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Abstract. Image super-resolution reconstruction is a mothed to generate clear images from vague images. Due to the image reconstructed by depth learning method has better display effect. Therefore, this paper proposes an image super-resolution reconstruction method based on residual sub-pixel convolutional network. Firstly, aiming at the problem that adding residual network will reduce the network efficiency, the batch normalization layer is deleted in the residual structure to increase the model efficiency. Then, so as to fully utilize the feature details of the vague image, the multi-layer feature image information is extracted and filtered through the jump connection and global feature multiplexing module. The network can take advantage of image details with different depths. Finally, so as to increase the speed of network reconstruction, the sub-pixel convolution network enlarges and rearranges the obtained feature images to obtain clear images. It can be concluded from the experimental results that this method obtains better subjective visual effect, and the experimental data of objective evaluation standard peak signal to noise ratio (PSNR) and Structural similarity (SSIM) are also larger than the classical bicubic interpolation method, super-resolution convolutional neural network (SRCNN) algorithm and super-resolution using very deep convolutional network (VDSR) algorithm show advantages of this method.

Keywords: batch normalization, residual network, sub-pixel convolution, super- resolution reconstruction

1 Introduction

Due to the gradual improvement of people's requirements for image definition, there are endless methods to improve image definition. In the past, the optical devices in the image acquisition system were mainly improved to improve the resolution of images. Because of the high production price and insufficient production level, this method of improving hardware is no longer applicable. As a result, researchers have devoted a great deal of enthusiasm to image enhancement technology. As a method of image enhancement, image super-resolution reconstruction is usually used in medical imaging [1], satellite remote sensing [2], security monitoring [3] and other fields due to its features of low cost, convenience and good reconstruction effect. Image super-resolution reconstruction technology includes two categories. One is multi-image super-resolution reconstruction, that is, synthesizing a clear image from multiple vague images. Another is single image processing, due to the long training time of multi-image reconstruction and the huge database required, single image reconstruction is more welcomed by researchers. Therefore, the single-image super-resolution reconstruction has been more welcomed by researchers. Due to the progress of artificial intelligence and deep learning, the results of single-image super-resolution reconstruction has also been greatly improved.

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Nowadays, Single image reconstruction methods mainly include the following three categories: one is the interpolation-based method, that is, the adjacent pixels of a pixel are predicted by mathematical method, and the predicted unknown pixels are filled into the corresponding position. Common interpolation-based methods have the nearest neighbor interpolation method [4], bilinear interpolation method [5], bicubic interpolation method [6], etc. The interpolation-based method has a small amount of computation and fast speed; however, the reconstructed image is relatively vague, the details of the image are seriously lost at high-frequency, which cannot enhance the clarity of the image. The second method is based on reconstruction, which is also a method using mathematical formulas to recover high-frequency information by combining with prior knowledge, and then reconstructing high-resolution images. Common reconstruction-based methods include convex set projection method [7], maximum a posteriori probability estimation method [8], iterative back projection method [9], etc. Although the image reconstruction effect is somewhat enhanced compared with interpolation method, this method has higher requirements for low-resolution images, in addition the display effect of reconstructed image is not great when the magnification is large. The third method is based on learning method, through analyzing and learning a large amount of data, the direct correspondence between vague image and clear image is reached, extract key parameters, and continuously optimize them, and finally reconstruct clear image. Previous learning-based methods such as sparse representation [10], etc. Due to the progress of deep learning and convolutional neural networks, in 2014, Dong et al. [11] took the lead in combining convolutional neural network with super-resolution reconstruction, SRCNN algorithm was presented, and image reconstruction effect is obviously superior than the interpolation-based and reconstruction-based methods. Since then, the combination of convolutional neural network and image reconstruction has turned into the mainstream direction in this field.

SRCNN algorithm is the simplest application of convolutional neural network. It inputs the vague image into the network after bicubic interpolation, and only fits the nonlinear mapping between vague image and clear image through three-layer network, so as to output high-resolution image. Yet, if the magnification factor is larger, the image output by the system is not good. So as to improve the display effect of SRCNN, Dong et al. [12] proposed a fast super-resolution convolutional neural network (FSRCNN) model in 2016. This algorithm uses deconvolution layer to effectively accelerates the training process of the network. Shi et al. [13] proposed an efficient sub-pixel convolutional neural network (ESPCN) model. This algorithm directly inputs the vague image into the model without magnification, but rearranges the feature image at the last layer to obtain clear image. Thus, it can tremendously shorten the training time of the model. However, the network depth of the above method is shallow, resulting in a large loss of detailed features, and the reconstruction effect is not ideal. Nevertheless, only adding the number of convolution layers of the neural network will lead to difficulties in network training. He et al. [14] proposed a residual network in 2016, which just solves this problem. Kim et al. [15] proposed a very deep super-resolution (VDSR) model, the algorithm increase the number of convolution layer to 20 layers, using recursive convolution network iteration and residual connection on the input vague image, prevented the emergence of the gradient dispersion and degradation problems caused by the network layer of the increase in the number, and realized the dual purpose of accelerating convergence speed and improve the quality of image reconstruction. Although the algorithm is able to enhance the reconstruction results of the network, the utilization of the detailed feature information of vague images is still limited, and the reconstructed image still has some problems, such as blurring, virtual edge and so on. When the network depth is deepened, the time of model training is extended accordingly. In 2017, Lai et al [16] Proposed the deep Laplacian pyramid networks (lapSRN) structure, which includes multi-level, step-bystep up sampling and prediction residuals, so it has achieved good results. And because the size is enlarged step by step, all operations are not carried out on the large-size feature image, so the speed is faster. In 2018, Haris et al. [17] proposed the deep back-projection networks (DBPN) algorithm to guide the reconstruction process through an error feedback mechanism that iteratively calculates the up sampling and down sampling projection errors, so as to obtain better results.

