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A Novel CSW Method for Data Envelopment Analysis Based on DM's Preference Information

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Abstract. A popular method for determining common set of weights (CSW) is to minimize the deviations of the CSW from all optimal weights of decision making units (DMUs). As this optimal weights are derived from the data set itself, leading the CSW methods fail to consider the decision maker's (DM's) preference information for some indicators. In this paper, we propose a novel CSW method based on the decision maker's preference information and cross weights. First, a multi-objective benevolent linear programming model is showed to overcome the problems of multiple decision making units being evaluated as "DEA efficient" and the optimal weights being non-unique for each decision making unit. Second, we propose a preference weights restriction method, which can better reflect the decision maker's preference information, and ensures that all variables have non-zero weights. Again, we utilize five steps to rescale the cross weights with decision maker's preference information to achieve comparability among decision making units. Then, we present a novel CSW model which combines two "Euclidean Distance" norms to determine the common weights. Finally, a numerical example is used to illustrate the validity of the models and show their significant role in achieving the uniqueness, comparability and non-zero weights.

Keywords: common set of weights, cross weights, data envelopment analysis, decision maker preference

1 Introduction

Data envelopment analysis (DEA), initially developed by Charnes, Cooper and Rhodes in 1978 [1], is a non-parametric methodology for evaluating the relative performance of decision making units (DMUs) that use multiple inputs to produce multiple outputs. In conventional DEA models, "total weights flexibility" is recognized as either a weakness or a strength [2]. On the one hand, with this flexibility, we can choose the advantageous weights for each DMU to obtain its best possible score. On the other hand, this flexibility inevitably leads to these drawbacks: (i) non-unique set of input/output weights available for each DMU; (ii) some input/output weights to take the value of 0 or ε; (iii) deters the comparison among DMUs on a common base.

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To overcome these drawbacks, scholars proposed many methods from different perspectives. Among these methods, a very popular one is to determine a common set of weights (CSW), which was initially introduced by Cook et al. [3] and subsequently developed by Roll et al. [4], which aims to fairly expose all DMUs to the same frontier facet and, thereby providing a common evaluation basis. The academic research on the CSW-DEA primarily focused on how to determine a CSW. For example, Roll and Golany developed a weighting algorithm to look for the center values of all DEA weights to generate a CSW [5]. Kao and Hung found the CSW by using the compromise solution approach to minimize the distance between DMUs and the ideal solution [6]. Ramon et al. and Ramon et al. expanded their algorithm on the cross-efficiency evaluation into the CSW calculation by minimizing the deviations between the common weights and the profiles of weights [7-8]. Sun et al. proposed two models to generate common weights by using both ideal and anti-ideal DMU [9]. Other approaches to obtain common weights include Regression Analysis [10], Goal Programming [11], Shannon's Entropy [12], Robust Optimization [13-14] and so on. In addition, some scholars have attempted to consider more factors and situations in obtaining the common weights. The representatives are as follows: Hajiagha et al. proposed a new method to determine CSW in a multi-period DEA [15]. Hu et al. utilized a common base to measure the relative efficiency of several homogeneous DMUs in a fuzzy environment [16]. Hatami-Marbini et al. proposed DMU satisfaction degree in relation to a common set of weights [17]. Sun al. proposed three new interval CSW-DEA models from distinct perspectives to obtain precise data [18].

Although there has been much research about the CSW-DEA, only a few scholars have taken into account the DM's preference information. Jahanshahloo (2011) introduced a concept of "preference common weights" to reflect the preferences of the decision makers (DMs) [19]. Abbasian-Naghneh aslo used the preference common-weights method proposed by Jahanshahloo et al. (2010) for time-series evaluations to determine the global Malmquist productivity index (MPI) [20]. In their studies, an implicit function on an interactive basis is utilized to resolve the conflicts inherent in the given multiple objectives, which means that an answer to the 'yes' or 'no' questions on feasible tradeoffs for DM. But sometimes the different importance of some inputs (salaries, staff numbers) and outputs (net profit, market share) to the decision maker needs to be reflected by a specific scores.

In this study, we propose a novel CSW method based on the DM's preference information with some specific scores, to better incorporate subjective preference into objective evaluation. Firstly, based on the idea of value judgment, we incorporate the DM's preference information into the weight restrictions by introducing interval variables. It's known that setting the boundaries for the weights based on the expert opinion sounds simple, but it involves a complex process in transforming information into mathematical form. For this reason, this paper will describe the calculation process of the preference weights boundaries in detail and will also show how to convert the expert opinions into weight restrictions in practical applications. Secondly, by considering DM's preference information, we create a novel CSW method that combines two "Euclidean Distance" norms to determine the CSW by minimizing the deviations of the CSW from all cross weights of DMUs. Additionally, we use five specific steps to modify the cross-weights evaluation with the DM's preference information, to enhance the comparability among the DMUs.

The rest of this paper is organized as follows: Section 2 gives a brief introduction of the CCR-DEA model and a multi-objective benevolent linear programming (MOBLP) model. Our proposed common weights method based on the DM's preference information is described in detail in section 3. An illustrative example is demonstrated in section 4 to illustrate our method and to compare it with the traditional DEA methods. The conclusions are presented in section 5.

2 Preliminaries

2.1 The CCR Model

Assume that there are n DMUs to be evaluated with m inputs and s outputs. Let $x_{ij}(i=1,\cdots,m)$ and $y_{rj}(r=1,\cdots,s)$ represent the ith input and the rth output of DMU_j $(j=1,\cdots,n)$, respectively. Consider a DMU, say, DMU_k $(k=1,\cdots,n)$, whose efficiency can be obtained by the CCR model [1], the linear programming (LP) is as follows:

$$
\begin{aligned}
& E_k = Max \sum_{r=1}^{s} u_{rk} y_{rk} \\
& s.t. \ \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \ j = 1, \cdots, n, \\
& \qquad u_{rk}, v_{ik} \ge 0, \quad r = 1, \cdots, s, \quad i = 1, \cdots, m.
\end{aligned} \tag{1}
$$

where $v_{ik}(i=1,\cdots,m)$ and $u_{rk}(r=1,\cdots,s)$ are the weights of the inputs and outputs, respectively. And the second constraint guarantees all efficiency values in the range (0, 1]. DMU_k is said to be "DEA efficient" or "optimistic efficient" when there is an optimal set of weights $u_{rk}^{*}(r=1,\cdots,s)$ and $v_{ik}^{*}(i=1,\cdots,m)$ that make $E_k^{*}=1$; otherwise, DMU_k is said to be "DEA non-efficient" or "optimistic non-efficient".

