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A Multi-target Tracking Algorithm for Pseudo Missed-detection Based on the PHD Filter

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Abstract. As the promising and efficient approximation of the Bayesian filter for multi-target tracking, the probability hypothesis density (PHD) filter iteratively propagates the first-order statistical moment of the multi-target states other than the multi-target density. However, the PHD filter cannot cope with the pseudo missed-detection problem caused by the improper position distribution of target-originated measurements in scenarios. To address the problem, we propose a multi-target filtering algorithm by integrating a missed-detection renovation scheme and an improved component fusion scheme into the PHD filter. Specifically, the PHD filter is used to estimate time-varying number of targets and their states, and the missed-detection renovation scheme is used to redistribute the PHD of the pseudo missed detections from the multi-target posterior PHD. In addition, the improved component fusion scheme is used to reduce and optimize the components of targets in multi-target posterior PHD. Experiment results demonstrate that the proposed algorithm can achieve better estimation accuracy and reliability in possible pseudo missed-detection tracking scenarios when compared against the related existing multi-target PHD filters.

Keywords: multi-target tracking, probability hypothesis density filter, pseudo missed-detection, random finite sets

1 Introduction

Due to its capability of handling an unknown and time-varying number of targets, the random finite set (RFS)-based multi-target Bayesian tracking algorithm has been receiving considerable attention worldwide over the last decade. The RFS-based PHD [1-2] filter is a promising and efficient suboptimal approximation for the multi-target Bayesian filter. Instead of propagating the full multi-target posterior density in multi-target Bayes filter, the PHD filter recursively propagates the first-order statistical moment of multi-target state during each filtering recursion. The Gaussian mixture (GM) and particle filter (PF) are two implementations to the non-analyticity of the PHD filter, which are called GM-PHD [3] and PF-PHD [4], respectively. Due to the efficiency and simplicity of the PHD filter, the standard PHD filter and PHD-based improved versions [5-10] have been extensively applied in target tracking [11-13], computer vision [14-15], mobile robot [16-17] and vehicle tracking [18-19].

Instead of using the explicit data association in the traditional multi-target filtering algorithm, a soft association method is integrated into the PHD filter such that the computational cost of the PHD filter has been significantly reduced. However, the number of targets in the PHD filter is assumed to follow the Poisson distribution, which exaggerates the effect of the estimated cardinality of the multi-target set when missed-detection problem occurs. Generally, there are two types of miss-detection, which are pseudo missed-detection and true missed-detection. The former is not true missed-detection in nature, which is formed by the random improper position distribution of target-originated measurements in the

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target state space. The true missed-detection is caused by the fact that the target-originated measurements cannot be received by sensors during the sampling periods. In [2], a simple scenario is utilized to illustrate the pseudo missed-detection problem in the PHD filter. When the distance between two targets is far away in tracking scenarios, the multi-target posterior intensity of the PHD filter is bimodal at each time step. As the targets move close together, the bimodal multi-target posterior intensity cannot be established at each filtering period. Therefore, state estimates of real targets cannot be extracted entirely in the PHD filter. Based on the maximum likelihood probability multi-hypothesis tracker(ML-PMHT) [20], Schoenecker et al. developed a non-Bayesian theory framework to analyze how close the two targets can get to one another and still be resolvable as two distinct targets in recent paper [21]. In theory, the proposed approach would be used in vehicle or pedestrian tracking scenarios on a pair-wise basis, but that would become computationally very burdensome very quickly.

To overcome the pseudo missed-detection problem, a marginal distribution based multi-target Bayes (MDB) filter is proposed in [22]. The MDB filter replaces joint distribution of multi-target states with the distribution of individual target state, which recursively propagates both the marginal distribution and existence probability of each individual target during the filtering process. Owning to the fact that the independence of individual target can be maintained, the MDB filter can provide higher filtering accuracy in terms of the states of targets and their number when tracking crossing targets in low clutter rate environments. Unfortunately, the MDB filter has the drawback that the estimated number of targets may be over-estimated when the intensity of clutter is comparatively heavy. Especially, the filtering performance of the MDB filter degrades significantly in tracking paralleling targets in dense cluttered circumstances. In [23], Zhou et al. proposed an improved PHD filter with the Gaussian mixture implementation to solve the pseudo missed-detection problem in video target scenarios. The measurement origin uncertainty can be solved properly by integrating the multi-feature of targets which are composed of spatial color information, gradient histogram and target contour. By using multi-feature fusion method, the measurements originated from different targets can be better distinguished, and the multi-target posterior intensity can be accurately approximated with these correct measurements in the update of the PHD filter. However, the proposed video multi-target GM-PHD filter is only applicable to track targets in the fields of computer vision. Aiming to solve the missed detection problem, Yazdian-Dehkordi et al. proposed a heuristic method named Refined GM-PHD filter [24]. The proposed filter jointly propagates the survival model and probability of confirmation of each target in the filter recursion. Based on the survival model and probability of confirmation, a novel state extraction method is proposed to estimate the states of missed detections. The results show that the Refined GM-PHD filter improves the filtering accuracy of the GM-PHD filter in missed-detection scenarios. Owning to initializing some crucial parameters, the Refined GM-PHD filter suffers from extremely terrible filtering accuracy during initial time steps. In addition, the Refined GM-PHD filter also fails to accurately track multiple targets in dense clutter or low detection probability scenarios. In [25], a Multi-scan GM-PHD filter is proposed to track nearby targets in the presence of data association uncertainty, noise and false alarms. Based on the history of targets in multiple last steps and exponential decay function, the proposed algorithm utilizes a multi-scan state estimate approach to extract the states of undetected targets. Compared with the GM-PHD and Refined GM-PHD filters, the Multi-scan GM-PHD filter improves the tracking performance in terms of the number of targets and their states. However, the length threshold of the multiple time steps is a crucial factor to the Multi-scan GM-PHD filter, which cannot provide better filtering performance if the length threshold is set to a smaller value. The complexity of the proposed filter increases with the use of the multi-scan technology. Lastly, a large number of parameters are used for the first time in the Multi-scan GM-PHD filter, and thresholds of these parameters are empirically selected and cannot change with various tracking environments.

