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The Study and Analysis of System Performance in Cooperative NOMA System

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Abstract. This research investigated a non-orthogonal multiple access (NOMA) system with relay in downlink cellular communication. We assumed that the communication between the base station (BS) and the user is via a half-duplex relay based on a decode-and-forward (DF) protocol. There is also a direct link in this cooperative NOMA system. The exact closed-form expression of user outage probability based on the cooperative NOMA network was derived through the analysis of this model. The outage probability when multiplexing two users were approximated in this NOMA system to analyze the system performance more deeply. The results showed that, compared to the orthogonal multiple access (OMA) system, the cooperative NOMA system performed better outage performance and greater diversity gain. Through simulations, we verified the influence of the relay position on outage performance. Meanwhile, the closer to the source node, the lower probability of outage. These results provide a theoretical basis for practical applications in communications.

Keywords: non-orthogonal multiple access, cooperative network, outage probability, sumrate

1 Introduction

Multiple access technology is a critical technology in wireless communication systems. The purpose of this technology is to differentiate signals from different users and provide communication service and support to multiple users. An orthogonal multiple access (OMA) system provides signal distinction through assigning independent channel resources in time, frequency, codeor space. OMA was adopted in the early wireless communication systems and is still in use today. OMA is based on single-user communication. Each communication process only involves one single user or data stream. Through years of development, the performance of single-user wireless communication has been greatly improved and is already very close to its theoretical upper bound. Recently, non-orthogonal multiple access (NOMA) has received extensive attention for further improve the performance of wireless communication systems [1-2]. NOMA breaks the limitation of orthogonalization in the OMA system, allowing multiple users to be supported simultaneously within the same channel resource. With advanced technologies in transmitters and receivers, NOMA multiplexes the signals of different users in the power domain and processes them as a whole. The multi-user communication method using NOMA provides a new room for growth in the performance of the wireless communication system [3].

In conventional direct link communication systems, information energy dispersed in the air has been treated as interference and cannot be utilized. However, wireless relay communication can effectively use the broadcasting character of wireless transmission to realize higher efficiency information

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communication, increase system capacity and enhance system coverage [4]. This technology has received attention in the standardization of the fourth-generation mobile communication system and has been adopted by LTE-Advanced and 802.16j/m standards. In relay communication systems, the relay or user with relay can send received signals to the corresponding target user, indirectly utilizing the information energy that is dispersed in the air. The diversity combination (such as maximal ratio combination) method is used at the receiving end, which effectively enhances communication efficiency, reliability, and energy efficiency. In this system, relay protocols with close to optimal performance come at the cost of complexity. Some simple relay protocols such as decode-and-forward (DF) and amplify-and-forward (AF) might achieve a better compromise between performance and complexity. The performance of the relay system is greatly affected by the state of each link in the relay system. It is usually close to the optimal performance in a symmetrical channel, but the gap with the optimal performance will become larger in other cases [5].

In 5G systems, relay technology will play an even more important role. Through cooperation between users or between users and relays, a more flexible networking method can be realized to support diversified services and access requirements. The application of a NOMA transmission system in relay technology has great potential in the next generations of wireless communication systems. The concept of NOMA is to allocate more power to the users with poor channel conditions and less power to users with better channel conditions. However, this approach still provides a limited improvement in capacity to cell-edge users with exceptionally poor channel conditions. As users are differentiated by power, the NOMA system enables multiple users to share all the channel resources, thereby effectively enhancing the spectral efficiency of the system [6-8].

On the other hand, the cooperative NOMA network refers to the improvement of the reliability of the NOMA network through cooperation between users or dedicated relay-assisted information transmission [9]. In a relay cooperative NOMA network, relay selection is the most direct and effective way to obtain space diversity and increase spectral efficiency. However, system performance might vary with different choices in relay schemes. A cooperative NOMA network utilizes the broadcasting characteristics of wireless communication, along with dedicated relay stations or multiple user cooperation, to form a virtual array of distributed antennas and achieve multiple input multiple output (MIMO). This antenna space diversity feature improves the overall performance of the system. The application of NOMA with relay systems is significant as it could further enhance the performance of cell-edge users [10].

Ding et al. proposed the concept of a cooperative NOMA. In this scheme, the users with better channel conditions need first to decode the signals of users with inferior channel conditions and act as a relay to assist the base station in serving the user with inferior channel conditions [11]. This scheme would improve the throughput of cell-edge users. Choi et al. studied a protocol involving the cooperation of two cell base stations with NOMA technology, which significantly improved the throughput of cell-edge users [5]. Men et al. applied the downlink NOMA technology to a multiple antenna amplify-and-forward (AF) relay cooperative network and studied the outage probability under Rayleigh fading conditions [12-13]. Kim et al. used a cooperative NOMA with DF at the relay, in which the relay assisted the cell-edge users and elevated the spectral efficiency [3]. Ding et al. studied the NOMA cooperative relay selection scheme and proved that a two-stage relay selection scheme could achieve optimal diversity gain with the lowest outage probability [11]. Zhang et al. studied downlink access systems with cooperative full-duplex relays and established a fundamental relationship between the base station, relay power allocation, and outage probability. They also numerically simulated the relationship between relay position and outage probability [14]. However, the above researches also have limitations. There is still no complete mathematical model that describes the working mechanism of relay NOMA systems. Therefore, the study of the NOMA cooperation scheme is essential in the application of relay nodes to increase transmission effectiveness and reliability for users with poor channel conditions.