2.2 Secondary Goal Based on Mehdi and Saeid's Idea

In traditional DEA models, the self-evaluation and non-unique optimal weights possibly undermine the discrimination and stability of the efficiency evaluation. As a result, some secondary goal methods in cross-efficiency evaluation were introduced by scholars [21-24] (Sexton et al.1986, Doyle and Green, 1994, Liang et al., 2008, Mehdi and Saeid, 2013, and so on). In this section, based on the idea of Mehdi and Saeid (2013) [24], we show a secondary goal model by considering the priorities between the two objective functions. Firstly, the DEA model (1) can be expressed equivalently in the following deviation variable form:

$$
\begin{aligned}
& Min \ \lambda_k \\
& s.t. \ \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \lambda_j = 0, \ j = 1, \ldots, n, \\
& \qquad u_{rk}, v_{ik}, \lambda_j \ge 0, \ \forall r, i, j.
\end{aligned} \tag{2}
$$

where λ_k is the deviation variable for DMU_k and λ_j the deviation variable for the jth DMU. Under this model, DMU_k is efficient if and only if $\lambda_k^{*}=0$. If DMU_k is not efficient, then its efficiency score is 1-λ_k^{*} (λ_k can be regarded as a measure of "inefficiency"). We refer to the deviation variable λ_j as the k-inefficiency of DMU_j.

We get the following model as a secondary goal to minimize the sum of "inefficiencies" for DMU_k in cross-efficiency evaluation.

$$
\begin{aligned}
& Min \ \sum_{j=1}^{n} \lambda_j \\
& s.t. \ \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rk} = E_k^{*} \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \lambda_j = 0, \ j = 1, \ldots, n \\
& \qquad u_{rk}, v_{ik} \ge 0, \quad r = 1, \cdots, s, \quad i = 1, \cdots, m
\end{aligned} \tag{3}
$$

where E_k^* is obtained from model (1). To make sure the feasible region is unchanged and at the same time to insure that the efficiency of the unit under evaluation reaches its maximum, we combine model (1) with model (3) into model (4) by introducing a sufficiently small non-negative number ε_1.

$$\begin{aligned} &E_k{}' = Max\left\{\sum_{r=1}^{s} u_{rk} y_{rk} \text{-} \varepsilon_1 \frac{1}{n}\sum_{j=1}^{n}\lambda_j\right\} \\ &s.t.\ \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ &\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \lambda_j = 0,\ j = 1,...,n \\ &\qquad u_{rk}, v_{ik} \geq 0, \quad r = 1,\cdots,s,\ i = 1,\cdots,m \end{aligned} \tag{4}$$

Model (4) satisfies the criteria of the benevolent model. In the objective function, the first priority is to maximize the efficiency of the evaluated unit DMU_k, which is achieved through $Max \sum_{r=1}^{s} u_{rk} y_{rk}$, the second priority is to maximize the average efficiency of the other units, which is achieved through $Max\left(\text{-}\varepsilon_1 \frac{1}{n}\sum_{j=1}^{n}\lambda_j\right)$, where ε_1 can take any adequately small non-negative numbers (e.g. $\varepsilon_1 = 0.0001$). By solving model (4), a unique optimal set of input and output weights could be obtained for every evaluated unit.

3 Methodology

3.1 Restricting the Weights with DM's Preference Information

In conventional DEA models, the "total weights flexibility" allows a DMU to seek maximum efficiency by selecting a mix of weights that either is implausible because it may cause some input/output weights to take the value of 0 or ε. In this case, one or more variables will been ignored, which result in the related DMUs can not be fully reflected. Moreover, all the input/output variables are "free specialization", which means an implicit assumption that there is without any priority among variables, that is unacceptable. Ignoring the DM's preference on the inputs/outputs variables will evidently lead to biased efficiency results. To ensure that the efficiency evaluation are more reliable and distinguishable, it is necessary and practical to reflect the DM's preference information by restricting the input and output weights properly. The specific process is shown in Fig. 1.

The simplest type of weight restrictions is the absolute weight restrictions [25], which limits the flexibility weights to certain defined bounds, as follows:

$$\begin{aligned} &U_r^l \leq u_r \leq U_r^u \qquad \forall r \\ &V_i^l \leq v_i \leq U_i^u \qquad \forall i \end{aligned} \tag{4}$$

where U_r^l, U_r^u, V_i^l and V_i^u are lower and upper bounds on output and input weights, respectively. In the formula (4), the upper and lower bounds for the weights are predetermined constants to reflect decision maker preferences. In applications, this method provides a simple way to include value judgments and ensures that all variables have non-zero weights.

However, it is worth noting that this method has two limitations: First, it is difficult to directly restrict and compare the sets of weights because the weights depends on the measurement units of the inputs or outputs and their magnitude (always different). Second, the absolute weight restrictions method has multiple feasible solutions, and it is impossible to determine a set of optimal solutions.