It is desirable therefore to have an effective and simple multi-target algorithm that can jointly estimate the number of targets and their states from noisy measurements in the pseudo undetected target scenarios. This gives the motivation for the multi-target PHD tracking algorithm with pseudo missed-detection scheme. In this paper, an improved PHD filtering algorithm with the implementation of Gaussian mixture for multi-target tracking is proposed, where two schemes, namely pseudo missed-detection renovation and improved pruning and merging of components, are introduced into the GM-PHD filter. The GM-PHD filter is responsible to propagate and update the multi-target intensity at each filter recursion step. The improved pruning and merging scheme of Gaussian components is used to generate the comparatively optimal components for approximating the multi-target posterior intensity as well as control the number of the component in the multi-target posterior intensity. The pseudo missed-detection renovation scheme is proposed to detect pseudo missed-detection problem and renovate the PHD of the undetected targets by sufficiently considering the effective weight distribution of the components of targets.

The key contributions of our approach are threefold. First, the pseudo missed-detection renovation scheme is incorporated into the GM-PHD filter to restore the weights of the undetected targets, which can make target posterior intensity more accurate. Second, the proposed pruning and merging scheme of Gaussian components only merges eligible components belonging to each individual target, and propagates only one comparatively optimal component of each target to the next time step. Third, using a number of simulations mimicking pseudo missed-detection scenarios, we show that the proposed algorithm outperforms the related existing multi-target PHD filters in terms of the number of targets and their states.

The remainder of this paper is organized as follows. Section 2 briefly described the PHD filter as well as its Gaussian mixture implementation. The overall steps of the proposed multi-target PHD algorithm with the pseudo missed-detection scheme are elaborated in detail in Section 3. In Section 4, the performance evaluations of the proposed algorithm are provided in two scenarios. Our closing remarks are offered in Section 5.

2 The PHD Filter and Its Gaussian Mixture Implementation

2.1 The PHD Filter

Let N_k and M_k be the respective number of targets and measurements at time k, the multi-target states and sensor measurements can be represented by finite sets as $X_k = \{x_{k,1}, x_{k,i}, \ldots, x_{k,N_k}\}$ and $Z_k = \{z_{k,1}, z_{k,j}, \ldots, z_{k,M_k}\}$ in the RFS-based multi-target PHD filter, where $x_{k,i}$ is the state of the ith target and $z_{k,j}$ is the jth measurement. In the PHD filter, the first-order statistical moment is named PHD or intensity, and the target states and its number can be jointly estimated by recursively computing the multi-target posterior PHD. The filter recursion of the PHD filter is composed of equations of prediction and update.

Suppose that $\mathcal{D}_{k-1}(x_{k-1} \mid Z^{(k-1)})$ is the multi-target posterior PHD at time $k-1$, where $Z^{(k-1)}$ is a union of measurement sets between time 1 and $k-1$. The equation of prediction is

$$\mathcal{D}_{k|k-1}(x_k \mid Z^{(k-1)}) = \gamma_k(x_k) + \int \mathcal{H}_{k|k-1}(x_k, x_{k-1}) \mathcal{D}_{k-1}(x_{k-1} \mid Z^{(k-1)}) d x_{k-1}, \tag{1}$$

$$\mathcal{H}_{k|k-1}(x_k, x_{k-1}) = p_{s,k} f_{k|k-1}(x_k \mid x_{k-1}) + \beta_{k|k-1}(x_k \mid x_{k-1}), \tag{2}$$

where $p_{s,k}$ is survival probability, $f_{k|k-1}(x_k \mid x_{k-1})$ is transition probability of target state, $\gamma_k(x_k)$ is the PHD of newborn targets, and $\beta_{k|k-1}(x_k \mid x_{k-1})$ is the PHD of spawned targets at time k.

The equation of update is

$$\mathcal{D}_k\left(x_k \mid Z^{(k)}\right) = \left[1 - p_{d,k} + \sum_{z \in Z_k} \frac{p_{d,k} g_k(z \mid x_k)}{\lambda_c c(z) + \int p_{d,k} g_k(z \mid x_{k-1}) \mathcal{D}_{k|k-1}(x_{k-1} \mid Z^{(k-1)}) d x_{k-1}}\right] \mathcal{D}_{k|k-1}(x_k \mid Z^{(k-1)}), \tag{3}$$

where $g_k(z \mid x)$ is the target likelihood function, $p_{d,k}$ is the detection probability, and the PHD of clutter is given by $\lambda_c c(z)$, where λ_c is the mean of Poisson clutter points per scan and $c(\cdot)$ is the spatial distribution of clutter point.

2.2 The Gaussian Mixture Implementation of the PHD Filter

As can be seen from the equations of prediction and update, there are several integrals involving in the recursion of the PHD filter, which makes it difficult to obtain closed-form solutions. Therefore, numerical approach is needed to approximate the equations of prediction and update of the PHD filter. Under the linear Gaussian assumption, the GM-PHD filter provides a closed-form solution of the PHD filter, which uses the weighted summation of components of targets to approximate the PHD of targets.

Let $\mathcal{N}(\cdot;m,P)$ be a Gaussian density with mean m and covariance P, and $\mathcal{D}_{k|k-1}(x)$ be the abbreviation $\mathcal{D}_{k|k-1}(x_k \mid Z^{(k-1)})$. The Gaussian mixture implementation of the multi-target predicted PHD $\mathcal{D}_{k|k-1}(x)$ in Eq. (1) can be given by

$$\mathcal{D}_{k|k-1}(x) = \sum_{j=1}^{J_{k|k-1}} w_{k|k-1}^{j} \mathcal{N}(x; m_{k|k-1}^{j}, P_{k|k-1}^{j}), \tag{4}$$

where $J_{k|k-1}$ is the number of Gaussian components, and $w_{k|k-1}^{j}$ is weight of the jth Gaussian component.

The Gaussian mixture implementation of multi-target posterior PHD $\mathcal{D}_k(x)$ in Eq. (3) can be represented by

$$\mathcal{D}_k(x) = (1 - p_{d,k})\mathcal{D}_{k|k-1}(x) + \sum_{z \in Z_k} \sum_{j=1}^{J_{k|k-1}} w_k^j(z) \mathcal{N}(x; m_{k|k}^{j}(z), P_{k|k}^{j}(z)), \tag{5}$$

where the weight of the jth component can be obtained by

$$w_k^j(z) = \frac{p_{d,k} w_{k|k-1}^{j} g_k(z \mid x^j)}{\lambda_c c(z) + p_{d,k} \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{i} g_k(z \mid x^i)}. \tag{6}$$

To keep the efficiency of the recursion of the GM-PHD filter as time progresses, the number of Gaussian components should be reduced by pruning the components with weak weights and merging the components that are so close together in the multi-target posterior PHD. For simplicity here, the detailed formulation of the GM-PHD filter is available in [3].

