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Image Sparse Representation Based on Improved K-SVD Algorithm

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Abstract. Images have a variety of geometric structures, including edges, corners, contours, and textures. Images with different structures can use different transforms to accomplish sparse representation. For more details of the changes, an image's rich edge information can be divided into blocks, with sparse basis representing different sparse. In this study, researchers propose a sparse representation algorithm based on improved K-Singular Value Decomposition (K-SVD) for image edges, corners, and contours. The improved algorithm breaks through restrictions on the orthogonal basis and uses different orthogonal bases in different feature regions of the image to construct a frame based on the combination of different regions. This paper analyzes the sparse K-SVD algorithm, concluding that the dictionary is more compact, that the sparsity factor is lower, and that it overall has a better effect on sparse image features. The experiments demonstrate that the improved K-SVD algorithm has a better effect on image smoothing, edge contours, and texture features.

Keywords: dictionary construction and optimization, image sparse representation, improved K-SVD algorithm, over-complete sparse representation

1 Introduction

Previous studies have demonstrated [1-2] that the wavelet transform performs better in sparse representation for images characterized by one-dimensional point singularity and piece wise smooth features. In contrast, the multi-scale geometric analysis method has a better effect on sparse representation for images with geometric features and edge singularities. The Fourier transform algorithm is superior in sparse representation for images with smooth features, while the Curvelet transform algorithm is better in sparse representation for image signals with discontinuous edges. For oscillatory signals, Gabor transform coefficients are sparse, and images with obvious color changes can be expressed by the wavelet transform. For a relatively smooth image, the Fourier transform or finite difference transform can be used to perform sparse representation. Different features of the signal can be used to obtain optimal sparse representation in different transformations.

In image analysis, images often have a variety of geometric structures, such as edges, corners, contour, and texture, image sparse representation algorithms are mainly reflected in the multi-resolution, multi direction and anisotropy and time-frequency localization; images of different structures can use different transform sparse representation without transformation fixed; for more details of the changes, the image edge information rich can be divided into blocks, with sparse basis different sparse representation.

This paper is built on the geometric features of images to accomplish multi-component smoothing, edge contouring, and texture dictionary based on the K-Singular Value Decomposition (K-SVD) algorithm. The improved algorithm is then used to classify and update the multi-component dictionary. Finally, the improved K-SVD algorithm is applied for sparse representation in an image. The improved

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K-SVD algorithm has a better effect on image smoothing, edge contours, and texture features, and the dictionary is more compact, that the sparsity factor is lower, and that it overall has a better effect on sparse image features.

2 K-SVD Algorithms

The dictionary learning method has become an important part of the prevailing trends in sparse representation. Dictionary construction has a direct impact on sparse decomposition in images. Currently, there are two main ways to construct a dictionary: in one, the learner constructs the dictionary according to prior knowledge, and in the other, the learner builds the dictionary by learning the sample in the way of learning.

Building on the K-Means algorithm, Aharon [3] et al. proposed the K-SVD method to construct dictionaries. In the K-SVD algorithm, dictionary atoms are updated by sparse representations of the image, and the algorithm is used to accomplish the sample training and learning. The K-SVD algorithm can effectively reduce the number of atoms in a dictionary, and the dictionary can represent the image information from the original dictionary after training.

The basic principle of the K-SVD algorithm can be represented as follows:

The K-SVD algorithm uses D to express the complete dictionary, while δ indicates the degree of sparsity, Y represents the sample set of dictionary training $\{y_i\}_{i=1}^{N}$, N is the sample dimension, $N >> K$, and X represents the sparse matrix. The dictionary training can be expressed as the following optimization problem [4],

$$\min_{D,X}\{\| Y - DX \|_F^2\}, s.t. \forall i, \| x_i \|_0 < \alpha \ . \quad \textbf{(1)}$$

Under the assumption that D has been established, the optimization problem from Formula 1 can be transformed into the sparse representation of sample set Y. The algorithm assumes that column k is d_k of dictionary D, and column k is also x_T^k of sparse matrix X. Then the following is true,

$$\| Y - DX \|_F^2 = \| E_k - d_k x_T^k \|_F^2 \ . \quad \textbf{(2)}$$

The sparse representation of sample set Y can be solved using the tracking algorithm, and the atoms of the dictionary can be updated using the sparse decomposition factor.

The K-SVD algorithm can obtain the optimal sparse representation of an image. The algorithm consists of two steps, sparse solution and dictionary update, which are carried out alternately. The algorithm is implemented by an iterative solution. The description and implementation steps of the K-SVD algorithm are as follows [3-4]:

Step 1: Dictionary D initialization, $D^{(0)} \in R^{n \times N}$, sparse matrix X is set as O, the upper limit of the iteration number is set to M, and i is set to 0.