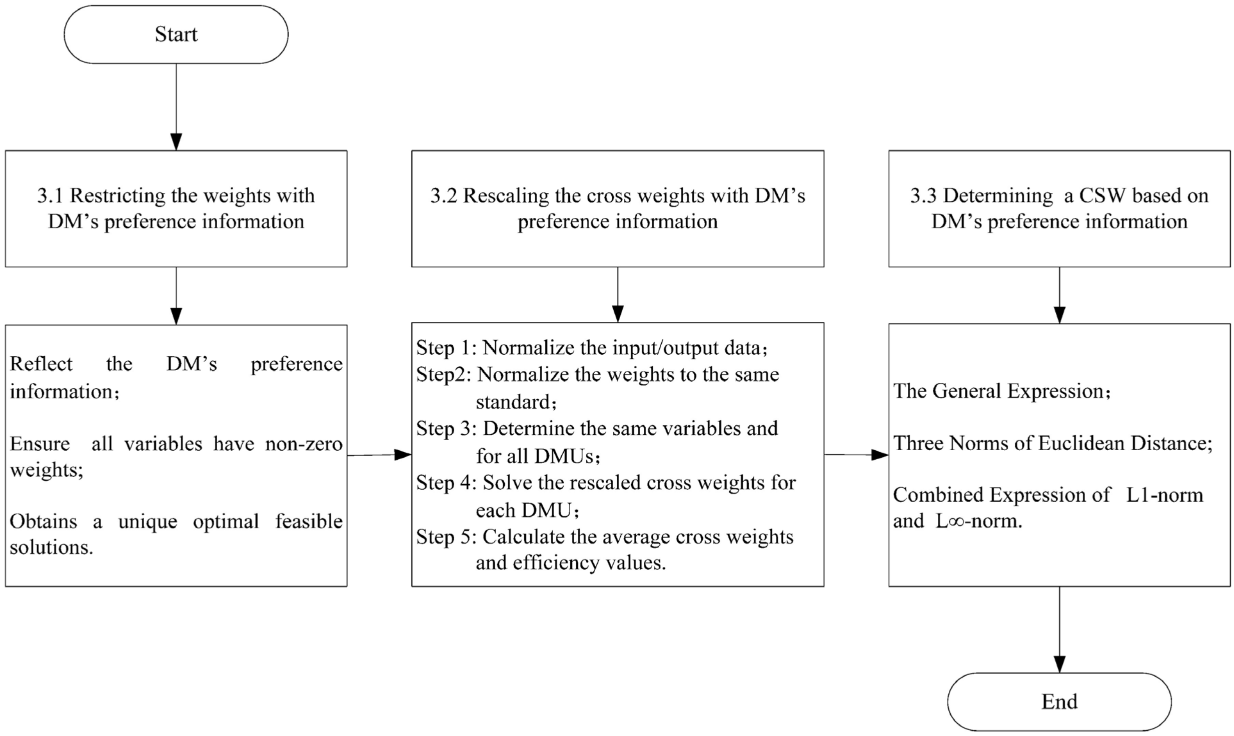

Fig. 1. The research process of the novel CSW method based on the DM's preference information and cross weights

For the first limitation, researchers have begun to tend to restrict the virtual weights, which refer to the product of the multiplier placed on a particular input or output and the value of the observed data for that input or output, which can be expressed as $v_{ik}x_{ik}$ ($i=1,\cdots,m,\ \ k=1,\cdots,n$) for the inputs or $u_{rk}y_{rk}$ ($r=1,\cdots,s$, $k=1,\cdots,n$) for the outputs. This product is units invariant within the set of inputs or outputs, thus allowing direct comparison of the weights within the input/output sets. There are managerial implications of this feature that allow a decision maker to see exactly how much each input/output is contributing towards efficiency.

Here, we focus in particular on eliminating the second limitation, and propose a preference weight restrictions method, which not only can better reflect the DM's preference information, but also ensure that the evaluation model has a set of optimal feasible solutions. The preference weights restrictions are as follows:

$$\begin{gathered}\alpha_r \cdot h_k \le u_{rk}y_{rk} \le \alpha_r \cdot H_k, r=1,\cdots,s \\ \beta_i \cdot h_k \le v_{ik}x_{ik} \le \beta_i \cdot H_k,\ i=1,\cdots,m\end{gathered} \tag{5}$$

Where α_r and β_i represent the subjective preference multipliers of the outputs and inputs, respectively, and were predetermined based on the prior information, observations, expert opinion or actual application requirements. The variables h_k and H_k serve as the lower and upper bounds, respectively, with h_k, $H_k \ge 0$ and $h_k \le H_k$, and their values are different among DMUs.

Theorem 1. Formula (5) ensures that the evaluation model obtains a unique optimal feasible solutions.

Proof. Considering the reliability of reality, α_r and β_i need to satisfy the constraints of $\alpha_r, \beta_i \ge 0$ and $\sum_{r=1}^{s}\alpha_r + \sum_{i=1}^{m}\beta_i = 1$. The quantities $u_{rk}y_{rk}$ and $v_{ik}x_{ik}$ are the products of the inputs/outputs and the corresponding weights. And α_r , β_i , y_{rk} and v_{ik} are known constants, which do not affect the feasibility of the formula (6).

$$\begin{gathered}h_k \le u_{rk} \le H_k, r=1,\cdots,s \\ h_k \le v_{ik} \le H_k,\ i=1,\cdots,m\end{gathered} \tag{6}$$

Therefore, essentially, the formula (5) are equivalent to the formula (6). Compared with the formula (4), the difference of formula (6) is that the weights bounds are variables. When the interval variables h_k and H_k take the extreme case $h_k = H_k$, the evaluation model can obtain a set of optimal feasible solutions.

Theorem 2. Formula (5) can better reflect the DM's preference information.

Proof. As the variables h_k and H_k are interval variables to restricts the input and output weights. In the extreme case $h_k = H_k$, the $\alpha_r = u_{rk} y_{rk}$, $r = 1,\cdots,s$ and $\beta_i = v_{ik} x_{ik}$, $i = 1,\cdots,m$. And the y_{rk} and v_{ik} are known constants. It is easy to get the conclusion that when $h_k = H_k$, the optimal weights and the DM's preferences are completely related. Therefore, the formula (5) can better reflects the DM's preference information.

Based on the above discussion and analysis, for the evaluated DMU_k $(k = 1,\cdots,n)$, we incorporate the proposed DM's preference weight restrictions into model (3) to obtain model (7):

$$
\begin{aligned}
& E_k'' = Max\left\{(h_k - H_k) + \varepsilon_2 \sum_{r=1}^{s} u_{rk}\, y_{rk} - \varepsilon_1 \frac{1}{n}\sum_{j=1}^{n}\lambda_j)\right\} \\
& s.t.\ \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\
& \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} + \lambda_j = 0,\ j = 1,\cdots,n \\
& \quad \alpha_r \cdot h_k \le u_{rk}\, y_{rk} \le \alpha_r \cdot H_k,\ r = 1,\cdots,s \\
& \quad \beta_i \cdot h_k \le v_{ik}\, x_{ik} \le \beta_i \cdot H_k,\ i = 1,\cdots,m \\
& \quad v_{ik}, u_{rk}, h_k, H_k \ge 0, i = 1,\cdots,m, r = 1,\cdots,s, k = 1,\cdots,n
\end{aligned}
\tag{7}
$$

Specifically, x_{ik} and y_{rk} represent the values of the input data of type i and the output data of type r of DMU_k $(k = 1,\cdots,n)$, respectively.

