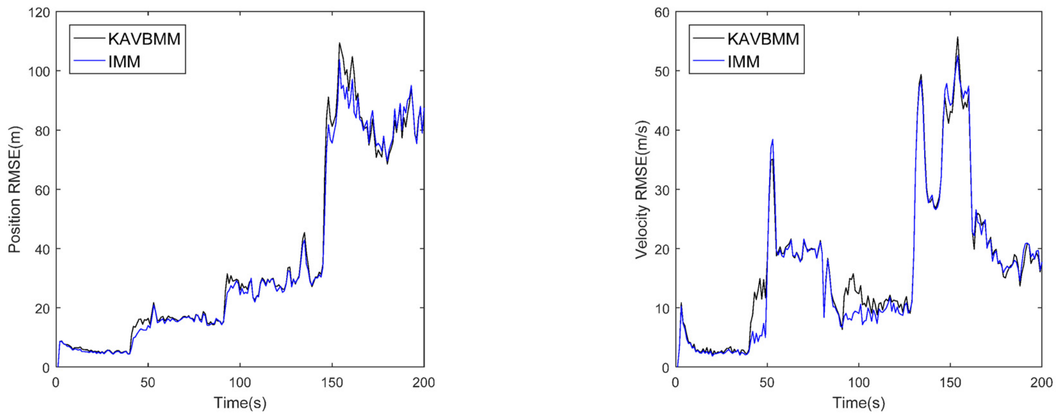
Fig. 7(a) presents a comparison between the measurement noise standard deviation and the real noise standard deviation estimated by the KAVBMM algorithm. Fig. 7(b) demonstrates the comparison between the IMM algorithm and the KAVBMM algorithm in terms RMSE of the position and velocity. When the variance of the IMM differs significantly from the real value, the estimation effect is poor. The fixed standard deviation is preset to be close to the real value during the 146s to 200s period, for which the estimated performance of the IMM algorithm is similar to that of the KAVBMM algorithm.



(a) Position RMSE (b) Velocity RMSE

Fig. 7. Comparison of KAVBMM in first situation

Experiment 2. It is assumed that the IMM algorithm with known real measurement noise variance is compared against the KAVBMM algorithm with unknown measurement noise variance. Fig. 8 shows a comparison between the KAVBMM algorithm with unknown noise variance and the IMM algorithm with the known variance of the measurement noise in terms of position (Fig. 8(a)) and velocity (Fig. 8(b)) RMSE. It can be seen from the figure that the performance of the two is broadly the same. Therefore, it can be concluded that the KAVBMM algorithm is capable to achieve a performance that matches the IMM algorithm of known measurement noise variance when the variance of measurement noise is unknown as the KAVBMM algorithm applies the variational Bayesian method to estimate the noise variance by using NACp values.



(a) Position RMSE (b) Velocity RMSE

Fig. 8. Comparison of the KAVBMM and the IMM in second situation

From the results of these two simulation experiments, they the KAVBMM algorithm can obtain good tracking performance compared with other tracking methods.

4.2 Experiment on the Real ADS-B Measurement and Discussion

In order to demonstrate the performance of the KAVBMM algorithm, a real tracking data generated from an ADS-B measurement record is applied. The trajectory is formed by 160 observations made by the ADS-B equipment. The sampling interval is 1 second. The NACp of this record is shown in Table 3.

Table 3. The real NACp value

Time of duration(s)	NACp	VEPU	EPU
1s~17s	9	(15m 45m]	(10m 30m]
18s-160s	10	(4m 15m]	(3m 10m]

Fig. 9 demonstrates that the measurement noise standard deviations obtained by the KAVBMM algorithm are all within the range of the NACp. As the real value of the target position is incapable to be obtained in the measurement experiment, the tracking performance of the KAVBMM algorithm is assessed by observing the filtered state estimation error variance. The estimation result of the estimated error variance of the KAVBMM algorithm is indicated in Fig. 10. It can be seen that the measurement noise variance estimation is convergent, as a result of which the measurement noise variance estimation is accurate and smooth. The filtering result obtained from the ADS-B measurement demonstrates that the KAVBMM algorithm is capable to track the target accurately with excellent and consistent filtering performance.

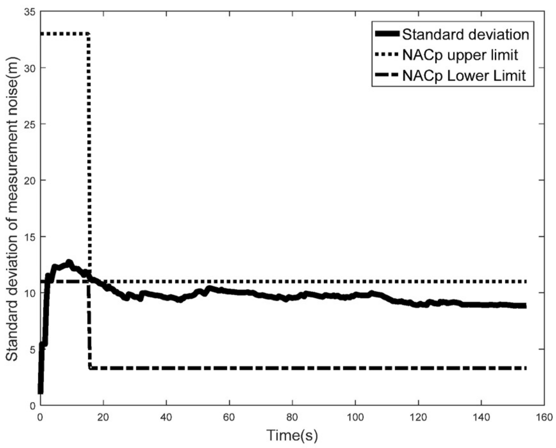


Fig. 9. The standard deviation range of measurement noise

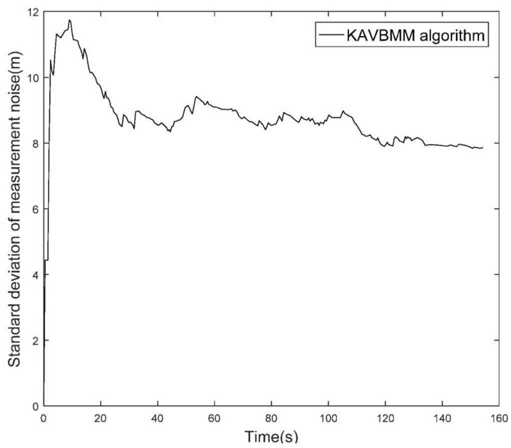


Fig. 10. The estimated measurement noise variance

The measurement noise variance estimation lies in the NACp corresponding interval (see Table 3) and is in convergence in Fig. 9. The variational Bayesian approximation can estimate the unknown measurement noise variance and adapt to its change. Fig. 10 illustrates that the KAVBMM algorithm can track the target accurately in the real situation. Therefore, the KAVBMM algorithm has an excellent tracking performance.

5 Conclusion

To track a maneuvering target using ADS-B equipment, a KAVBMM algorithm is developed in this paper. In the proposed KAVBMM algorithm, the IMM method is first introduced for adaptation to the change of target maneuver modes. Meanwhile, it adopts the accuracy category information of ADS-B measurements to get the dynamic changes in measurement noise and then estimates the noise variance on measurements by using screening. As illustrated in the experimental results of the simulation experiment and the real-data experiment, the proposed KAVBMM algorithm is effective, and it can keep track of a maneuvering target when the noise variance is unknown.

In further work, we will extend the proposed KAVBMM algorithm in multi-sensor multi-target tracking, and further improve the tracking performance for maneuvering targets.

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