Step 2: Use the OMP algorithm to solve the sparse matrix X of training samples Y [12], $\min_{X} \| Y - D^{(i)} X^{(i)} \|_F^2, s.t. \| x_n^i \|_0 \leq T$, $n = \{1, 2, ..., N\}$.

Step 3: Dictionary learning using formula (3),

$$\| Y - D^{(i)} X^{(i)} \|_F^2 = \| Y - \sum_{j=1}^{N} D_j^{(i)} x_T^i(i) \|_F^2 = \| E_k - d_k^{(i)} x_T^{k(i)} \|_F^2 \ . \quad \textbf{(3)}$$

Where $D^{(i)}$ is the dictionary of the i iteration, $X^{(i)}$ is the sparse matrix of the i iteration, d_k is column k of dictionary D, and x_T^k is the sparse matrix X in line k.

Step 4: Update atoms, replace the atomic $d_k^{(i+1)}$ in dictionary D with the first column of U_k, $k = \{1, 2, ..., N\}$ as follows,

$$E_k = U_k \Delta_k V_k^T \ . \quad \textbf{(4)}$$

Step 5: The number of iterations increases by 1, $i = i + 1$, and the iterative conditional judgment occurs. If $\| Y - D^{(i)} X^{(i)} \|_F^2 \leq \delta$, or if the number of iterations is set to the upper limit M, then the iteration is terminated, and the training dictionary D' is obtained. Otherwise, Step 2 will continue to be executed until the iteration termination condition is satisfied.

3 Image Sparse Representation Based On Improved K-SVD Algorithm

An image contains a variety of features such as edges, corners, and contours, and it is difficult to effectively represent a multi-feature image with only one transform. This paper proposes the sparse representation of images based on the improved K-SVD algorithm, which eliminates the restriction on the orthogonal basis by using the orthogonal basis to different image characteristics in the region. By combining the different areas to form a framework, the algorithm uses an over-complete representation of the image.

In the over-complete sparse representation algorithm, the design of the dictionary plays a decisive role in the sparse representation of the image. Whether the atoms in the dictionary match the geometric structure of the image has a decisive influence on the results of the sparse representation. The dictionary can be constructed using existing orthogonal bases such as tight frame or multi-scale geometric analysis. In addition, the dictionary can be constructed through orthogonal combinations. The improved algorithm proposed in this paper takes into account smoothness, edges, and texture features of the image. According to the geometric characteristics of the image, the improved algorithm can construct a multi-component dictionary.

Use X to represent the original image, $X = \{x_i \mid x_i \in R^k, i = 1,2,...n\}$. Use D to represent the dictionary, $D = [d^1, d^2, ...d^m]$, $D \in R^{k \times m}$, where d^i is a list, d^i is an atom of $R^{k \times m}$, λ is used to balance the reconstruction error, A is a coefficient matrix, α^i is a column of the coefficient matrix, and α^i is the coefficient vector of image X^i. This turns the sparse representation of the image into the optimization problem in formula (5), which can be solved by an iterative alternating algorithm,

$$\min_{D,\alpha} \| X - DA \|_F^2 + \lambda \| A \|_{1,1} \quad s.t. \| d^j \|_2 \leq 1, \forall j \in 1,2,...m . \tag{5}$$

Where $\| . \|_F$ is the Frobenius norm [3]. When dictionary D and coefficient matrix A are fixed, equation (5) is a convex optimization function; when dictionary D is fixed, solving coefficient matrix A can be accomplished by the l_1 norm. The formula for solving coefficient matrix A is as follows,

$$\min_{\alpha^i} \| x^i - D\alpha^i \|_2^2 + \lambda \| \alpha^i \|_1 . \tag{6}$$

Dictionary D can be obtained through learning of the training sample library; the purpose of the dictionary learning is to obtain dictionary D and the coefficient matrix A product, in order to approach the training sample X. In over-complete sparse representation, dictionary D is often optimized by the MOD algorithm [5-6]. First, the partial derivative of the function $\| X - DA \|_F^2$ is obtained; then, the equation $(X - DA)A^T = 0$ is obtained; and finally, the dictionary is obtained by the inverse solution D: $D = XA^T (AA^T)^{-1}$.