Remark 1. Model (7) can ensure there is no zero weights in feasible solution. Because the quantities x_{ik}, y_{rk}, α_r and β_i are known and the variables h_k and H_k are greater than or equal to 0, the weights of the input/output indicators v_{ik} and u_{rk} can be guaranteed to be non-zero.

Remark 2. Obtain the values of h_k, H_k and the unique optimal preference weights for each DMU by solving the model (7). In the objective function, the first goal $Max\,(h_k - H_k)$ is equivalent to $Min\,(H_k - h_k)$, which means that the goal is to minimize the interval between the upper bound H_k and the lower bound h_k. The second goal is maximizing the efficiency of the evaluated unit DMU_k, with $\varepsilon_2 = 0.01$, and the third goal is maximizing the average efficiency of the other units, with $\varepsilon_1 = 0.0001$. In this case, we first satisfy the preference information of decision makers on the importance of input/output indicators, and then maximize the average efficiency of the other units under the insurance of maximizing the efficiency of the evaluated unit. Model (7) is solved n times, each time for one different DMU. As consequence, we can obtain the values of h_k, H_k and the unique optimal preference weights for each DMU.

3.2 Rescaling the Cross Weights with DM's Preference Information

In all above models, the evaluated DMU self-evaluates its efficiency with the most favorable weights assigned to itself. As a result, often, too many DMUs are self-evaluated as the best and cannot be distinguished and compared. To resolve this problem, cross-efficiency evaluation was proposed to ensure that each DMU is not only self-evaluated but also peer-evaluated by the other DMUs. Wang and Chin (2010) proposed that the n sets of weights be used to generate an average set of weights for the n DMUs and then that a set of average weights be used for evaluating and ranking DMUs; this is referred to as cross weights evaluation [26].

In this section, based on the ideas of Wang and Chin (2010), we use five steps to modify the cross weights evaluation with the DM's preference information to achieve the comparability among different DMUs.

Step 1: Normalize the input/output data

It is generally known that the input/output indicators have different metrics or dimensions in the DEA efficiency evaluation. For convenience of comparison, we should eliminate the dimension influence among the different evaluation indicators. The input/output indicators $x_{ij}(i=1,\cdots,m)$ and $y_{rj}(r=1,\cdots,s)$ of the $DMU_j(j=1,\cdots,n)$ are normalized as follows:

$$\begin{cases} \hat{x}_{ij} = x_{ij} / \max\limits_{j=1,\cdots,n} \{x_{ij}\} \\ \hat{y}_{rj} = y_{rj} / \max\limits_{j=1,\cdots,n} \{y_{rj}\} \end{cases} \tag{8}$$

Step 2: Normalize the weights to the same standard

It is easy to see that the input and output weights derived from model (7) are not comparable among DMUs because they meet the same condition of $\sum_{i=1}^{m} v_{ik} x_{ik} = 1$, whereas $x_{ik}(i=1,\cdots,m)$ vary from one DMU to another. To strengthen the quantitative comparability among all DMUs, Wang and Chin (2010) [26] normalized the weights derived from their proposed neutral DEA model to the same standard: $\sum_{i=1}^{m} v_{ik}(\sum_{j=1}^{n} x_{ij}) = 1$. In their study, if $\sum_{j=1}^{n} x_{ij}$ is infinite, the weight will be not only extremely small and inconsistent with common sense but also inconvenient for calculating and processing. To avoid these shortcomings, we rescale the weights derived from model (7) to the same standard: $\sum_{i=1}^{m} \hat{v}_{ik} \bar{\hat{x}}_i = 1$, $\bar{\hat{x}}_i = \frac{1}{n}\sum_{j=1}^{n} \hat{x}_{ij}$, which ensures that different DMUs share the same benchmark for efficiency evaluation.

Step 3: Determine the same variables h and H for all DMUs

In model (7), each DMU has its own h_k and H_k, to make the restricted weights better reflect the DM's preference information. To enhance comparability among DMUs, we determine the same h and H for all decision units. The variables h and H are calculated as follows:

$$\begin{aligned} & Min \ \ (H-h) \\ & s.t. \ \ \sum_{i=1}^{m} \hat{v}_{ik} \bar{\hat{x}}_i = 1 \\ & \qquad \sum_{r=1}^{s} \hat{u}_{rk} \hat{y}_{rj} - \sum_{i=1}^{m} \hat{v}_{ik} \hat{x}_{ij} + \hat{\lambda}_j = 0, \ j=1,\cdots,n \\ & \qquad \alpha_r \cdot h \le \hat{u}_{rk} \hat{y}_{rj} \le \alpha_r \cdot H, \quad r=1,\cdots,s, \ k=1,\cdots,n \\ & \qquad \beta_i \cdot h \le \hat{v}_{ik} \hat{x}_{ij} \le \beta_i \cdot H, \quad i=1,\cdots m, \ k=1,\cdots n \\ & \qquad \hat{v}_{ik}, \hat{u}_{rk}, h, H \ge 0, \qquad i=1,\cdots,m, r=1,\cdots,s \end{aligned} \tag{9}$$

In model (9), the first constraint is to guarantee the comparability by the same standard, and the last three constraints are restrictions on DM's preference weights after being rescaled, in order to enable all DMUs to find the same lower bound h and upper bound H.

Step 4: Solve the rescaled preference cross weights for each DMU

After we compute h^* and H^* for all DMUs by using model (9), we can then obtain the preference cross weights of each DMU by solving model (10):

$$
\begin{aligned}
&\hat{E}_k = Max\left\{\sum_{r=1}^{s}\hat{u}_{rk}\hat{y}_{rk} - \varepsilon_2\frac{1}{n}\sum_{j=1}^{n}\hat{\lambda}_j)\right\} \\
&s.t.\ \sum_{i=1}^{m}\hat{v}_{ik}\bar{\hat{x}}_i = 1 \\
&\quad \sum_{r=1}^{s}\hat{u}_{rk}\hat{y}_{rj} - \sum_{i=1}^{m}\hat{v}_{ik}\hat{x}_{ij} + \hat{\lambda}_j = 0,\ j = 1,\cdots,n \\
&\quad \alpha_r \cdot h^* \le \hat{u}_{rk}\hat{y}_{rk} \le \alpha_r \cdot H^*,\ r = 1,\cdots,s \\
&\quad \beta_i \cdot h^* \le \hat{v}_{ik}\hat{x}_{ik} \le \beta_i \cdot H^*,\ i = 1,\cdots m \\
&\quad \hat{v}_{ik}, \hat{u}_{rk} \ge 0, i = 1,\cdots,m, r = 1,\cdots,s
\end{aligned}
\tag{10}
$$

In model (10), the objective function attempts to maximize the average efficiency of the other units under the insurance of maximizing the efficiency of the evaluated unit, with $\varepsilon_2 = 0.01$. Since the values of α_r, β_i, h^* and H^* are all known, the rescaled preference cross weights $\hat{u}_{rk}$ and $\hat{v}_{ik}$ can be obtained. Model (10) is solved n times, and we can obtain the rescaled cross weights for each DMU.