3 The Proposed Multi-target PHD Algorithm for Pseudo Missed-detection

3.1 The Pseudo Missed-detection Renovation Scheme

The PHD of targets in Eqs. (4) and (5) is approximated via the weighted summation of components, and each component is expressed by the weight w, mean m and covariance P. To distinguish the components of different targets in multi-target PHD, an auxiliary parameter, namely label ℓ, is introduced into each component. Therefore, each component of a target can be represented by state parameter set $x_k = \{w_k, m_k, P_k, \ell_k\}$. The multi-target predicted PHD $\mathcal{D}_{k|k-1}(x)$ in Eq. (4) can be given in form of the component set $\aleph_{k|k-1}$ as

$$\aleph_{k|k-1} = \{x_{k|k-1}^{i}\}_{i=1}^{J_{k|k-1}} = \{w_{k|k-1}^{i}, m_{k|k-1}^{i}, P_{k|k-1}^{i}, \ell_{k|k-1}^{i}\}_{i=1}^{J_{k|k-1}}, \tag{7}$$

where the component $x_{k|k-1}^{i}$ in component set $\aleph_{k|k-1}$ belonging to the same target has the same label value, otherwise the component $x_{k|k-1}^{i}$ should be assigned with a different label value.

It can be seen from the Eq. (5) that the multi-target posterior PHD $\mathcal{D}_k(x)$ is composed of both the PHD of missed-detection term $(1 - p_{d,k})\mathcal{D}_{k|k-1}(x)$ and the PHD of measurement-update term $\sum_{z \in Z_k} \sum_{j=1}^{J_{k|k-1}} w_k^j(z) \mathcal{N}(x; m_{k|k}^{j}(z), P_{k|k}^{j}(z))$. The PHD of the missed-detection term preserves the PHD of undetected targets that have no measurements in current measurement set. The PHD of the measurement-update term provides the PHD of detected targets with the corresponding measurements in current measurement set. Owning to the fact that the corresponding measurements of undetected targets are not preserved in the measurement set Z_k at time k, the PHD of these targets only exists in

$(1-p_{d,k})\mathcal{D}_{k|k-1}(x)$. However, the multi-target posterior PHD $\mathcal{D}_k(x)$ preserves the PHD of the pseudo undetected targets in that the measurements of these undetected targets are in the measurement set Z_k at time k.

The importance of each component in the PHD of targets is reflected by its weight, which is updated during the update step of the GM-PHD filter at time k. Due to the improper location distribution of target-originated measurements in the target state space, the corresponding components of the pseudo undetected targets cannot obtain correct weights via Eq. (6). Additionally, uncertain association of targets with measurements makes it more difficult to distinguish the origin of the target-originated measurements. At time k, the PHD filter uses the Euclidean distance between targets and measurements to decide which measurement in the measurement set Z_k a target belongs to. Therefore, the PHD of the pseudo missed detections is occupied by the PHD of several other targets. By making full use of the multi-target posterior PHD $\mathcal{D}_k(x)$, the incorrect PHD of the pseudo undetected targets can be renovated, and the pseudo missed-detection problem can be solved.

When the pruning and merging step of the GM-PHD filter terminates, assume that the multi-target posterior PHD $\mathcal{D}_k(x)$ in form of component set $\aleph_{mer,k}$ can be approximated as

$$\aleph_{mer,k} = \left\{x_k^i\right\}_{i=1}^{J_k} = \left\{w_k^i, m_k^i, P_k^i, \ell_k^i\right\}_{i=1}^{J_k}. \tag{8}$$

A possible target set $\mathcal{L}_k$ for the multi-target posterior PHD $\mathcal{D}_k(x)$ can be obtained as

$$\mathcal{L}_k = \left\{\ell_k^i \middle| x_k^i \in \aleph_{mer,k}, \forall i = 1 : J_k\right\}. \tag{9}$$

Then, a pseudo missed-detection indicator $f_{pmd,k}$ at time k can be computed by

$$f_{pmd,k} = \begin{cases} true \quad , & round(S_{w,k}) \geq count(\mathcal{L}_k) \\ flase \quad , & otherwise \end{cases}, \tag{10}$$

where $round(\cdot)$ is the rounding function, $count(\cdot)$ is the function that computes the number of elements in a set, $S_{w,k}$ is the weight summation of targets which can be obtained via Eq. (28).

Given the classical weight threshold of target state extraction ϖ_{th} defined in [3], the number of the effective targets with weights greater than ϖ_{th} denoted by $N_{g,k}$, and the weight summation of components of the effective targets $S_{e,k}$ can be represented as

$$N_{g,k} = count\left(\Theta_k\right), \tag{11}$$

$$S_{e,k} = \sum w_k^j \text{ , } \forall j \in \Theta_k, \tag{12}$$

$$\Theta_k = \left\{i \middle| w_k^i > \varpi_{th}, \forall i = 1 : J_k\right\}. \tag{13}$$

The weight summation of components of targets with weights below ϖ_{th} represented by $S_{b,k}$ in the multi-target posterior PHD can be computed as

$$S_{b,k} = \sum w_k^n \text{ , } \forall n \in \zeta_k, \tag{14}$$

$$\zeta_k = \left\{j\right\}_{j=1}^{J_k} \setminus \Theta_k. \tag{15}$$

If the condition towards $f_{pmd,k} = true$ at time k, the multi-target posterior PHD is incorrectly updated in the update of the GM-PHD filter. The weights of components of all targets in the multi-target posterior PHD should be redistributed such that the PHD of the pseudo undetected targets can be renovated. Finally, the multi-target posterior PHD can be accurately approximated. Assume that the possible pseudo undetected target set $\psi_{pmd,k}$ can be represented according to

$$\psi_{pmd,k} = \left\{ \ell_k^i \middle| w_k^i \le \omega_{th}, \forall i = 1 : J_k \right\}. \tag{16}$$

For the target with the label ℓ_k^i in the set $\psi_{pmd,k}$, a new weight $w_{new,k}^i$ can be redistributed by

$$w_{new,k}^i = \begin{cases} 1 & , \ \delta_w^i > (1 - w_k^i) \\ w_k^i + \delta_w^i, & otherwise \end{cases}, \tag{17}$$

$$\delta_w^i = \frac{w_k^i \times (S_{w,k} - S_{e,k})}{S_{b,k}}. \tag{18}$$

where δ_w^i is the compensation weight of the target with the label ℓ_k^i.