The K-SVD algorithm was proposed by Aharon et al., as Farley has a singular value decomposition of dictionary update [7], in order to reduce the error of sparse representation. The K-SVD algorithm includes two components, sparse coding and the dictionary update. Using D to express the learning dictionary, Y to represent the training sample, and X to represent the Y in dictionary D in the representation of the coefficient, the mathematical optimization of the objective function can be expressed using formula (7) as follows,

$$\min_{D,X} \| Y - DX \|_F{}^2 \ subjectto \| x_i \|_0 \leq T . \tag{7}$$

Where T is a constant and x_i corresponds to coefficient y_i in training sample Y. It is a column of the X matrix.

Based on the K-SVD algorithm and the geometric features of images, this paper builds multi-component smoothing, edges, and textures into the dictionary, using the orthogonal basis to different image characteristics of the region. The orthogonal combinations of different areas form a framework for over-complete sparse representation of an image. Because dictionary D is a multi-component dictionary, dictionary updating is implemented by way of classification updating.

Let d_{Lk} denote the k atom in the L dictionary, let X_T^k be the sparse coefficient of the k line in the sparse matrix X, let X_R^k be a vector composed of the non-zero element of X_T^k, and let E_k denote the approximation error matrix. Ultimately, we classify and update the atoms in the D dictionary. Formula (7) can be converted to the target optimization function in formula (8) as follows,

$$\begin{aligned}\| Y - DX \|_F{}^2 &= \| Y - (\sum_{j\neq k} d_{Lj} X_T{}^j + d_{Lk} X_T{}^k) \|_F{}^2 \\ &= \| (Y - \sum_{j\neq k} d_{Lj} X_T^j) - d_{Lx} X_T^k \|_F^2 = \| E_k - d_{Lk} X_T^k \|_F^2 \end{aligned} . \tag{8}$$

The improved algorithm can be used to update each atom d_{Lk}^j in dictionary D_L in order to obtain a subset of the signal represented by d_{Lk}^j; then, the residual error can be calculated using formula (9),

$$R_{Lk}^j = \Omega_{Lk}^j - d_{Lk}^j . X^j(k) . \tag{9}$$

Where X^j is a sparse factorization matrix. Select the d_{Lk} associated with the column from E_k to form a new matrix E_{LK}^R, and then conduct SVD decomposition of E_{LK}^R, $E_{LK}^R = U\Delta V^T$. Obtain the updated atom $\overline{d_{Lk}}$ (first column of U); then, the first column of V and singular value Δ(1,1) are multiplied to obtain a new coefficient $\overline{X}_R^k$. Next, construct a gradient operator ∇, where $\nabla = -R_k^j . pinv(X^j(k))$, in order to update each atom d_{Lk}^j in the classification dictionary D_L, where μ is the step factor. Finally, according to the updated atomic structure, the repetition gradient operator represents an iterative way to complete the update.

The improved K-SVD algorithm for sparse representation is described and implemented as follows.

Step 1: The sparse coefficient matrix X is set to O, the number of iterations is set to M, and the initial value of i is set to 0.

Step 2: First, an image (Y) is divided into $\sqrt{n} \times \sqrt{n}$ sub-blocks, which are rearranged into a vector $Y_H \in R^n$. According to the geometric characteristics of the image, the improved algorithm is used to classify different features such as smoothing, edge contouring, and texture structure. Different types of sub-blocks adopt different sparse dictionaries, using a set of basis Φ_r corresponding to one of the characteristics. Based on the total Φ_r portfolio into a redundant basis set, represented by the D dictionary, $D \in R^{n\times k}, k > n$, $D^0{}_L = \{d_{Lk}^0 ; k = 1,2,...,K\}$.

Step 3: For the sparse representation of image vectors, $Y_H \approx DX$. This error satisfies the condition $\| Y_H - DX \|_p \leq \varepsilon$. The improved algorithm for sparse decomposition on the block of image signal is $X = OMP(Y_H, D^j)$, where X is the coefficient matrix of $K \times M$. The sparse coefficient matrix X can be obtained by solving $\hat{X} = \arg\min_x \| Y_H - DX \|_2^2 + \mu \| X \|_0$.

Step 4: According to formula (9) of the target optimization function, the algorithm updates each atom d_{Lk}^j of dictionary $D^j{}_L$. Obtain a subset of the signal d_{Lk}^j, $\Omega_k^j = \{f_i \mid X^j(k,i) \neq 0\}, i = 1,2,...,M$. Then, use the formula $R_{Lk}^j = \Omega_{Lk}^j - d_{Lk}^j . X^j(k)$ to calculate the residuals. Construct the gradient operator ∇, $\nabla = -R_{Lk}^j . pinv(X^j(k))$, updating each atom d_{Lk}^j one by one, $d_{Lk}^{j+1} = d_{Lk}^j - \mu . \nabla$, where μ is the set of step factors, according to the new atomic repeat constructed gradient operator.