Step 5: Calculate the average preference cross weights and efficiency values

After the n sets of cross weights are obtained from model (10), the average preference cross weights $(\bar{\hat{v}}_i^*,\ \bar{\hat{u}}_r^*)$ for the n DMUs based on the method of Wang and Chin (2010) [26] are generated as:

$$
\begin{aligned}
\bar{\hat{v}}_i^* &= \frac{1}{n}\sum_{k=1}^{n}\hat{v}_{ik}^*, \quad i = 1,\cdots,m \\
\bar{\hat{u}}_r^* &= \frac{1}{n}\sum_{k=1}^{n}\hat{u}_{rk}^*, \quad r = 1,\cdots,s
\end{aligned}
\tag{11}
$$

The efficiency score of DMU_k can be computed by using the average set of preference cross weights; then all DMUs can be ranked:

$$
\bar{E}_k = \frac{\sum_{r=1}^{s}\bar{\hat{u}}_r^*\hat{y}_{rk}}{\sum_{i=1}^{m}\bar{\hat{v}}_i^*\hat{x}_{ik}}, k = 1,\cdots,n
\tag{12}
$$

3.3 Determining a CSW Based on DM's Preference Information

Although the average cross weights can be used to distinguish and rank all DMUs, it is irrational to regard the average cross weights as the basis of efficiency evaluation for all DMUs. The common set of weights method is popular for providing the same evaluation basis for all DMUs. For instance, Ramon, Ruiz and Sirvent (2012) proposed a CSW model that considered the deviations of the CSW from the profiles of weights of the DMUs in E (the set of efficient DMUs) provided by the CCR model [27]. In their study, they only considered the distances of the CSW from the weights of the efficient units, which may lead the CSW to be unreliable if there are very few efficient DMUs in DEA. What is more, the efficiencies calculated using this CSW may further deviate from their true values so that their corresponding rankings, to some extent, lack reliability.

3.3.1 The General Expression

In this section, we propose a general expression formula that tries to determine a CSW for all units by minimizing the deviations of the CSW from all preference cross weights of DMUs in order to solve the above problems.

The distance functions between the CSW and all cross weights of DMUs are defined through a family of p-metrics. preference

$$Lp_i = \left(\left(\sum_{j=1}^{n} \left| \hat{v}_{ij}^* - v_i \right| \hat{x}_{ij} \right)^p \right)^{1/p}, \quad i = 1, \cdots, m$$
$$Lp_r = \left(\left(\sum_{j=1}^{n} \left| \hat{u}_{rj}^* - u_r \right| \hat{y}_{rj} \right)^p \right)^{1/p}, \quad r = 1, \cdots, s \tag{13}$$

Where v_i and u_r are the common set of weights for all DMUs; $\hat{v}_{ij}^*$ and $\hat{u}_{rj}^*$ represent the optimal preference cross weights of each DMU; $\left|\hat{v}_{ij}^* - v_i\right|$ and $\left|\hat{u}_{rj}^* - u_r\right|$ serve as the distance between the CSW and the cross weights; $\hat{x}_{ij}$ and $\hat{y}_{rj}$ are the normalized input/output indicators; and p represents the distance parameter, i.e., a real number belonging to the closed interval $[1, \infty]$.

We can minimize the deviations of the CSW from all preference cross weights by the multiple objective programming:

$$Min\{Lp_i, Lp_r \mid i = 1, \cdots, m, r = 1, \cdots, s\} \tag{14}$$

If an optimal solution of the following single objective programming (15) exists, then this optimal solution will be an efficient solution of the multiple objective programming (14) (Chiang et al., 2011).

$$\begin{aligned} & Min\left\{ \sum_{i=1}^{m} Lp_i + \sum_{r=1}^{s} Lp_r \right\} \\ & s.t. \ \sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \le 0, \ j = 1, \cdots, n \\ & \quad u_r, v_i \ge 0, i = 1, \cdots, m, r = 1, \cdots, s \end{aligned} \tag{15}$$

In model (15), the constraints imply that the efficiencies obtained from the CSW of all units are still not more than 1.

3.3.2 Three Norms of Euclidean Distance

To minimize the deviations of the CSW from the profiles of preference cross weights of all DMUs, we can use different distance norms, e.g., p=1, 2 and ∞.

(1) $L_1 - norm$

By considering p = 1 in model (15), the distance measures the sum of individual deviations, as follows:

$$\begin{aligned} & Min\left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \left| \hat{v}_{ij}^* - v_i \right| \hat{x}_{ij} + \sum_{r=1}^{s} \sum_{j=1}^{n} \left| \hat{u}_{rj}^* - u_r \right| \hat{y}_{rj} \right\} \\ & s.t. \ \sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \le 0, \ j = 1, \cdots, n \\ & \quad u_r, v_i \ge 0, i = 1, \cdots, m, r = 1, \cdots, s \end{aligned} \tag{16}$$