3.2 The Pruning and Merging Scheme of the Gaussian Components

The number of Gaussian components that approximate the multi-target PHD increases exponentially as time progresses, making the computational load very high. Thus, an effective pruning and merging scheme for Gaussian components is proposed to manage the number of Gaussian components to increase efficiency. The biggest differences between the component fusion method of the standard GM-PHD filter and our proposed scheme are twofold. First, the merged components are limited to the components belonging to each individual target. Second, only one comparatively optimal component that better approximates the PHD of a target is selected and propagated to the next time step. The pruning and merging method of the proposed algorithm is described in Table 1.

Table 1. Pruning and merging method of the proposed algorithm

Given the component set $\aleph_k = \left\{ w_k^i, m_k^i, P_k^i, \ell_k^i \right\}_{i=1}^{J_k}$, the threshold T_{pru}, U and $J_{\max}$.
$l = 0$, $\Xi_{pru} = \left\{ i \mid w_k^i > T_{pru}, \forall i = 1 : J_k \right\}$.
Repeat
$l = l + 1$.
$i^* = \arg\max_{i \in \Xi} (w_k^i)$.
$\Xi_{sam} = \left\{ i \mid \ell_k^i = \ell_k^{i^*}, \forall i \in \Xi_{pru} \right\}$, $\Xi_{mer} = \left\{ i \mid (m_k^i - m_k^{i^*})^T (P_k^i)^{-1} (m_k^i - m_k^{i^*}) \le U, \forall i \in \Xi_{sam} \right\}$.
$w_{mer,k}^l = \sum_{i \in \Xi_{mer}} w_k^i$, $m_{mer,k}^l = \frac{1}{w_{mer,k}^l} \sum_{i \in \Xi_{mer}} w_k^i m_k^i$,
$P_{mer,k}^l = \frac{1}{w_{mer,k}^l} \sum_{i \in \Xi_{mer}} w_k^i (P_k^i + (m_{mer,k}^l - m_k^i)(m_{mer,k}^l - m_k^i)^T)$, $\ell_{mer,k}^l = \ell_k^{i^*}$.
$\Xi_{pru} = \Xi_{pru} \setminus \Xi_{sam}$.
Until $\Xi_{pru} = \emptyset$.
If $l > J_{\max}$, *then*
replace $\left\{ w_{mer,k}^i, m_{mer,k}^i, P_{mer,k}^i, \ell_{mer,k}^i \right\}_{i=1}^{l}$ by $J_{\max}$ components with greatest weighs
Output $\aleph_{mer,k} = \left\{ w_{mer,k}^i, m_{mer,k}^i, P_{mer,k}^i, \ell_{mer,k}^i \right\}_{i=1}^{l}$

3.3 The Detailed Steps of the Proposed Algorithm

Fig.1 shows the overall processes of the proposed algorithm, where the steps with the mark Δ are our newly proposed scheme in this paper. In addition, the detailed characteristics of all the steps are given subsequently.

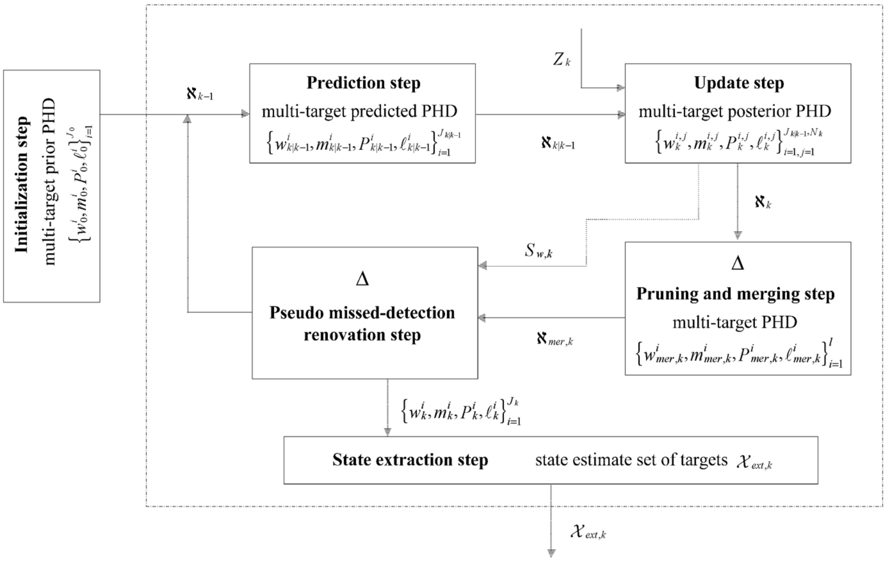


Fig. 1. The overall processes of the proposed algorithm

Initialization step. At time $k=0$, given the multi-target prior PHD $\mathcal{D}_0(x)$ in form of J_0 components as

$$\aleph_0 = \left\{ w_0^i, m_0^i, P_0^i, \ell_0^i \right\}_{i=1}^{J_0}. \tag{19}$$

Prediction step. At time $k-1$, assume that the multi-target posterior PHD $\mathcal{D}_{k-1}(x)$ is represented by the set $\aleph_{k-1} = \left\{ w_{k-1}^i, m_{k-1}^i, P_{k-1}^i, \ell_{k-1}^i \right\}_{i=1}^{J_{k-1}}$, then the multi-target predicted PHD $\mathcal{D}_{k|k-1}(x)$ can be approximated by a set as

$$\aleph_{k|k-1} = \left\{ w_{k|k-1}^i, m_{k|k-1}^i, P_{k|k-1}^i, \ell_{k|k-1}^i \right\}_{i=1}^{J_{k|k-1}}, \tag{20}$$

where the parameters in set $\aleph_{k|k-1}$ are

$$w_{k|k-1}^i = p_{s,k} w_{k-1}^i,\ m_{k|k-1}^i = F_{k-1} m_{k-1}^i, \tag{21}$$

$$P_{k|k-1}^i = Q_{k-1} + F_{k-1} P_{k-1}^i (F_{k-1})^T,\ \ell_{k|k-1}^i = \ell_{k-1}^i, \tag{22}$$

where F_{k-1} and Q_{k-1} are the state transition matrix and process noise covariance, respectively.