2 Model and Transmission Protocol for NOMA Based on Relay Systems

2.1 System Model for Cooperative NOMA

In this study, we investigate the downlink wireless communication system that contains a relay node(RN) and uses NOMA as the access method. For the convenience of analysis, it is assumed that the system is composed of a base station (BS), a half-duplex RN, and two user equipment (UE) configured with a single antenna. Fig. 1 shows the cooperative NOMA system based on the DF protocol. There will be obstacles and shadow fading problems associated with larger cell, where the direct channel conditions are poor between the base station (BS) and the two user equipment. In this situation, one user could use RN for multi-hop coordinated transmission and adopt half-duplex and decode-and-forward (DF) for relay transmission.

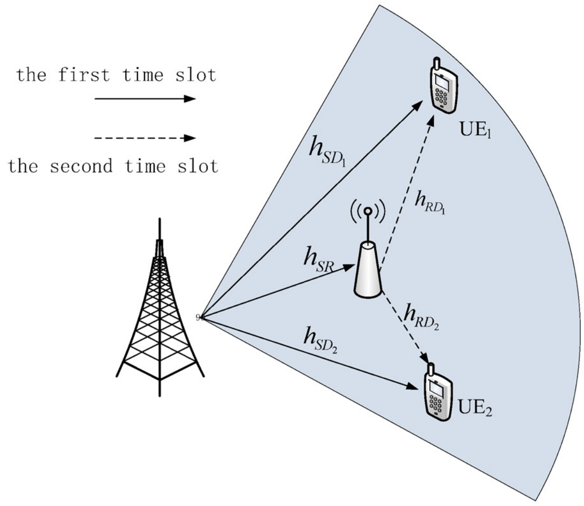

Fig. 1. NOMA relay transmission system model

The transmission and retransmission power of BS and RN are P_s and P_r, respectively. For the convenience of analysis, we use S, R, D_1, D_2 to denote BS, relay node, UE$_1$ and UE$_2$ respectively. We also assume that the channels $h_{SD_i}, h_{RD_i} (i=1,2)$, and h_{SR} between the BS and RN, RN and D$_1$or D$_2$, which all obey complex Gaussian distribution, with the respective parameters $\Omega_{SD_i}, \Omega_{RD_i} (i=1,2)$, and Ω_{SR}.

2.2 Signal Model of Cooperative NOMA

After the relay receives the information sent by the source correctly, it will use the same encoding method as the source to re-encode and transmit the part of the information to be forwarded. This is similar to conventional relay and forward methods, where each NOMA cooperative transmission includes two time slots.

In the first time slot, the BS uses superimposition coding technology for the transmission signals x_1 and x_2 required by the users UE$_1$ and UE2 and broadcasts them to RN. The transmission signals are in the form of $x_s = \sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2$, where α_1, α_2 are the power allocation factors of each user. Assuming that the users in the cell are sorted according to the channel quality of their direct links between the source and the sink, the channel gain should satisfy $|h_{SD_1}|^2 < |h_{SD_2}|^2$. Therefore, the transmission power allocated for x_1 should be greater than that of x_2, i.e. $\alpha_1 > \alpha_2$, where $\alpha_1 + \alpha_2 = 1$. The signals received at RN, and the receiving ends D$_1$ and D$_2$, could be expressed by the following equations:

$$y_{SR} = h_{SR}(\sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2) + n_{SR} \tag{1}$$

$$y_{SD_1} = h_{SD_1}(\sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2) + n_{SD_1}. \tag{2}$$

$$y_{SD_2} = h_{SD_2}(\sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2) + n_{SD_2}. \tag{3}$$

where n_{SR}, $n_{SR}, n_{SD_1}, n_{SD_2}$ are additive white Gaussian noise (AWGN), and $n_{SR}, n_{SD_1}, n_{SD_2} \sim CN(0, N_0)$.

In the direct links, the signal to interference plus noise ratio (SINR) is defined by $\gamma_{SD_1}^{x_1} = \frac{\alpha_1 P_s |h_{SD_i}|^2}{\alpha_2 P_s |h_{SD_i}|^2 + N_0^2}$ for signal x_1 at the UE$_1$. In order to decode signal x_2 at the UE$_2$, one needs to demodulate signal x_1 firstly, with $\gamma_{SD_{1\to2}}^{x_1} = \frac{\alpha_1 P_s |h_{SD_i}|^2}{\alpha_2 P_s |h_{SD_i}|^2 + N_0^2}$ denoting its instantaneous SINR. This is followed by demodulating the signal x_2, along with its post demodulation SINR $\gamma_{SD_2}^{x_2} = \alpha_2 \rho |h_{SD_i}|^2$.

For cooperative links that contain a relay, successive interference cancellation (SIC) is used at RN to decode and forward the signal. For example, one could first treat x_1 as interference and decode x_1. Then eliminate x_1 to decode x_2. The decoded SINR for x_1 could be expressed as: $\gamma_R^{x_1} = \frac{\alpha_1 P_s |h_{SR}|^2}{\alpha_2 P_s |h_{SR}|^2 + N_0}$.

In the second time slot, if signals x_1 and x_2 could be accurately decoded at the relay, the relay could use the same coding method as the source, and use superposition coding (SC) to recode the two signals into one signal x_s. Then forward signal x_s to a sink with power P_R. This forwarded signal could be expressed as $x_s = \sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2$. The specific signal received at thereceiving end is:

$$y_{RD1} = \sqrt{P_R} h_{RD_1} x_S + n_{RD_1} = \sqrt{P_R} h_{RD_1} [h_{SR}(\sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2) + n_{SR}] + n_{RD_1}. \quad \textbf{(4)}$$

$$y_{RD_2} = \sqrt{P_R} h_{RD_2} x_S + n_{RD_2} = \sqrt{P_R} h_{RD_2} [h_{SR}(\sqrt{\alpha_1 P_s} x_1 + \sqrt{\alpha_2 P_s} x_2) + n_{SR}] + n_{RD_2}. \quad \textbf{(5)}$$

where $n_{RD_1}, n_{RD_2} \sim CN(0, N_0)$ are AWGN.