Step 5: The number of iterations is increased by 1, $i=i+1$, and the iterative conditional judgment occurs. If $\| Y_H - D^{(i)}X^{(i)} \|_F^2 \le \delta$, where δ is the reconstruction error, or if the number of iterations is set to the upper limit M, then the iteration is terminated, and the training dictionary D' is obtained. Otherwise, Step 3 continues to be executed until the iteration termination condition is satisfied.

4 Experimental Analysis

4.1 Evaluation Criteria for Image sparse Representation

In recent research, the sparsity factor has been used to evaluate the advantages and disadvantages of sparse representation of an image. The sparse factor is calculated as shown in the following formula [7, 11],

$$S = \frac{number\ of \quad nonzero\ coefficient\mathrm{s\ in} \quad W}{number\ of \quad samples \quad in\ Y}. \tag{10}$$

Where Y is the source image and W is the coefficient of the transform domain image. The nonzero element in a molecule is the element whose coefficient is less than a given threshold. The larger the sparsity factor S, the worse the sparse representation of the image. Similarly, the smaller the sparsity factor S, the better the sparse representation of the image.

In addition to the sparsity factor, the nonlinear approximation error is often used to evaluate the degree of sparse representation. The nonlinear approximation error reflects the energy concentration degree and the sparse degree of the decomposition coefficient, and it is defined in equation (11),

$$\varepsilon(M) = \| f - f_M \|^2 = \sum_{i \in \mathrm{I}_M} |< f, g_i >|^2 . \tag{11}$$

Where $G = \{g_i\}, i = 1, 2, \ldots N$, G is a set of standard orthogonal bases, $f = \sum_{i=0}^{+\infty} < f, g_i > g_i$, I_M is the M vector of the maximum coefficient amplitude, and f_M is the nonlinear approximation of the source signal f, so that $f_M = \sum_{i \in I_M} < f, g_i >$.

In this paper, we use the sparse factor and the nonlinear approximation error as two indicators by which to analyze sparse representation under the improved algorithm.

4.2 Experimental Analysis

Experimental analysis involved two 256*256 pixel images, a remote sensing image and a Lena image. Experiments were carried out based on sparse representation of the wavelet transform, sparse representation based on the multi-scale transform, and sparse representation based on the improved K-SVD algorithm.

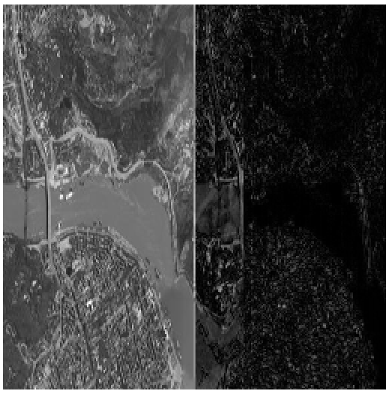
Fig. 1 shows the effects of the wavelet image decomposition experiment. Fig. 1(a) is a remote sensing image of the original G1, Fig. 1(b) is the result of wavelet row-transform in the remote sensing image, Fig. 1(c) is a small wave column-transform of the remote sensing image results, and Fig. 1(d) is the first order wavelet decomposition result of the remote sensing image.

Fig. 2 also shows the effects of the wavelet image decomposition experiment. Fig. 2(a) is the Lena image of the original G2, Fig. 2(b) shows the result of wavelet row-transform for the Lena image, Fig. 2(c) is the wavelet column-transform of the Lena image, and Fig. 2(d) shows the first order wavelet decomposition results of the Lena image.

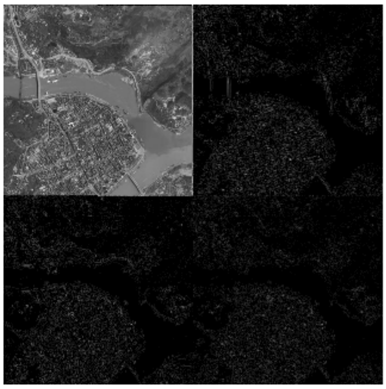
In the Contourlet transform, experiments were conducted on the Laplasse tower-type decomposition of the image. The top of the image LPn contains the image’s low-frequency information, while the other layer contains the high-frequency characteristics of the image at different scales on the performance.