Update step. Given the multi-target predicted PHD in form of component set as Eq. (20) and the measurement set $Z_k = \left\{ z_k^j \right\}_{j=1}^{N_k}$, the multi-target posterior PHD $\mathcal{D}_k(x)$ in form of component set at time k can be denoted by

$$\aleph_k = \left\{ w_k^{i,j}, m_k^{i,j}, P_k^{i,j}, \ell_k^{i,j} \right\}_{i=1, j=1}^{J_{k|k-1}, N_k}, \tag{23}$$

where the corresponding parameters of each component are

$$w_k^{i,j} = \frac{p_{d,k} w_{k|k-1}^i \mathcal{N}(z_k^j; H_k m_{k|k-1}^i, H_k P_{k|k-1}^i (H_k)^T + R_k)}{\lambda_c c\left(z_k^j\right) + p_{d,k} \sum_{n=1}^{J_{k|k-1}} w_{k|k-1}^n \mathcal{N}(z_k^j; H_k m_{k|k-1}^n, H_k P_{k|k-1}^n (H_k)^T + R_k)}, \tag{24}$$

$$m_k^{i,j} = m_k^i + P_{k|k-1}^i (H_k)^T (H_k P_{k|k-1}^i (H_k)^T + R_k)^{-1} (z_k^j - H_k m_{k|k-1}^i), \tag{25}$$

$$P_k^{i,j} = (I - P_{k|k-1}^i (H_k)^T (H_k P_{k|k-1}^i (H_k)^T + R_k)^{-1} H_k) P_{k|k-1}^i, \tag{26}$$

$$\ell_k^{i,j} = \ell_{k|k-1}^{i} . \tag{27}$$

In addition, the weight summation of the components in set $\aleph_k$ denoted by $S_{w,k}$ can be obtained as

$$S_{w,k} = \sum_{i=1}^{J_{k|k-1}} \sum_{j=1}^{N_k} w_k^{i,j} . \tag{28}$$

Pruning and merging step. The components that approximate multi-target posterior PHD are pruned and merged by using the proposed pruning and merging scheme as illustrated in Section 3.2.

Pseudo missed-detection renovation step. The PHD of the pseudo missed-detection is renovated by using the method as illustrated in Section 3.1. If some targets cannot have effective weights after the pseudo missed-detection renovation step, these targets don't necessarily disappear. In the proposed algorithm, the targets with smaller weights more than three subsequent scans are considered to have disappeared. Otherwise, the weights of these targets are assigned to the maximum predicted weight at each time step.

State extraction step. Assume that the multi-target PHD are component set of the form $\left\{w_k^i, m_k^i, P_k^i, \ell_k^i\right\}_{i=1}^{J_k}$, the state estimate set of targets can be represented as

$$\mathcal{X}_{ext,k} = \left\{m_k^i \middle| w_k^i > \omega_{th}, \forall i = 1 : J_k\right\} . \tag{29}$$

4 Simulation Results

The effectiveness of the proposed multi-target PHD algorithm is evaluated in several simulated multi-target tracking environments. We compare the proposed algorithm against the GM-PHD, Refined GM-PHD, Multi-scan GM-PHD and MDB filters. At time k, the target state $x_k = \left[x_{1,k}, x_{2,k}, x_{3,k}, x_{4,k}\right]^T$ is composed of the position $\left[x_{1,k}, x_{2,k}\right]^T$ and velocity $\left[x_{3,k}, x_{4,k}\right]^T$. The models of the target dynamic and measurement are given by

$$f_{k|k-1}(x_k \mid x_{k-1}) = \mathcal{N}(x_k; F_{k-1}x_{k-1}, Q_{k-1}) , \tag{30}$$

$$g_k(z_k \mid x_k) = \mathcal{N}(z_k; H_k x_k, R_k) , \tag{31}$$

where

$$F_{k-1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, Q_{k-1} = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \sigma_w^2 , H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, R_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sigma_v^2 , \tag{32}$$

where the sampling time $T = 1s$, and the standard deviations of the process noise and measurement noise are set to $\sigma_w = 0.2$ and $\sigma_v = 50$, respectively.

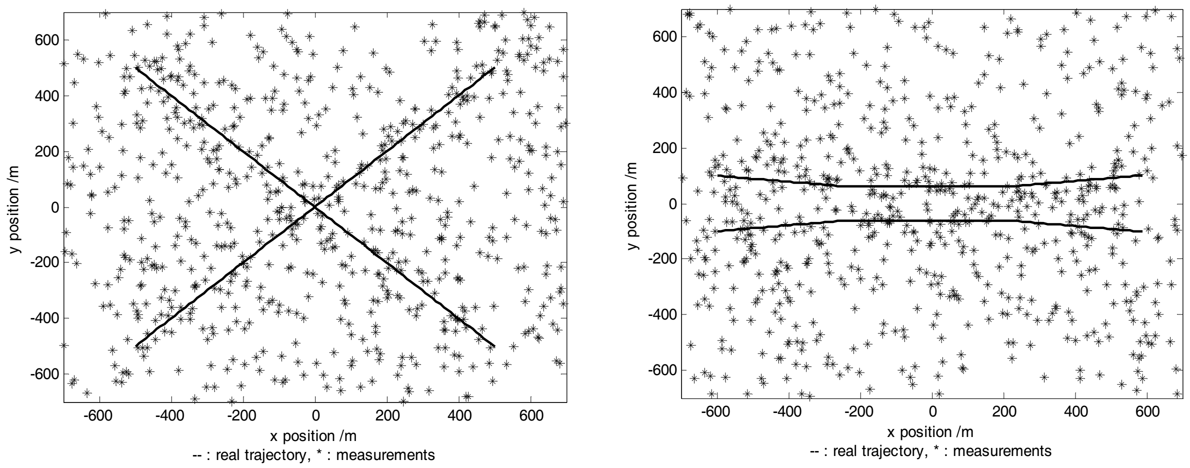
The survival probability and detection probability are set to $p_{s,k} = 0.99$ and $p_{d,k} = 0.99$, respectively. The three classical thresholds in the pruning and merging scheme of the proposed algorithm are set to $T_{pru} = 10^{-5}$, $U = 4$, $J_{\max} = 100$. The filtering performance of different filters in each experiment is obtained by 100 Monte Carlo runs. The optimal sub-pattern assignment (OSPA) distance [26] and number of target estimation error (NTE) [3] are used to evaluate the tracking performance.