Viewed from the receiving end, in relay link $R \to D_1$, the instantaneous SINR for the first user could be expressed as:

$$\gamma_{RD_1}^{x_1} = \frac{\alpha_1 P_R |h_{SR}|^2 |h_{RD_1}|^2}{\alpha_2 P_R |h_{RD_1}|^2 |h_{SR}|^2 + P_R(|h_{RD_1}|^2 + |h_{SR}|^2) + N_0^2}. \quad \textbf{(6)}$$

After demodulating the signal of the first user, the instantaneous SINR through the relay link $R \to D_2$ for the second user could be expressed as

$$\gamma_{RD_{1\to2}}^{x_1} = \frac{\alpha_1 P_R |h_{RD_2}|^2 |h_{SR}|^2}{\alpha_2 P_R |h_{SR}|^2 |h_{RD_2}|^2 + P_R(|h_{SR}|^2 + |h_{RD_2}|^2) + N_0^2}. \quad \textbf{(7)}$$

and the receiving signal SINR for the second user will be expressed as

$$\gamma_{RD_2}^{x_2} = \frac{\alpha_2 P_R |h_{SR}|^2 |h_{RD_2}|^2}{P_R(|h_{RD_1}|^2 + |h_{SR}|^2) + N_0^2}. \quad \textbf{(8)}$$

Finally, a selective combination scheme is adopted to process the signal at the receiving end.

2.3 Analysis of Achievable Transmission Rates with Cooperative NOMA Systems

Reasonable values for power allocation factor α_1 and α_2 could assure that UE$_1$ and UE$_2$ are properly received and decoded by the relay and the ending. As the cooperative NOMA system is based on the DF, satisfactory conditions for the signal receiving rates R_1 and R_2 (for UE$_1$ and UE$_2$ respectively), and for coefficients α_1 and α_2 could be deduced [15-16].

$$\begin{aligned} R_1 &= \frac{1}{2}\min\{\log_2(1+\gamma_{SD_1}^{x_1}),\log_2(1+\gamma_{SR}^{x_1})\} \\ &= \frac{1}{2}\log_2\left(1+\frac{\min\{|h_{SD_1}|^2,|h_{SR}|^2\}\rho\alpha_1}{\min\{|h_{SD_1}|^2,|h_{SR}|^2\}\rho\alpha_2+1}\right) \\ &= \frac{1}{2}\log_2(1+\min\{|h_{SD_1}|^2,|h_{SR}|^2\}\rho)-\frac{1}{2}\log_2(\min\{|h_{SD_1}|^2,|h_{SR}|^2\}\rho\alpha_2+1) \end{aligned} \quad . \qquad \textbf{(9)}$$

where $\rho=\frac{P}{N_0^2}$, and 1/2 indicates that two time slots are required to complete one information transmission.

As the point to point DF signaling rate at the relay depends on the weaker channel, the achievable rate for signal x_2 of UE_2 could be expressed as:

$$\begin{aligned} R_2 &= \frac{1}{2}\min\{\log_2(1+\gamma_{SD_2}^{x_2}),\log_2(1+\gamma_{RD_2}^{x_2})\} \\ &= \frac{1}{2}\log_2(1+\min\{|h_{SR}|^2\ \rho\alpha_2,|h_{RD_2}|^2\ \rho\}) \end{aligned} \quad . \qquad \textbf{(10)}$$

where $\gamma_{SD_2}^{x_2}=\alpha_2\rho|h_{SD_i}|^2$, $\gamma_{RD_{1\to 2}}^{x_2}=\frac{\alpha_1\rho^2|h_{RD_2}|^2|h_{SR}|^2}{\alpha_2\rho^2|h_{RD_2}|^2|h_{SR}|^2+\rho(|h_{RD_2}|^2+|h_{SR}|^2)+1}$, $\gamma_{RD_2}^{x_2}=\frac{\alpha_2\rho^2|h_{RD_2}|^2|h_{SR}|^2}{\rho(|h_{RD_2}|^2+|h_{SR}|^2)+1}$.

3 Analysis of the System Outage Probability

Outage probability is a key index in assuring quality of system (QoS) and deserves special attention during system design and optimization. The analysis above reveals that user outage might arise from two situations as stated here:

(1) The relay cannot correctly decode; this will lead to the failure of correct decoding at the receiving end.

(2) The relay can be decoded correctly, but the ending cannot recover the information contained in the source when decoding according to the signal forwarded by the relay.

In this section, we will analyze and deduce the closed-form expression of user outage probability and derive anapproximation expression for high SINR. The outage probability analyzed below is based on the above two scenarios.

3.1 Analysis of Outage Probability within the NOMA Cooperation Mechanism

This part analyzes the conditions for the outage of the cooperative NOMA system. Then we deduce the relationship between outage probability and power allocation factor, and propose an optimal solution for power allocation.

We assume that the relay participates in the entire transmission process, from beginning to end, as we analyze the condition where $|h_{SR}|^2>|h_{SD}|^2$. The analysis above shows that during the cooperation transmission process, outage occurs only when the direct link S-D and the relay link R-D are simultaneously outage. It is obvious that system outage will occur only when the SINR for both links is below the decision threshold.