(a) Remote sensing image G1

(b) Wavelet row-transform for G1



(c) Wavelet column-transform for G1



(d) First order wavelet transform for G1

Fig. 1. Effect of remote sensing image wavelet decomposition

(a) Lena image G2

(b) Wavelet row-transform for G2

(c) Wavelet column-transform for G2

(d) First order wavelet transform for G2

Fig. 2. Wavelet decomposition results for Lena image

This study decomposed the remote sensing image (G1) and Lena image (G2) into three LP layers. The LP tower decomposition effect on the Lena image is shown in Fig. 3. The LP tower decomposition effect on the remote sensing image (G1) is shown in Fig. 4. In Fig. 3 and Fig. 4, LP0, LP1, and LP2 represent the numbers of columns that are decomposed.

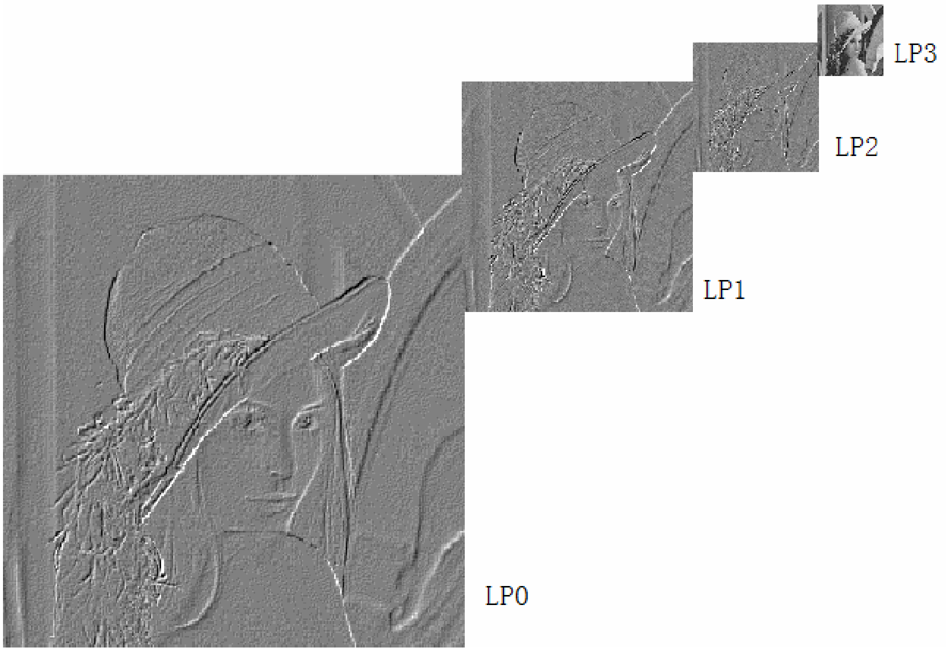


Fig. 3. Tower decomposition results of Lena image G2

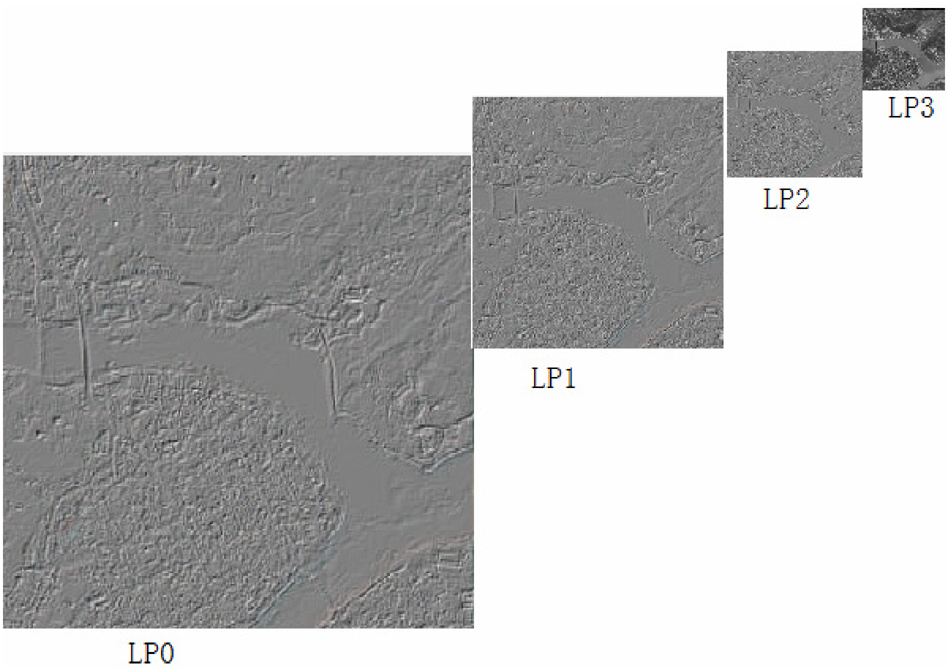


Fig. 4. Tower decomposition results of remote sensing image G1

This study conducted experimental analysis of the nonlinear approximation of the remote sensing image (G1) based on the wavelet transform, Coutrvelet transform, Ridgelet transform, and improved K-SVD algorithm. In the experiment, the coefficient of the maximum amplitude of M in the transform coefficient was preserved, and the remaining transform coefficients were set to 0. The image reconstruction was performed using the coefficients of M maximum amplitude, and the effect of sparse representation was analyzed by reconstructing the error. When the reconstruction error is small, the sparse representation of the algorithm is better.