$$OSPA_{p,c}\left(X_k, \hat{X}_k\right) = \left(\frac{1}{\left|\hat{X}_k\right|} \min_{\pi \in \Pi_{\left|\hat{X}_k\right|}} \sum_{i=1}^{\left|X_k\right|} \left(d_c\left(x^i, \hat{x}^{\pi(i)}\right) \right)^p + c^p \times \left(\left|\hat{X}_k\right| - \left|X_k\right| \right) \right)^{1/p} , \tag{33}$$

$$NTE\{X_k, \hat{X}_k\} = E\{|\hat{X}_k| - |X_k|\}, \quad \textbf{(34)}$$

where X_k and $\hat{X}_k$ are the respective ground truth and estimation of target state set. Two parameters of the OSPA distance are set to $p=1$ and $c=200$, respectively.

Example 1. This example is used for demonstrating the performance comparison of different filters in the cluttered scenarios with two crossing/paralleling targets. Fig. 2 shows the real trajectories of two crossing/paralleling targets and measurements over 100 times, where the clutter rate is set to $\lambda_c = 5$.

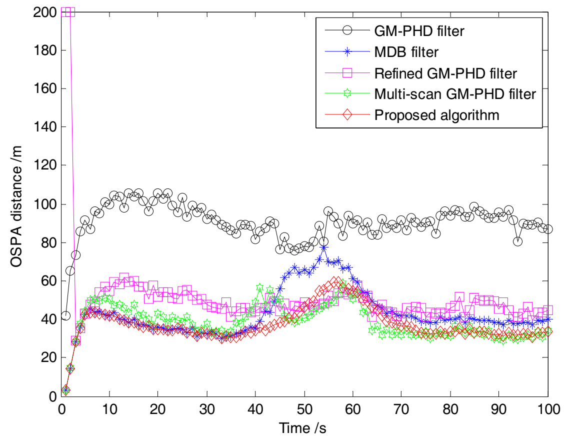


(a) The crossing target scenario (b) The paralleling target scenario

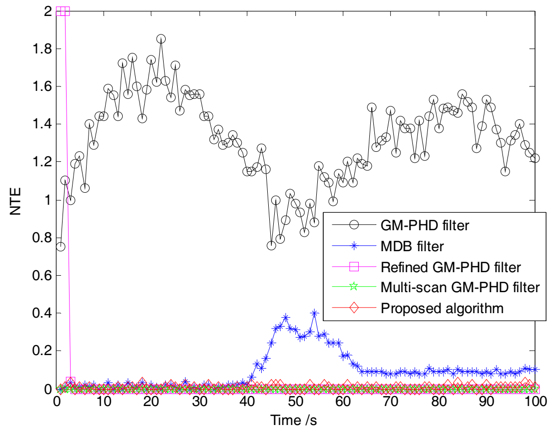
Fig. 2. Measurements and trajectories of targets

Fig. 3 shows the OSPA distances and NTE of different filters in crossing/paralleling target scenarios. When targets are well separated in crossing target scenario, the OSPA distances and NTE of the MDB filter and three GM-PHD-based algorithms are lower than those of the GM-PHD filter. It is evidently that the proposed algorithm achieves better performance than the other filters. When two targets move near each other in the scenario, the proposed algorithm not only offers better performance than the MDB filter, but also has similar performance to the Refine GM-PHD and Multi-scan GM-PHD filters. Due to clutter and pseudo missed-detection caused by closely spaced targets, the GM-PHD offer poor filtering performance in term of OSPA distance and NTE. It is difficult for the MDB filter on its own to reasonably solve the pseudo missed detections. Therefore, there is a peak on OSPA distance and NTE between the time steps 40 and 65. This phenomenon indicates the filtering accuracy of the MDB filter is comparatively low. Benefiting from two proposed methods, the proposed algorithm achieves accurate target number estimation and low OSPA distance between the time steps 40 and 65 shown in Fig. 3(b). Although the Refined GM-PHD filter can deal with the missed-detection problem, it provides relatively poor filtering performance because of clutter and parameter initialization problem. Affected by the fixed and empirical thresholds of auxiliary parameters, the Multi-scan GM-PHD filter gives two peaks in OSPA distance shown in Fig. 3(a).

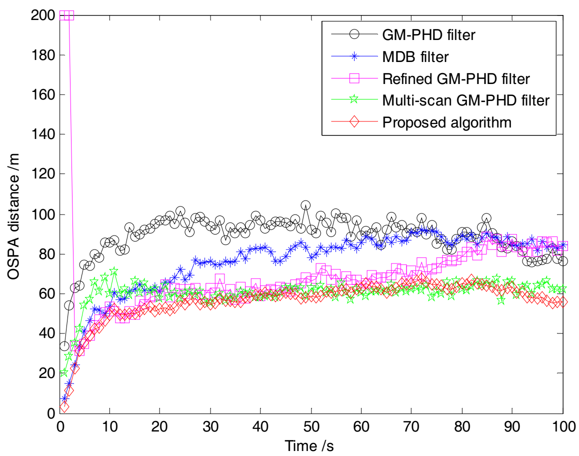
When tracking paralleling targets in cluttered scenario shown in Fig. 2(b), it can be seen from the Fig. 3(c) and Fig. 3(d) that the filtering performances of the MDB filter and three GM-PHD-based algorithms are better than that of the GM-PHD filter. Additionally, the proposed algorithm outperforms the other filters again. The reason for larger OSPA distance and NTE of the MDB filter is that the filter cannot resolve pseudo missed-detection caused by the closely spaced targets with parallel motion. Due to the deficiency of the Refined GM-PHD filter in handling the paralleling targets, the filter cannot effectively track the targets between the time steps 80 and 100. With the help of the exponential decay function, the tracking performance of the Multi-scan GM-PHD filter is similar to the proposed algorithm. Better filtering performance provided by the proposed algorithm is mainly attributed to the high resolution of pseudo undetected targets by using two proposed schemes, which are able to recoup and refine the incorrect target posterior PHD caused by the pseudo missed-detection problem.