Let $\gamma_{th1},\gamma_{th2}$ denote the minimum SNR predefined by UE_1 and UE_2, respectively. When the user's actual SNR is less than the predefined minimum SNR, a system outage occurs. In other words, a system outage occurs when UE_1 cannot decode signal x_1, or when UE_2 cannot decode signal x_2. Since signal x_1 of UE_1 is detected by SIC, signal x_2 of UE_2 must be decoded firstly.

Signal x_2 of UE_2 was used as an example for an analysis at the first time slot. UE_2 can accurately demodulate the preceding UE_1 signal, and then SIC was used to demodulate UE_2. This event can be defined as:

$$\begin{aligned} B_{2,1}^{c} &= \{\gamma_{SD_{1\to 2}}^{x_2} > \gamma_{th1}^{x_2}\} \\ &= \left\{ \frac{\alpha_1 \rho |h_{SD_2}|^2}{\alpha_2 \rho |h_{SD_2}|^2 + 1} > \gamma_{th1} \right\} \\ &= \left\{ |h_{SD_2}|^2 > \frac{\gamma_{th1}}{\alpha_1 \rho - \alpha_2 \rho \gamma_{th1}} \right\} \end{aligned} \quad . \tag{11}$$

For UE$_2$, this signal can be directly demodulated. This event is defined as:

$$\begin{aligned} B_{2,2}^{c} &= \{\gamma_{SD_2} > \gamma_{th2}\} \\ &= \{\alpha_2 \rho |h_{SD_2}|^2 > \gamma_{th2}\} \\ &= \left\{ |h_{SD_2}|^2 > \frac{\gamma_{th2}}{\alpha_2 \rho} \right\} \end{aligned} \quad . \tag{12}$$

Then the probability of user UE$_2$ being interrupted on the direct link is expressed as:

$$\begin{aligned} P_{out\_direct}^{x_2} &= 1 - \Pr(B_{2,1}^{c} \mathrm{U} B_{2,2}^{c}) \\ &= 1 - \Pr(\gamma_{SD_{1\to 2}} > \gamma_{th1}, \gamma_{SD_2} > \gamma_{th2}) \\ &= 1 - \Pr\left(\frac{\alpha_1 \rho |h_{SD_2}|^2}{\alpha_2 \rho |h_{SD_2}|^2 + 1} > \gamma_{th1}, \alpha_2 \rho |h_{SD_2}|^2 > \gamma_{th2} \right) \\ &= 1 - \Pr\left(|h_{SD_2}|^2 > \frac{\gamma_{th1}}{\alpha_1 \rho - \alpha_2 \rho \gamma_{th1}}, |h_{SD_2}|^2 > \frac{\gamma_{th2}}{\alpha_2 \rho} \right) \end{aligned} . \tag{13}$$

Let $\zeta_1 = \max[\frac{\gamma_{th1}}{\alpha_1 \rho - \alpha_2 \rho \gamma_{th1}}, \frac{\gamma_{th2}}{\alpha_2 \rho}]$, and the outage probability for the direct link can be expressed as:

$$P_{out\_direct}^{x_2} = 1 - \Pr(|h_{SD_2}|^2 > \zeta_1) . \tag{14}$$

In the second time slot, which is the relay link, the following equation defines the event where the second user can accurately demodulate the preceding signal of the first user [17]:

$$C_{2,1}^{c} = \{\gamma_{RD_{1\to 2}} > \gamma_{th3}\} . \tag{15}$$

$$\begin{aligned} C_{2,1}^{c} &= \left\{ \frac{\alpha_1 \rho^2 |h_{SR}|^2 |h_{RD_2}|^2}{\alpha_2 \rho^2 |h_{SR}|^2 |h_{RD_2}|^2 + \rho(|h_{SR}|^2 + |h_{RD_2}|^2) + 1} > \gamma_{th3} \right\} \\ &= \left\{ [\rho^2(\alpha_1 - \gamma_{th3}\alpha_2)|h_{RD_2}|^2 - \rho\gamma_{th3}] |h_{SR}|^2 > \gamma_{th3}(1 + \rho |h_{RD_2}|^2) \right\} \\ &= \left\{ |h_{RD_2}|^2 > \frac{\gamma_{th3}}{\alpha_1 \rho - \alpha_2 \rho \gamma_{th3}} = \tau, |h_{SR}|^2 > \frac{\tau(1 + \rho |h_{RD_2}|^2)}{\rho(|h_{RD_2}|^2 - \tau)} \right\} \end{aligned} . \tag{16}$$

For the second user, the event where it directly demodulates itself can be defined as:

$$\begin{aligned}C_{2,2}^{c} &= \{\gamma_{RD_2} > \gamma_{th4}\} \\ &= \left\{\frac{\alpha_2\rho^2 |h_{SR}|^2 |h_{RD_2}|^2}{\rho(|h_{SR}|^2 + |h_{RD_2}|^2) + 1} > \gamma_{th4}\right\} \\ &= \left\{[(\alpha_2\rho |h_{RD_2}|^2 - \gamma_{th4})|h_{SR}|^2 > \gamma_{th4}(1 + \rho |h_{RD_2}|^2)\right\} . \\ &= \left\{|h_{RD_2}|^2 > \frac{\gamma_{th4}}{\alpha_2\rho} = \lambda, |h_{SR}|^2 > \frac{\lambda(1 + \rho |h_{RD_2}|^2)}{\rho(|h_{RD_2}|^2 - \lambda)}\right\}\end{aligned} \tag{17}$$