The results of the nonlinear approximation experiment are shown in Fig. 5, which includes data for the improved K-SVD algorithm, Rigedet transation, Wavelet transation, and Coutrvelet transation algorithm.

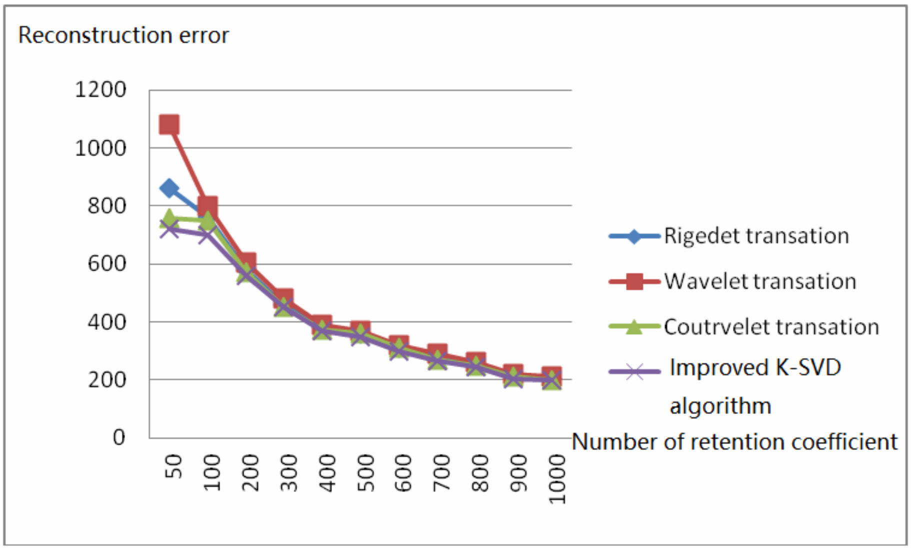


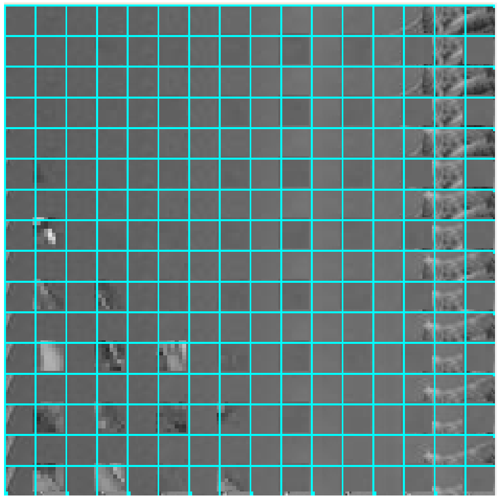
Fig. 5. Experimental results of nonlinear approximation

The experimental results indicate that the reconstruction error of the improved K-SVD algorithm is significantly smaller when the number of retention coefficients is small. Additionally, when the number of the maximum amplitude coefficient M is the same, the reconstruction error of the improved K-SVD algorithm is small. These results demonstrate that the improved algorithm yields good nonlinear approximation results.

In the sparse representation experiment based on the improved K-SVD algorithm, we classified the existing remote sensing image database according to the main features of the scenes. The experiment selected a representative classification scene and used the improved K-SVD algorithm to classify the dictionary. The remote sensing image was divided into 16*16 sub-blocks. In order to better highlight the main features of each block, the pixels between the blocks and the blocks can be overlapped. The 16*16 sub-block image was then transformed from two dimensions to a one-dimensional image, which was then transformed into a one dimension image signal of 256*1. Finally, all of the selected sub-block images were composed of the training set X. In this way, the improved K-SVD algorithm was used to construct and optimize the classification dictionary.