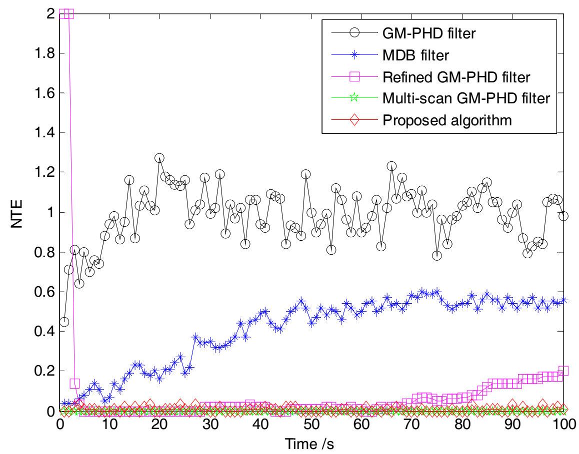
(a) OSPA distance in crossing target scenario



(b) NTE in crossing target scenario



(c) OSPA distance in paralleling target scenario

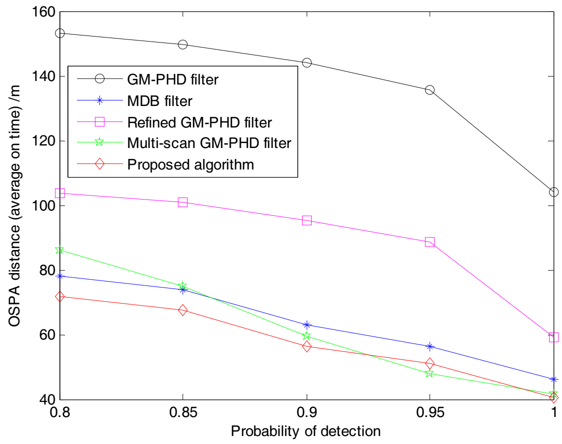


(d) NTE in paralleling target scenario

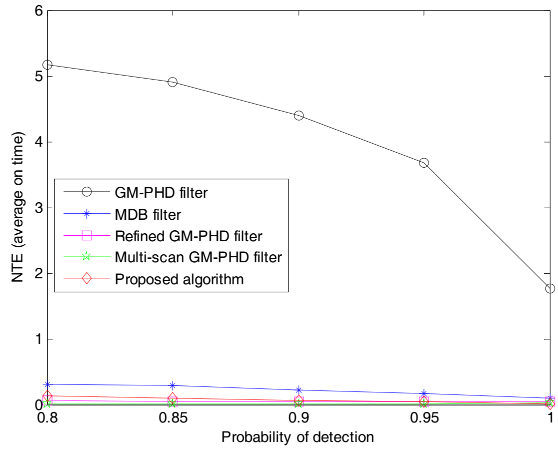
Fig. 3. Performance comparison of different algorithms in both scenarios

Example 2. This example is used to study the effectiveness of the proposed algorithm in two scenarios men-tioned above with various detection probabilities, where the probabilities of detection are set to $p_{d,k} = 0.8, 0.85, 0.9, 0.95, 1$, and the other parameters of both scenarios remain unchanged.

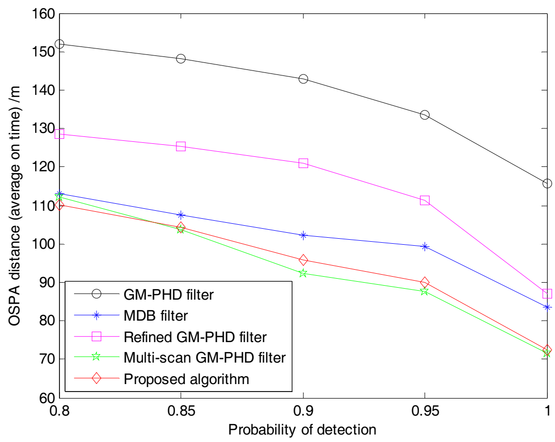
As can be observed that the proposed algorithm provides relatively reliable and accurate state estimates of targets reflected by the low OSPA distances for all the probabilities of detection shown in Fig. 4(a) and Fig. 4(c). Additionally, the NTE of the proposed algorithm shown in Fig. 4(b) and Fig. 4(d) illustrates the estimated target number is close to the number of real targets in both scenarios. As shown in Fig. 4, when the probability of detection increases, the filtering performances of all algorithms tends to rise. For the same probability of detection, the proposed algorithm outperforms the MDB and Refined GM-PHD filters in Fig. 4(a), and achieves similar OSPA distance with the Multi-scan GM-PHD filter in Fig. 4(c). The reason for the better performance of the proposed algorithm is that the inaccurate weights of targets can be effectively revised by applying the pseudo missed-detection renovation, and finally a relative optimal target posterior PHD can be obtained by using the proposed pruning and merging scheme of the Gaussian components.