Let $\zeta_2 = \max[\frac{\gamma_{th3}}{\alpha_1\rho - \alpha_2\rho\gamma_{th3}}, \frac{\gamma_{th4}}{\alpha_2\rho}]$, and the probability of outage of the second user on the relay link can be expressed as:

$$\begin{aligned}P_{out\_relay}^{x_2} &= 1 - \Pr(C_{2,1}^{c} \mathbf{U} C_{2,2}^{c}) \\ &= 1 - \Pr(|h_{RD_2}|^2 > \zeta_2, |h_{SR}|^2 > \frac{\zeta_2(1 + \rho |h_{RD_2}|^2)}{\rho(|h_{RD_2}|^2 - \zeta_2)}) \quad .\end{aligned} \tag{18}$$

Therefore, during the entire relay transmission process, the probability of outage for the second user can be expressed as:

$$\begin{aligned}P_{out}^{x_2} &= P_{out\_direct}^{x_2} \times P_{out\_relay}^{x_2} \\ &= [1 - \Pr(\gamma_{SD_{1\to2}} > \gamma_{th1}, \gamma_{SD_2} > \gamma_{th2})][1 - \Pr(\gamma_{RD_{1\to2}} > \gamma_{th3}, \gamma_{RD_2} > \gamma_{th4})] \quad . \\ &= \left\{1 - \Pr(|h_{SD_2}|^2 > \zeta_1)\right\}\left\{1 - \Pr(|h_{RD_2}|^2 > \zeta_2, |h_{SR}|^2 > \frac{\zeta_2(1 + \rho |h_{RD_2}|^2)}{\rho(|h_{RD_2}|^2 - \zeta_2)})\right\}\end{aligned} \tag{19}$$

The derivation above shows that this event is true only when $\alpha_1 > \alpha_2\gamma_{th1}$.

It can be seen that when the direct link and the relay link are interrupted at the same time, the second user will be interrupted.

We assume that the relay link channel gain follows an exponential distribution. Since the channels of the two users are sorted, the probability density function (PDF) of channel $|\mathrm{h}_{\mathrm{SD}_2}|^2$ after statistical orderaddition is:

$$f_{|h_{SD_2}|^2}(x) = \frac{2}{\Omega_{SD}}(e^{-\frac{x}{\Omega_{SD}}} - e^{-\frac{2x}{\Omega_{SD}}}) . \tag{20}$$

hence the result from the first part of equation (19) is

$$\Pr(|h_{SD_2}|^2 < \zeta_1) = 1 - \Pr(|h_{SD_2}|^2 > \zeta_1) = 1 - 2e^{-\frac{\zeta_1}{\Omega_{SD}}} + e^{-\frac{2\zeta_1}{\Omega_{SD}}} . \tag{21}$$

On the other hand, the relay link channel between users has not been estimated nor sorted before the relay processes the signal. If we assume that the relay link channel gain $|h_{RD_2}|^2$ follows an exponential distribution, the second part of the outage probability equation could be expressed as:

$$\begin{aligned}&1 - \Pr(|h_{RD_2}|^2 > \zeta_2, |h_{SR}|^2 > \frac{\zeta_2(1 + \rho |h_{RD_2}|^2)}{\rho(|h_{RD_2}|^2 - \zeta_2)}) \\ &= 1 - \int_{\zeta_2}^{+\infty} \frac{1}{\Omega_{RD}} e^{-\frac{y}{\Omega_{RD}}} e^{-\frac{\zeta_2(1+\rho y)}{\rho(y-\zeta_2)\Omega_{SR}}} dy \quad . \\ &= 1 - e^{-\zeta_2(\frac{1}{\Omega_{SR}} + \frac{1}{\Omega_{RD}})} \sqrt{\frac{4\zeta_2(1 + \rho\zeta_2)}{\gamma\Omega_{SR}\Omega_{RD}}} K_1(\sqrt{\frac{4\zeta_2(1 + \rho\zeta_2)}{\gamma\Omega_{SR}\Omega_{RD}}})\end{aligned} \tag{22}$$

where $K_1(\cdot)$ is the first-order modified Bessel function of the second kind [18].

Following the derivation above, the outage probability for UE$_2$ could be expressed as:

$$\begin{aligned} P_{out}^{x_2} &= P_{out\_direct}^{x_2} \times P_{out\_relay}^{x_2} \\ &= \left(1-2e^{-\frac{\zeta^*}{\Omega_{SD}}}+e^{-\frac{2\zeta^*}{\Omega_{SD}}}\right)\times\left(1-e^{-\zeta^*(\frac{1}{\Omega_{SR}}+\frac{1}{\Omega_{RD}})}\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}}K_1(\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}})\right)\cdot \end{aligned} \quad \textbf{(23)}$$

where $\zeta = \max[\zeta_1, \zeta_2]$.

Likewise, the outage probability for UE$_1$ could also be derived and expressed as:

$$\begin{aligned} P_{out}^{x_1} &= P_{out\_direct}^{x_1} \times P_{out\_relay}^{x_1} \\ &= [1-\Pr(\gamma_{SD_1} > \gamma_{th1})][1-\Pr(\gamma_{RD_1} > \gamma_{th3})] \\ &= \left\{1-\Pr\left(\frac{\alpha_1\rho\,|h_{SD_2}|^2}{\alpha_2\rho\,|h_{SD_2}|^2+1} > \gamma_{th1}\right)\right\} \\ &\times\left\{1-\Pr\left(\frac{\alpha_1\rho^2\,|h_{SR}|^2|h_{RD_2}|^2}{\alpha_2\rho^2\,|h_{SR}|^2|h_{RD_2}|^2+\rho(|h_{SR}|^2+|h_{RD_2}|^2)+1} > \gamma_{th3}\right)\right\} \end{aligned} \quad . \quad \textbf{(24)}$$