Remark 3. Model (16) is non-linear because the objective has absolute values. However, its optimal solution can be found by transforming the non-linear model into a linear model. To convert the non-linear model into a linear model, we here introduce the lack variables δ_{ij}^-, δ_{ij}^+, δ_{rj}^- and δ_{rj}^+ based on the idea of goal programming, with $\delta_{ij}^-, \delta_{ij}^+, \delta_{rj}^-, \delta_{rj}^+ \ge 0$. Then, adding a set of constraints into model (17), the restrictions on the deviations are $\hat{v}_{ij}^* - v_i = \delta_{ij}^+ - \delta_{ij}^-$, $\hat{u}_{rj}^* - u_r = \delta_{rj}^+ - \delta_{rj}^-$. Therefore, minimizing the non-linear objective in model (16) is equivalent to minimizing the linear objective function $\sum_{i=1}^{m} \sum_{j=1}^{n} (\delta_{ij}^+ + \delta_{ij}^-) \hat{x}_{ij} + \sum_{r=1}^{s} \sum_{j=1}^{n} (\delta_{rj}^+ + \delta_{rj}^-) \hat{y}_{rj}$. Finally, the non-linear model (16) can be converted into the following linear model (17):

$$\begin{aligned}
& Min\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}(\delta_{ij}^{+}+\delta_{ij}^{-})\hat{x}_{ij}+\sum_{r=1}^{s}\sum_{j=1}^{n}(\delta_{rj}^{+}+\delta_{rj}^{-})\hat{y}_{rj}\right\} \\
& s.t.\ \sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{i=1}^{m}v_i\hat{x}_{ij}\le 0,\ j=1,\cdots,n \\
& \qquad \hat{v}_{ij}^{*}-v_i=\delta_{ij}^{+}-\delta_{ij}^{-},\ i=1,\cdots,m,\ j=1,\cdots,n \\
& \qquad \hat{u}_{rj}^{*}-u_r=\delta_{rj}^{+}-\delta_{rj}^{-},\ r=1,\cdots,s,\ j=1,\cdots,n \\
& \qquad u_r,v_i\ge 0, i=1,\cdots,m, r=1,\cdots,s
\end{aligned} \tag{17}$$

(2) L_2-norm

By considering p = 2 in model (15), the distance measures the quadratic sum of individual deviations, as follows:

$$\begin{aligned}
& Min\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}(\hat{v}_{ij}^{*}-v_i)^2\hat{x}_{ij}^2+\sum_{r=1}^{s}\sum_{j=1}^{n}(\hat{u}_{rj}^{*}-u_r)^2\hat{y}_{rj}^2\right\} \\
& s.t.\ \sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{i=1}^{m}v_i\hat{x}_{ij}\le 0,\ j=1,\cdots,n \\
& \qquad u_r,v_i\ge 0, i=1,\cdots,m, r=1,\cdots,s
\end{aligned} \tag{18}$$

(3) $L_\infty-norm$

When p = ∞ in model (15), the distance minimizes the maximum of individual deviations, as follows:

$$\begin{aligned}
& Min\left\{\sum_{i=1}^{m}\underset{j}{Max}(\left|\hat{v}_{ij}^{*}-v_i\right|\hat{x}_{ij})+\sum_{r=1}^{s}\underset{j}{Max}(\left|\hat{u}_{rj}^{*}-u_r\right|\hat{y}_{rj})\right\} \\
& s.t.\ \sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{i=1}^{m}v_i\hat{x}_{ij}\le 0,\ j=1,\cdots,n \\
& \qquad u_r,v_i\ge 0, i=1,\cdots,m, r=1,\cdots,s
\end{aligned} \tag{19}$$

Let $\omega_i=\underset{j}{Max}(\left|\hat{v}_{ij}^{*}-v_i\right|\hat{x}_{ij})$, $\varphi_r=\underset{j}{Max}(\left|\hat{u}_{rj}^{*}-u_r\right|\hat{y}_{rj})$, $\hat{v}_{ij}^{*}-v_i=\delta_{ij}^{+}-\delta_{ij}^{-}$ and $\hat{u}_{rj}^{*}-u_r=\delta_{rj}^{+}-\delta_{rj}^{-}$. Then, we have the single-objective linear programming model (20):

$$\begin{aligned}
& Min\left(\sum_{i=1}^{m}\omega_i+\sum_{r=1}^{s}\varphi_r\right) \\
& s.t.\ \sum_{r=1}^{s}u_r\hat{y}_{rj}-\sum_{i=1}^{m}v_i\hat{x}_{ij}\le 0,\ j=1,\cdots,n \\
& \qquad \hat{v}_{ij}^{*}-v_i=\delta_{ij}^{+}-\delta_{ij}^{-},\ i=1,\cdots,m,\ j=1,\cdots,n \\
& \qquad \hat{u}_{rj}^{*}-u_r=\delta_{rj}^{+}-\delta_{rj}^{-},\ r=1,\cdots,s,\ j=1,\cdots,n \\
& \qquad (\delta_{ij}^{+}+\delta_{ij}^{-})\hat{x}_{ij}\le\omega_i,\ i=1,\cdots,m,\ j=1,\cdots,n \\
& \qquad (\delta_{rj}^{+}+\delta_{rj}^{-})\hat{y}_{rj}\le\varphi_r,\ r=1,\cdots,s,\ j=1,\cdots,n \\
& \qquad u_r,v_i,\omega_i,\varphi_r\ge 0,\ i=1,\cdots,m,\ r=1,\cdots,s
\end{aligned} \tag{20}$$

3.4 The Combined Expression of L_1-norm and $L_\infty-norm$

Since there is mutual compensability of the deviations between one row and another in the L_1-norm of Euclidean Distance, and since the concept of $L_\infty-norm$ minimizes the maximum deviation, which will lead to multiple feasible solutions for CSW, we aim to avoid the defect of a single norm by introducing a priority-based combination of $L_1 - norm$ and $L_\infty - norm$ with $\varepsilon_2 = 0.01$, and hence to determine the common weights of all DMUs by the linear programming model (21).

$$
\begin{aligned}
& Min\left\{\left(\sum_{i=1}^{m}\omega_i + \sum_{r=1}^{s}\varphi_r\right) + \varepsilon_2\left(\frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n}(\delta_{ij}^{+}+\delta_{ij}^{-})\hat{x}_{ij} + \frac{1}{n}\sum_{r=1}^{s}\sum_{j=1}^{n}(\delta_{rj}^{+}+\delta_{rj}^{-})\hat{y}_{rj}\right)\right\} \\
& s.t.\ \sum_{r=1}^{s}u_r\hat{y}_{rj} - \sum_{i=1}^{m}v_i\hat{x}_{ij} \le 0,\ j=1,\cdots,n \\
& \quad \hat{v}_{ij}^{*} - v_i = \delta_{ij}^{+} - \delta_{ij}^{-},\ i=1,\cdots,m,\ j=1,\cdots,n \\
& \quad \hat{u}_{rj}^{*} - u_r = \delta_{rj}^{+} - \delta_{rj}^{-},\ r=1,\cdots,s,\ j=1,\cdots,n \\
& \quad (\delta_{ij}^{+}+\delta_{ij}^{-})\hat{x}_{ij} \le \omega_i,\ i=1,\cdots,m,\ j=1,\cdots,n \\
& \quad (\delta_{rj}^{+}+\delta_{rj}^{-})\hat{y}_{rj} \le \varphi_r,\ r=1,\cdots,s,\ j=1,\cdots,n \\
& \quad u_r, v_i, \omega_i, \varphi_r \ge 0,\ i=1,\cdots,m,\ r=1,\cdots,s
\end{aligned}
\tag{21}
$$