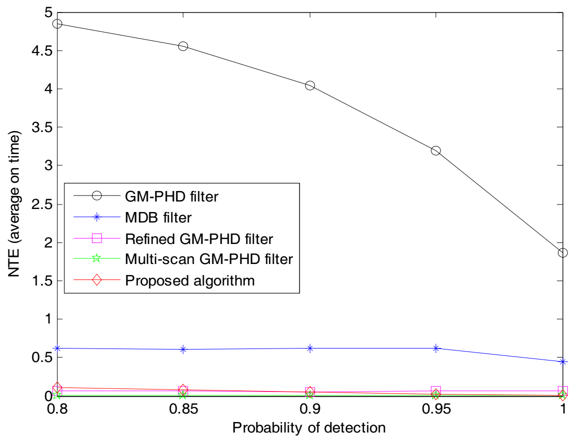
(a) OSPA distance in crossing target scenario



(b) NTE in crossing target scenario



(c) OSPA distance in paralleling target scenario

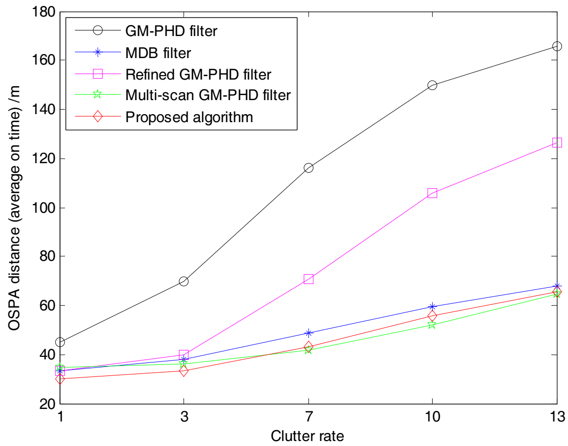


(d) NTE in paralleling target scenario

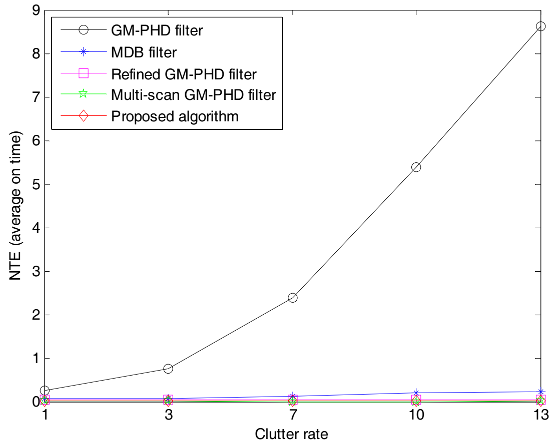
Fig. 4. Performance comparison of different algorithms with different detection probabilities

Example 3. This example is used to evaluate the filtering performance of the proposed algorithm in cross-ing/paralleling target scenarios with the clutter rate $\lambda_c = 1, 3, 7, 10, 13$, and the other parameters are the same as the crossing/paralleling scenarios.

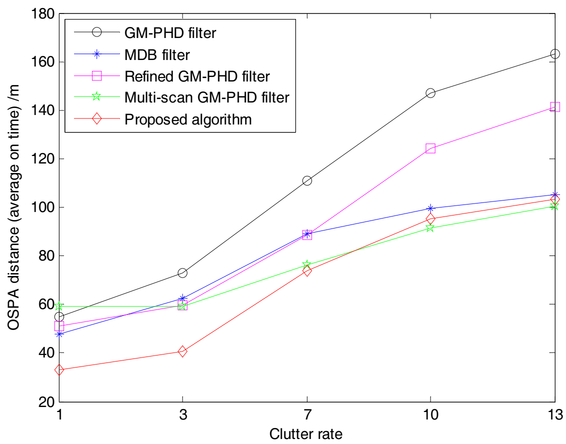
Fig. 5 shows the effect of various clutter rates on the OSPA distances and NTE of different filters. It can be seen that the OSPA distances of all the filters increase with clutter rate. The reason for this phenomenon is that these filters have to resolve higher detection uncertainty in distinguishing different targets and clutter in dense false alarm scenes which are more sophisticated. Except the GM-PHD and MDB filters, the NTE of the other filters almost maintain at zero, which shows relatively accurate cardinality estimate. As is shown in Fig. 5(a) and Fig. 5(c), at each clutter rate the proposed algorithm has consistently smaller OSPA distance than the GM-PHD, MDB and Refined GM-PHD filters. Compared with the Multi-scan GM-PHD filter, the proposed algorithm achieves short OSPA distance in relatively low clutter rate scenarios. When tracking targets in dense clutter rate scenes, the tracking performance in terms of the OSPA distance and NTE obtained from the proposed algorithm and Multi-scan GM-PHD filter is similar. The reason for the better performance of the proposed algorithm is that the target posterior PHD can be accurately refined at each time step by using the proposed algorithm with two effective schemes.



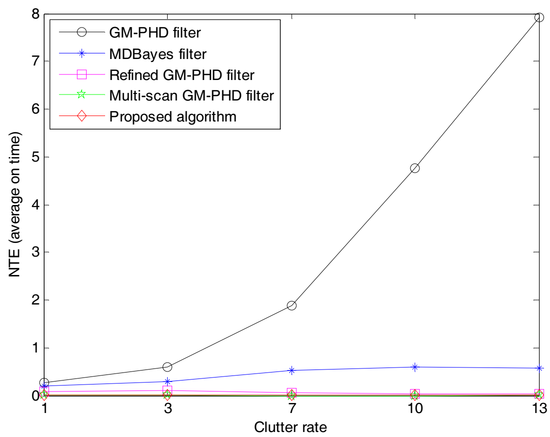
(a) OSPA distance in crossing target scenario



(b) NTE in crossing target scenario



(c) OSPA distance in paralleling target scenario



(d) NTE in paralleling target scenario

Fig. 5. Performance comparison of different algorithms with different clutter rates

5 Conclusions

For solving the pseudo missed-detection problem caused by incorrect position distribution of target-originated measurements in cluttered scenarios, a robust multi-target tracking algorithm is proposed within the framework of the GM-PHD filter. Compared with GM-PHD filter, the proposed algorithm not only integrates a novel PHD renovation scheme of pseudo undetected targets, but also improves the pruning and merging method of GM-PHD filter. By using the pseudo missed-detection renovation method, the PHD of pseudo undetected targets can be renovated from the multi-target posterior PHD at each time step. Moreover, the improved fusion scheme of Gaussian component can refine the components in the target posterior PHD by selecting the relative optimal components with a minimum number that can efficiently approximate multi-target PHD. Experimental results illustrate that the proposed algorithm is able to overcome the pseudo missed-detection problem in cluttered scenarios and has a relatively stronger filtering performance than the standard GM-PHD, Refined GM-PHD and MDB filters. However, the multi-target tracking experiment scenarios do not consider spawn targets, the performance of the proposed algorithm will be further verified in more complex environments. In the future, the proposed algorithm will be enhanced to estimate the trajectories of targets in the presence of data association uncertainty, detection uncertainty, noise and false alarms.

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