It is known that the PDF for channel $|\mathrm{h}_{\mathrm{SD}_2}|^2$ is

$$f_{|h_{SD_1}|^2}(x) = \frac{2}{\lambda_{SD}}e^{-\frac{2x}{\lambda_{SD}}}. \quad \textbf{(25)}$$

Likewise, as the square of the channel gain of the relay link obeys exponential distribution, the equation above is finally simplified into:

$$P_{out}^{x_1} = \left(1-e^{-\frac{2\zeta^*}{\Omega_{SD}}}\right)\times\left(1-e^{-\zeta^*(\frac{1}{\Omega_{SR}}+\frac{1}{\Omega_{RD}})}\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}}K_1(\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}})\right). \quad \textbf{(26)}$$

3.2 The Asymptotic Expression for the Outage Probability in the Cooperation NOMA under Higher SNR

The asymptotic expression of the outage probability under the condition of higher SNR is analyzed. Since $x \to 0$, the exponential function and the first-order modified Bessel function of the second kind have the following properties:

$e^x \approx 1+x, K_1(x) \approx 1/x$, also $\frac{\gamma_{th2}}{\alpha_2\rho} = \zeta^*$. Therefore, when the signal-to-noise ratio approaches infinity, $\rho \to \infty$, the outage probability is approximated as:

$$\begin{aligned} P_{out}^{1} &= \left(1-e^{-\frac{2\zeta_1^*}{\Omega_{SD}}}\right)\times\left(1-e^{-\zeta^*(\frac{1}{\Omega_{SR}}+\frac{1}{\Omega_{RD}})}\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}}K_1(\sqrt{\frac{4\zeta^*(1+\rho\zeta^*)}{\rho\Omega_{SR}\Omega_{RD}}})\right) \\ &= \left(1-e^{-\frac{2\zeta_1^*}{\Omega_{SD}}}\right)\times\zeta^*(\frac{1}{\Omega_{SR}}+\frac{1}{\Omega_{RD}}) \end{aligned} \quad . \quad \textbf{(27)}$$

$$P_{out}^{2} \approx \left(1-2e^{-\frac{\zeta_1^*}{\Omega_{SD}}}+e^{-\frac{2\zeta_1^*}{\Omega_{SD}}}\right)\times\zeta^*(\frac{1}{\Omega_{SR}}+\frac{1}{\Omega_{RD}}). \quad \textbf{(28)}$$

4 Simulation Results and Analysis

In this section, we use outage probability and sum rate of system to verify the performance of the cooperative NOMA system.

For convenience, it is assumed that the distribution of the BS, relays, and users are on a straight line. Without loss of generality, normalize the distance between S and D. The distance from the transmitter to the relay the first user could be expressed as $d_{SD_1} = d_{SR} + d_{RD_1} = 1$. The distance from the relay to the second user $d_{SD_2} = d_{SR} + d_{RD_2} = 3$, and the route loss index α is 4. Power allocation factor $\alpha_1 = \frac{1}{3}$ and $\alpha_1 = \frac{2}{3}$. The minimum SNR predefined by UE$_1$ and UE$_2$ are assumed to be $\gamma_{th1} = 1.5$, $\gamma_{th2} = 2$. In the NOMA system, the receiving end needs to apply SIC to distinguish each user.

Fig. 2 shows the system outage probability for a fixed power allocation situation of the BS and the relay in the cooperative NOMA system. It can be seen that the curve of the theoretical analysis matches the curve of the simulated results well. In addition, the outage probability decreases as the transmission power of the BS increases. This is consistent with prior analysis and is related to the outage probability, which is a decreasing function from the theoretical analysis. This shows that the outage probability of the cooperative NOMA system can be reduced by better self-interference cancellation technology.

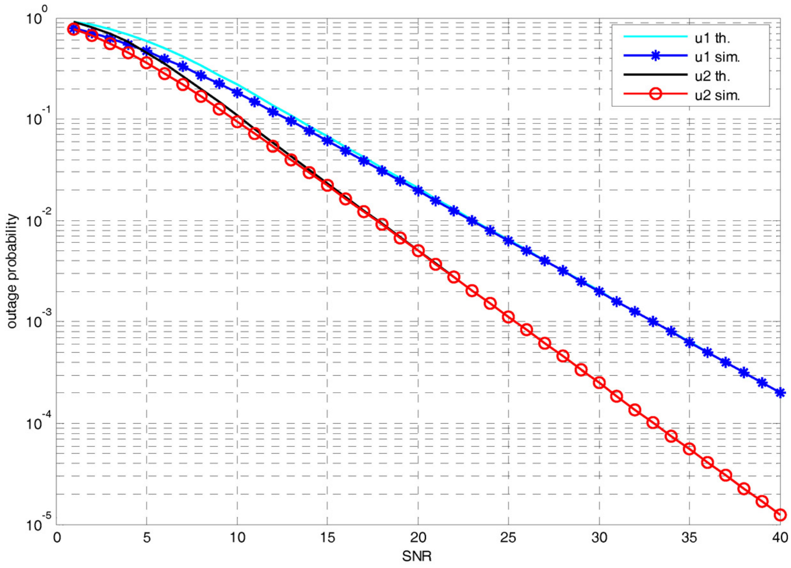

Fig. 2. The relationship between outage probability and transmission power in the cooperative NOMA system. (th. is an acronym for theoretical value. sim. is an acronym for simulated value)

Fig. 3 shows the change of the outage probability curve with relay position under different fading parameters. Fig. 3 shows that the optimal relay position for each user is different. The optimal relay location for users with better channel conditions is close to the base station. According to the principles of NOMA, the user with the best channel is allocated the least transmission power. In order to receive the largest SNR at the relay, the best user channel should have a relay that is closest to the base station.