We can obtain the CSW efficiency values in accordance with the determined common weights:

$$
E_j = \frac{\sum_{r=1}^{s}u_r\hat{y}_{rj}}{\sum_{i=1}^{m}v_i\hat{x}_{ij}},\ j=1,\cdots,n \tag{22}
$$

4 Numerical Example

In this section, we take the example used in Wong & Beasley (1990) [28]. concerning the efficiency evaluation of scientific research in a university's seven departments to illustrate and examine our proposed methodology. Each DMU is evaluated in terms of three inputs and three outputs. The inputs include the number of academic staff (X_1), academic staff salaries in thousands of pounds (X_2) and support staff salaries in thousands of pounds (X_3). The outputs refer to the number of undergraduate students (Y_1), the number of postgraduate students (Y_2) and the number of research papers (Y_3).

4.1 Efficiency Results of the Traditional Models

In Table 1 we show the inputs/outputs of seven departments in a university and the efficiency scores of DMUs calculated from CCR model. Obviously, only the DMU4's efficiency score is less than 1 among the seven decision units. The efficiency of the other six DMUs is 1, i.e., DEA is effective. This result reflects the multi-DMUs efficiency problem caused by the "total weights flexibility" in the basic DEA models, which makes it impossible to distinguish and rank the efficiency values of the efficient DMUs.

Table 1. The raw data set and the efficiency values by CCR model

DMU	Inputs			Outputs			Efficiency by model (1)
	X_1	X_2	X_3	Y_1	Y_2	Y_3	
1	12	400	20	60	35	17	1.0000
2	19	750	70	139	41	40	1.0000
3	42	1500	70	225	68	75	1.0000
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1.0000
6	19	730	50	132	45	45	1.0000
7	41	2350	600	305	159	97	1.0000

As shown in Table 2, the efficiency values of all DMUs obtained from our proposed model (3) are exactly the same as those of the CCR model. We show the unique optimal weights for each DMU in Table 2; there are zero values in these weights. This result shows that the efficiency values obtained by model (3) may ignore some corresponding weights of some DMUs, and so model (3) cannot truly reflect the efficiency values of the DMUs.

Table 2. The efficiency values and unique optimal weights for each DMU

DMU	The unique optimal weights based on introducing a secondary goal function (ε_1 =0.0001)						Efficiency and ranking by model (3)
	V_1	V_2	V_3	U_1	U_2	U_3	
1	0.0354	0.0014	**0.0000**	0.0058	0.0161	0.0051	1.0000
2	0.0418	0.0002	0.0003	0.0056	0.0040	0.0016	1.0000
3	**0.0000**	0.0004	0.0063	0.0044	**0.0000**	**0.0000**	1.0000
4	0.0642	0.0001	**0.0000**	0.0091	**0.0000**	**0.0000**	0.8197
5	0.0093	0.0003	0.0001	0.0012	0.0038	0.0011	1.0000
6	0.0404	0.0003	0.0005	0.0037	0.0061	0.0054	1.0000
7	0.0179	0.0001	0.0002	0.0020	0.0015	0.0016	1.0000

4.2 Efficiency Results of the Proposed DM's Preference Weights Model

We use the analytic hierarchy process (AHP) to obtain the DM's preference on the perception of the relative importance of input/output indicators, as illustrated in Table 3.

Table 3. The DM's preference on evaluation indicators by AHP

First-grade Index	First-grade Index Weight	Second-grade Index	Second-grade Index Weight	Synthetic weights
Inputs	0.3333	X_1	0.5396	0.1798
		X_2	0.1634	0.0545
		X_3	0.297	0.0990
outputs	0.6667	Y_1	0.114	0.0760
		Y_2	0.4054	0.2703
		Y_3	0.4806	0.3204

In model (7), we propose the DM's preference weight restrictions to reflect the preference information of decision makers on the importance of input/output indicators. Table 4 shows the n sets of h_k and H_k, the DM's preference weights and the efficiency values for each DMU given by model (7). We have obtained a complete ranking of all DMUs by model (7), as shown in the rightmost column of Table 4, which is more persuasive to decision makers. In Table 4, there is no zero value in the optimal weights for each DMU, indicating that the DM's preference weight restrictions have effectively solved the zero-weights problem in basic DEA models.

Table 4. The efficiency values and DM's preference weights for each DMU

DMU	h_k	H_k	The DM's preference weights for each DMU by model (7) ($\varepsilon_2 = 0.01,\ \varepsilon_1 = 0.0001$)						Efficiency and ranking by model (7)
			V_1	V_2	V_3	U_1	U_2	U_3	
1	1.3906	3.0003	0.0450	0.0004	0.0149	0.0018	0.0107	0.0262	0.9271 (3)
2	1.2488	3.0003	0.0284	0.0002	0.0042	0.0007	0.0082	0.0100	0.8326 (4)
3	1.2180	3.0003	0.0128	0.0001	0.0042	0.0004	0.0048	0.0052	0.8121 (5)
4	0.4901	3.0003	0.0360	0.0003	0.0030	0.0004	0.0110	0.0092	0.3267 (7)
5	1.4795	3.0003	0.0120	0.0001	0.0012	0.0004	0.0028	0.0036	0.9864 (2)
6	1.4999	3.0003	0.0284	0.0002	0.0059	0.0009	0.0090	0.0107	1.0000 (1)
7	1.1571	3.0003	0.0132	0.0001	0.0005	0.0003	0.0020	0.0038	0.7714 (6)

4.3 Efficiency Results of the Modified Cross Weights Model with DM's Preference Information

For convenience of comparison, the raw input/output data have been normalized by model (8), as shown in Table 5. The data normalization only eliminates the influence of dimension among input/output variables, and does not change the information content represented by the data.