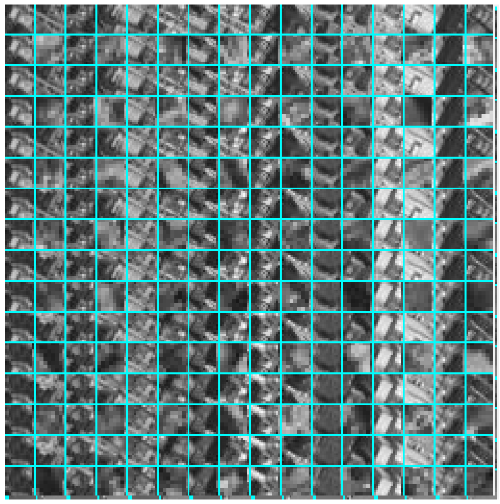
Fig. 6(a) shows a uniform scene on the Yangtze River, and Fig. 6(b) shows the trained dictionary. Fig. 6(b) shows atomic image fragments with homogeneous characteristics. Fig. 6(c) shows a complex, detailed scene of urban architecture, and its dictionary of training is shown in Fig. 6(d), where atomic image fragments have strong edge feature details. Fig. 6(e) shows an image of road traffic with strong texture features. The trained dictionary is shown in Fig. 6(f), where the atomic image fragments also have strong texture characteristics.

(a) Training image with uniform scene features



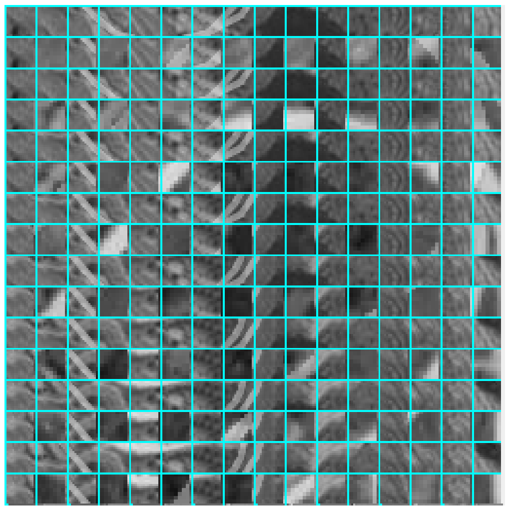
(b) Dictionary of training

(c) Training image with complex details



(d) Dictionary of training

(e) Training image with strong texture feature

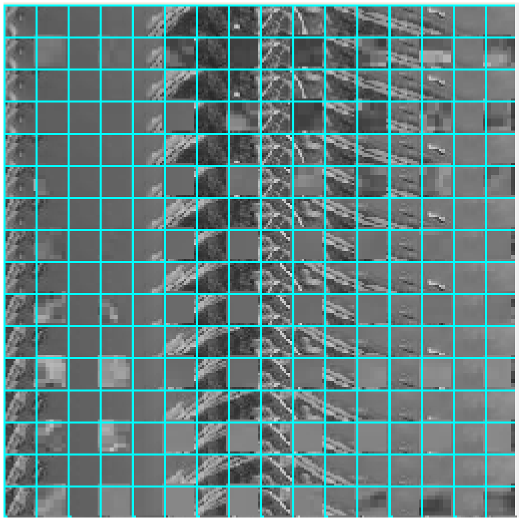


(f) Dictionary of training

Fig. 6. Dictionary of image classification training

In this experiment, dictionary optimization was accomplished through comprehensive dictionary training on various scenes, as well as comprehensive training on uniform, detail, texture, and complex scene edges. After obtaining the dictionary, sparse on the original said the use of dictionary training to reconstruct the 16*16 image pixel block, then was combined into the entire 256*256 image as shown in Fig. 7.

(a) Scene classification image



(b) Training dictionary optimization

Fig. 7. Dictionary optimization experiment

This paper used the wavelet transform, Ridgelet transform, Contourlet transform, Curvelet transform, over-complete sparse representation algorithm, and improved K-SVD algorithm based on the sparse representation of the experimental analysis. Various algorithms yielded the image sparsity factors shown in Fig. 8.