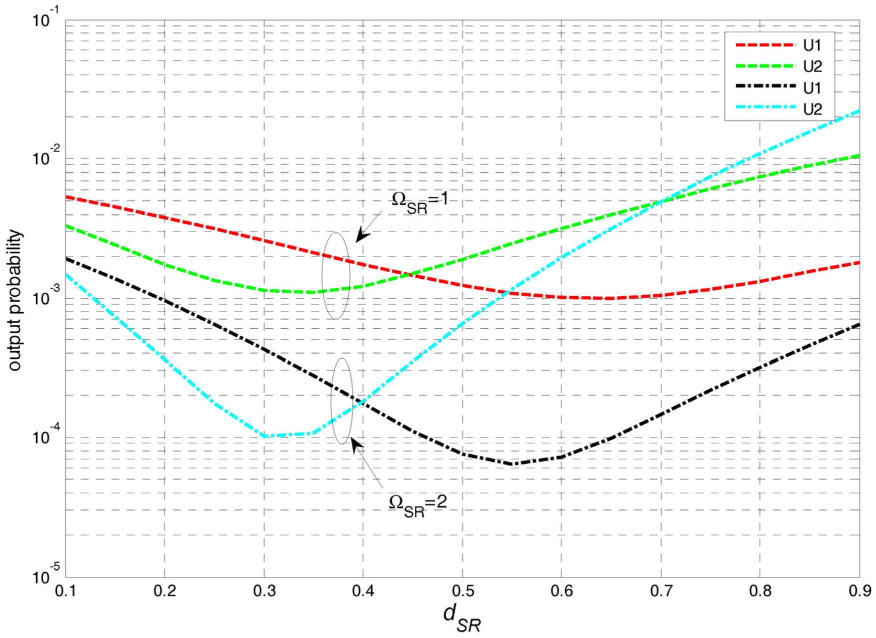

Fig. 3. The relationship between outage probability and d_{SR} in the cooperative NOMA system

Fig. 4 shows the change curve of user rate with SNR. It can be seenthat the approximate values of the ergodic rates of UE_1 and UE_2 obtained by numerical calculations agree well with the results from computer simulation. Fig. 4 illustrates that when the SNR is high enough, the rates of UE_1 and UE_2 remain unchanged, which is consistent with the result obtained from the respective equation. In addition, the simulation results show that the ergodic sum-rate of all users of the NOMA relay system is greater than the rate of a single user of the OMA relay system, which proves the effectiveness of the NOMA system.

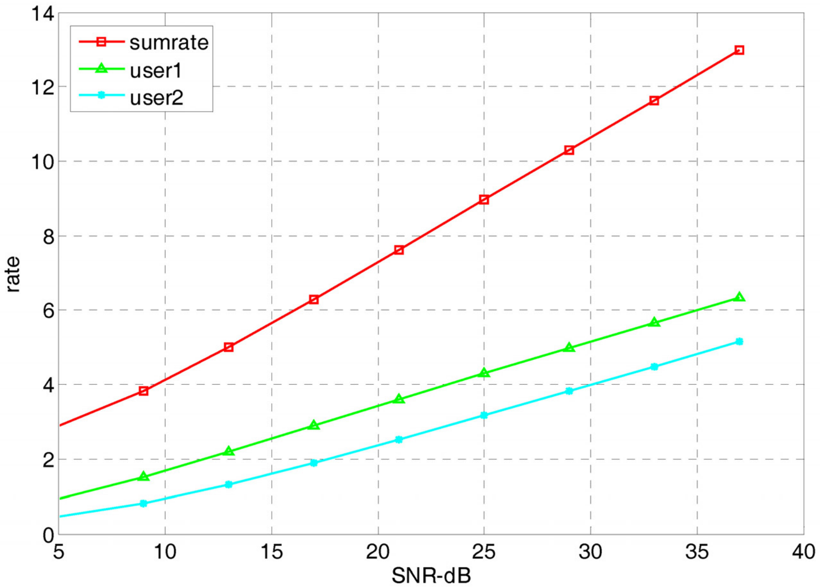

Fig. 4. The relationship between sum rate and SNR in thecooperative NOMA system

Fig. 5 shows the change curve of the user ergodic sumrate with relay position under different fading parameters. The figure illustrates that no matter where the relay is located, the ergodic sum-rates achieved by the NOMA system are better than the results from OMA, and the optimal relay position is around $d_{SR} = 0.5$. The rates of user 1 and user 2 do not change with the change in d_{SR}. In addition, when the RN is far from the midpoint position, the theoretical upper bound obtained agrees better with the simulation results.