Table 5. The Normalized input/output data set by model (8)

DMU	Normalized inputs			Normalized inputs		
	X_1	X_2	X_3	Y_1	Y_2	Y_3
1	0.2667	0.1702	0.0333	0.1967	0.2201	0.1308
2	0.4222	0.3191	0.1167	0.4557	0.2579	0.3077
3	0.9333	0.6383	0.1167	0.7377	0.4277	0.5769
4	0.3333	0.2553	0.1667	0.2951	0.0755	0.1308
5	1.0000	0.8511	0.4167	0.8295	0.9119	1.0000
6	0.4222	0.3106	0.0833	0.4328	0.2830	0.3462
7	0.9111	1.0000	1.0000	1.0000	1.0000	0.7462

We obtain the values h = 0.2019 and H = 6.0568 for all DMUs from model (9). Table 6 shows the optimal cross weights for each DMU obtained from model (10), the average cross weights obtained from model (11) and the average cross-efficiency for all DMUs obtained from model (12). Compared with the efficiency values of model (7), the deviations among the efficiency values obtained from model (12) are smaller, because model (12) considers the comparability among DMUs.

Table 6. The efficiency values and cross weights by the modified cross weights model

DMU	The optimal cross weights by model (10)						Average cross-efficiency and ranking by model (12)
	V_1	V_2	V_3	U_1	U_2	U_3	
1	0.6386	0.8751	0.5996	0.0780	1.1775	0.4947	0.8713 (5)
2	1.5264	0.0345	0.1713	1.0100	0.2197	0.4916	0.9181 (4)
3	1.1668	0.4696	0.1713	0.6240	0.1276	1.0014	0.7417 (6)
4	0.7195	1.0388	0.1199	0.6022	0.7231	0.4947	0.5363 (7)
5	1.0890	0.3879	0.4937	0.5549	1.1046	0.1572	0.9913 (1)
6	1.4947	0.0354	0.2398	0.7751	0.4050	0.6125	0.9745 (2)
7	1.1858	0.3301	0.3850	0.4603	1.2704	0.0867	0.9315 (3)
Average cross weights	1.1173	0.4530	0.3115	0.5864	0.7183	0.4770	

4.4 Efficiency Results of the Proposed Common Set of Weights Model

In Table 7 we show the common weights for all DMUs, which is generated from model (21). As is shown in Table 8, the common weights model (22) can obtain a full ranking of the efficiencies among DMUs.

Table 7. The common weights generated by model (21)

	V_1	V_2	V_3	U_1	U_2	U3
Common-weights ($\varepsilon_2 = 0.01$)	1.2722	0.4742	0.3869	0.6324	0.9281	0.4660

Table 8. The efficiency and ranking for each DMU by model (22)

DMU	1	2	3	4	5	6	7
Efficiency and Ranking ($\varepsilon_2 = 0.01$)	0.9002 (4)	0.9145 (5)	0.7376 (6)	0.5210 (7)	1.0000 (1)	0.9735 (2)	0.9446 (3)

In Table 9, we show the efficiency scores and related rankings of DMUs calculated by different models. The efficiency rankings of our proposed models are basically consistent with the results of the CCR model. Obviously, DMU5 and DMU6 have been evaluated as the more effective DMUs through our proposed model (7), model (12) and model (22), which are also CCR efficient. More importantly, DMU4 is evaluated as the most inefficient DMU by all of our proposed models, and it is also the most inefficient in the CCR model. Compared with model (1)/(3), model (7) has completely ranked the efficiencies of all DMUs by introducing the DM's preference Weights restriction. In addition, the deviations among the efficiency values obtained from model (12) are smaller than the deviations in model (7), and this result illustrates that the self-evaluation added to peer-evaluation in model (12) is more reliable. In model (12) and model (22), the relative efficiency rankings of DMU1 and DMU2 have been exchanged, mainly due to the common weights restrictions in model (22); this demonstrates that the common evaluation basis for all DMUs in the DEA evaluation is necessary.

Table 9. The efficiency values and associated rankings computed by different models

DMU	The CCR model (1) / The unique optimal weights model (3)	The proposed DM's preference weights model (7)	The average set of cross weights model (12)	The proposed common set of weights model (22)
1	1.0000 (1)	0.9271 (3)	0.8713 (5)	0.9002 (4)
2	1.0000 (1)	0.8326 (4)	0.9181 (4)	0.9145 (5)
3	1.0000 (1)	0.8121 (5)	0.7417 (6)	0.7376 (6)
4	0.8197 (7)	0.3267 (7)	0.5363 (7)	0.5210 (7)
5	1.0000 (1)	0.9864 (2)	0.9913 (1)	1.0000 (1)
6	1.0000 (1)	1.0000 (1)	0.9745 (2)	0.9735 (2)
7	1.0000 (1)	0.7714 (6)	0.9315 (3)	0.9446 (3)

5 Conclusions

To determine the common weights in DEA, this paper has proposed a new common weights selection approach based on the DM's preference information and cross weights. The proposed method shows a multi-objective benevolent linear programming (MOBLP) model, which can make each DMU have a unique optimal weights. Moreover, based on the idea of value judgment approaches, the proposed approach incorporates the expert opinion into weights restriction, which not only combines qualitative analysis with quantitative analysis but also overcomes the zero-value problem for the optimal weights. In addition, the novel common weights model is combined with two "Euclidean Distance" norms to determine the CSW by minimizing the deviations of the CSW from all preference cross weights of DMUs in order to strengthen the efficiency analysis with a fair and unbiased assessment of all the units using a common basis.

Our approach mainly has three advantages. First, it provides a novel method for incorporating the DM's preference information into a set of common weights. Second, When solving the optimal preference cross weights for each DMU, the quantitative analysis has the equal priority to the qualitative analysis. So the generated common weights based on the optimal preference cross weights is more scientific and acceptable. Third, the process of determining common weights overcomes the drawback of the non-uniqueness, zero weights and uncomparability. Therefore, when the preferences of input/output indicators are different in an evaluation (e.g., bidding scheme evaluation, supplier selection evaluation, etc.), using our approach would have an advantage in enhancing the quality of efficiency discrimination and ranking and in ensuring the fairness and impartiality of the evaluation.

Our shortcomings include two points. First of all, our approach has not verified again by using the real-world dataset and considering more realistic constraints, such as undesirable input / output factors, imprecise data or fuzzy environment. Secondly, if the proposed preference integration method is introduced into other DEA models or multi-stage DEA models, will there be complicated or even no solution? Future studies will use the proposed approach to make assessment decisions involving more realistic constraints.

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