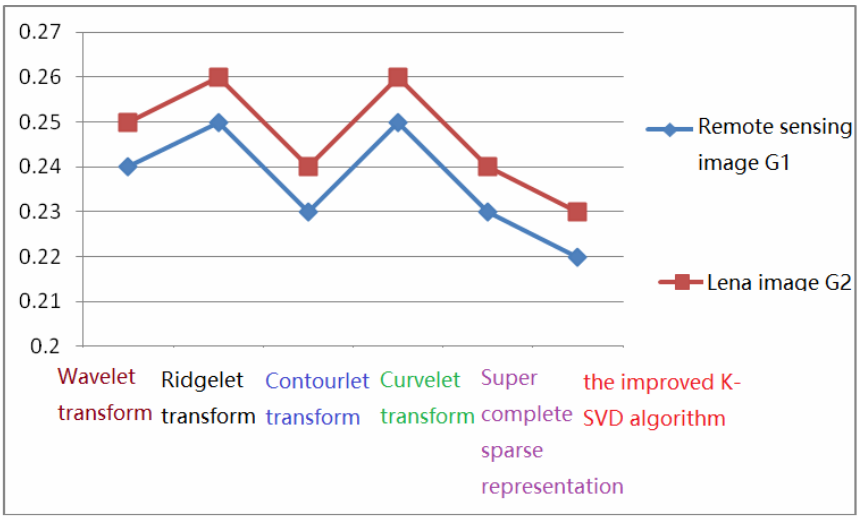


Fig. 8. Results of image sparse factors obtained by different algorithms

In Fig. 8, the sparsity factor of the image obtained by the improved K-SVD algorithm is smaller than those obtained by the wavelet transform, Ridgelet transform, Contourlet transform, Curvelet transform, or super complete sparse representation algorithm. These results demonstrate that the improved algorithm can produce better sparse representation.

For the selected remote sensing image G1 and Lena image G2, we compared and analyzed the M vector (I_M) of the maximum coefficient amplitude obtained by each algorithm. The I_M parameter analysis is shown in Fig. 9.

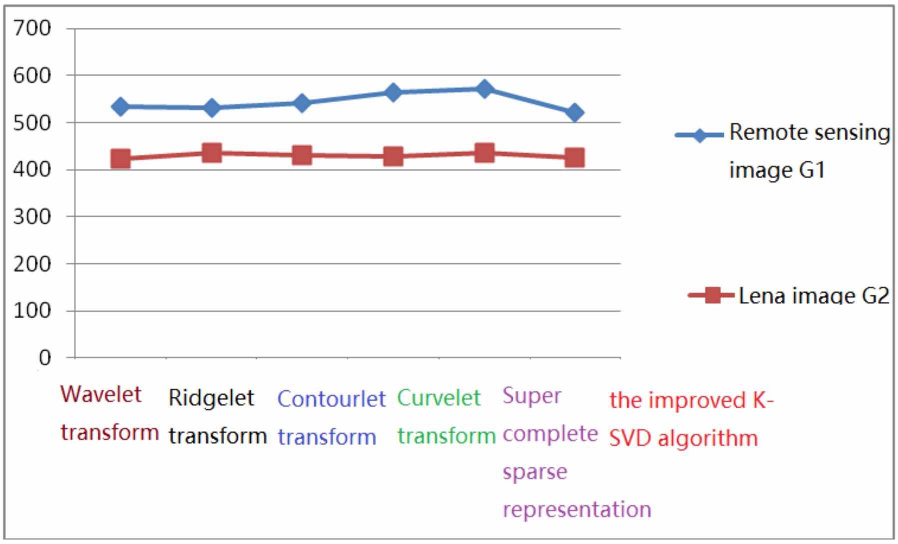


Fig. 9. Results of I_M parameters obtained by different algorithms

In Fig. 9, the I_M parameters obtained by the improved K-SVD algorithm are much smaller than those obtained by the wavelet transform, Ridgelet transform, Contourlet transform, Curvelet transform, and super complete sparse representation algorithm. These results demonstrate that the improved K-SVD algorithm performs better in terms of sparse representation. For the experimental Lena image G2, the improved K-SVD algorithm also yields a smaller I_M parameter.

The experimental results for the sparse factor and I_M parameters show that the combined dictionary can achieve better performance in terms of sparse representation. Compared with the traditional K-SVD algorithm, the sparse representation method based on the improved K-SVD algorithm is more compact. The sparse factor obtained by the algorithm is small, and the experimental results demonstrate that the algorithm performs better.

5 Conclusions

This paper proposed a sparse representation algorithm to address image edges, corners, and contours based on the improved K-SVD algorithm. The improved algorithm breaks through the restriction of the orthogonal basis, utilizes different orthogonal bases for different feature regions of the image, and forms a frame with the orthogonal bases from different regions. This paper then analyzed the sparse representation method based on the improved K-SVD algorithm, comparing it with the traditional K-SVD algorithm in order to construct a more compact dictionary. The sparsity of the improved algorithm is small, and it has a better effect on the sparse representation of multi-feature images. The experimental results demonstrate that the improved K-SVD algorithm performs better in image smoothing, edge contouring, and texture features.

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