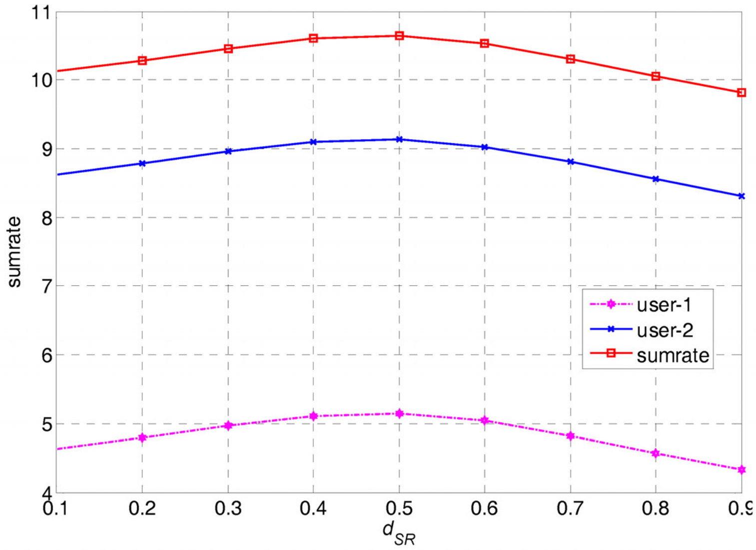

Fig. 5. The relationship between ergodic sum-rate and d_{SR} in the relay NOMA system

5 Conclusion and Discussion

In this study, we investigated a downlink NOMA system with a cooperative full-duplex relay, in which near users act as full-duplex relays for remote users. An accurate expression is obtained through complex mathematical derivation for the system outage probability with channels following an exponential distribution. As the expression equations obtained are complex, valuable information cannot be directly observed. To better understand the system outage behavior, the present paperprovides a lower limit for outage probability. The diversity order of the system was obtained through the approximation of this lower limit under high SNR conditions. The ergodic sum-rate of all users was also analyzed, providing a closed-form expression for the upper limits of the ergodicsumrate. All the results from theoretical analyses were verified through computer simulation. The properties of NOMA and OMA were also compared by computer simulations, which suggest that NOMA is superior to OMA and that NOMA provides higher spectral efficiency and user fairness.

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References

[1] X. Wang, Y. Chen, An effective interference mitigation scheme for downlink NOMA heterogeneous networks，in: Proc. of 2017 3rd IEEE International Conference on Computer and Communications, 2017.

[2] Z. Bai, J. Jia, C.X. Wang, D. Yuan, Performance Analysis of SNR-Based Incremental Hybrid Decode-Amplify-Forward Cooperative Relaying Protocol, IEEE Transactions on Communications 63(6)(2015) 2094-2106.

[3] J. Kim, I. Lee, Non-Orthogonal Multiple Access in Coordinated Direct and Relay Transmission, IEEE Communications Letters 19(11)(2015) 2037-2040.

[4] Z. Ding, M. Peng, H. V. Poor, Cooperative Non-Orthogonal Multiple Access in 5G Systems, IEEE Communications Letters 19(8)(2015) 1462-1465.

[5] J. Choi, Non-Orthogonal Multiple Access in Downlink Coordinated Two-Point Systems, IEEE Communications Letters 18(2)(2014) 313-316.

[6] X. Yue, Y. Liu, S. Kang, A. Nallanathan, Z. Ding, Spatially Random Relay Selection for Full/Half-Duplex Cooperative NOMA Networks, IEEE Transactions on Communications 66(8)(2018) 3294-3308.

[7] M. Aldababsa, O. Kucur, Outage and ergodic sumrate performance of cooperative MIMO- NOMA with imperfect CSI and SIC, International Journal of Communication Systems 33(11)(2020) e4405.

[8] Z. Mobini, M. Mohammadi, B.K. Chalise, H.A. Suraweera, Z. Ding, Design and Performance Analysis of Full-Duplex Cooperative NOMA Systems, IEEE Transactions on Wireless Communications 18(6)(2019) 3295-3311.

[9] Y. Zheng, Z. Ding, P. Fan, N. Al Dhahir, The Impact of Power Allocation on Cooperative Non-orthogonal Multiple Access Networks With SWIPT, IEEE Transactions on Wireless Communications 16(7)(2017) 4332-4343.

[10] M. Zeng, V. Fodor, Energy minimization for delay constrained mobile edge computing with orthogonal and non-orthogonal multiple access, Ad hoc networks 98 (2020) 1-13.

[11] Z. Ding, H. Dai, H. V. Poor, Relay Selection for Cooperative NOMA, IEEE Wireless Communication Letters 5(4)(2016) 416-419.

[12] J. Men, J. Ge, Non-Orthogonal Multiple Access for Multiple-Antenna Relaying Networks, IEEE Communications Letters 19(10)(2015) 1686-1689.

[13] J. Men, J. Ge, C. Zhang, Performance Analysis of Non-orthogonal Multiple Access for Relaying Networks Over Nakagami-m Fading Channels, IEEE Transactions on Vehicular Technology 66(2)(2017) 1200-1208.

[14] L. Zhang, J. Liu, M. Xiao, G. Wu, Y. Liang, S. Li, Performance Analysis and Optimization in Downlink NOMA Systems with Cooperative Full-duplex Relaying, IEEE Journal on Selected Areas in Communications 35(10)(2017) 2398-2412.

[15] S. Zhang, X. Xu, J. Peng, Y. Chen, S. Wang, Analysis of uplink NOMA in cellular IoT: a physical layer security perspective, in: Proc. of 2018 IEEE 88th Vehicular Technology Conference, 2018.

[16] C. Zhong, Z. Zhang, Non-orthogonal Multiple Access with Cooperative Full-duplex Relaying, IEEE Communications Letters 20(12)(2016)2478-2481.

[17] Z. Zhang, Z. Ma, M. Xiao, Z. Ding, P. Fan, Full-Duplex Device-to-Device-Aided Cooperative Nonorthogonal Multiple Access, IEEE Transactions on Vehicular Technology 66(5)(2017) 4467-4471.

[18] J. Kim, I. Lee, J. Lee. Capacity Scaling for D2D Aided Cooperative Relaying Systems Using NOMA, IEEE Wireless Communication Letters 7(1)(2017)42-45.