To solve these problems, an image super-resolution reconstruction method based on residual sub-pixel convolutional network is proposed in this paper. The main contributions of this method are as follows:

(1) On the premise of SRCNN model, the residual network with batch normalization layer deleted is added to solve the problems of network gradient disappearance, and the training speed of the network is accelerated.

(2) A global feature multiplexing structure is designed to extract the feature information of different convolution layers for reconstruction, so as to enrich the details of the output image.

(3) At the end of the whole network, the sub-pixel convolution module is applied to rearrange the feature images, so as to realize the reconstruction of clear images.

2 System Model

2.1 Super-resolution Convolutional Neural Network

Super-resolution Convolutional Neural Network (SRCNN) only learns the mapping relationship between vague image and clear image. It only uses three convolution layers, so the weight parameters contained are relatively few and the conversion speed is considerable. The processing process of this method is mainly composed of three parts: image block extraction and representation, non-linear mapping and reconstruction.

Image block extraction and representation is to enlarge a vague image to the size of the target of clear image by bicubic interpolation method. At this time, the enlarged image is still called the vague image, that is, the image to be input into the network. Then the feature of the vague image is extracted to form a high-dimensional feature image. This procedure uses formula (1).

$$F_1(Y) = max(0, W_1^*(Y) + B_1).$$
(1)

Where, W_1 represents the convolution kernel, B_1 is the deviation. $W_1 = c * f_1 * f_1 * num$, c represents the number of channels, f_1 is the size, num represents the kernel number. The activation function is *ReLU*.

The non-linear mapping refers to the fitting of the non-linear mapping through its two layers convolution. The first layer convolution: the size of the convolution kernel is $9 \times 9(f_1 \times f_1)$, the number of the convolution kernel is $64(n_1)$, as well as, 64 feature images are output. The second layer convolution: the size of convolution kernel is $1 \times 1(f_2 \times f_2)$, the number of convolution kernel is $32(n_2)$, as well as, 32 feature images are output. Nonlinear mapping is realized by mapping n_1 dimension features to n_2 dimension features. This procedure uses formula (2).

$$F_2(Y) = max(0, W_2 * F_1(Y) + B_2).$$
(2)

Where, W_2 is the convolution kernel. B_2 is the deviation. $W_2 = n_1 * 1 * 1_1 * n_2$. It adopts 1*1 convolution.

Reconstruction refers to the final high-resolution image output after the last convolution operation. The third layer convolution: the size of the convolution kernel is $5 \times 5(f_3 \times f_3)$, and the number of the convolution kernel is $1(n_3)$, the result of convolution is the final reconstructed clear image. This procedure uses formula (3).

$$F(Y) = (W_3 * F_2(Y) + B_3.$$
(3)

Where, W_3 is the convolution kernel. B_3 is the deviation. $W_3 = n_2 * f_2 * f_2 * c$. Fig. 1 shows the SRCNN network structure.

The loss function is expressed by mean square error (MSE) and is expressed as formula (4).

$$L(\theta) = \frac{1}{n} \sum_{i=1}^{n} ||F(Y_i; \theta) - X_i||^2 ..$$
(4)

Where, $\theta = \{W_1, W_2, W_3, B_1, B_2, B_3\}$, $F(Y; \theta)$ is the reconstructed image, X_i is the benchmark image, and Y_i is its corresponding low-resolution image.



Fig. 1. SRCNN network structure

2.2 Residual Network

Because the convolutional neural network can extract different levels of features, the more layers of the network, the richer the features that can be extracted at different levels. Therefore, the number of network layers of SRCNN network model is only three, and the display effect of the reconstructed clear image is blurred, virtual, as well as other problems. By deepening the depth of the network, researchers can find that the high-resolution image with better reconstruction effect can be obtained. Network layer is not, however, the deeper, the better. Simply increasing the network layers will lead to gradient dispersion or gradient explosion. Although these problems can be solved by regularization initialization and intermediate regularization layer, When the number of convolution layers increases to a certain value, the accuracy of the network model will deteriorate, that is, when the network becomes very deep, the depth of the network becomes more difficult to training, that is network directly bypasses the input to the output through jump connection, which is equivalent to the implementation of a simple identity mapping. It not only ensures the complete information, but also changes the learning goal and reduces the difficulty of learning because the whole network only learns the different part of input and output. The general formula of residual network function is formula (5):

$$H(x) = F(x) + x.$$
 (5)

Where, H(x) represents target mapping, F(x) represents residual mapping, and x is input. So, when f (x) = 0, an identity mapping function H(x) = X is formed. When other conditions remain unchanged, fitting the residual function H(x) = F(x) - x is more effective than fitting the identity mapping function H(x) = X. Therefore, while it is true that the output will not be optimized H(x), the residual network can help improve performance as well as solve degradation problems. Fig. 2 shows the basic structure of the residual network.



Fig. 2. Residual network structure

3 Methodology

3.1 Network Structure

This paper proposes an image super-resolution reconstruction method based on residual sub-pixel convolutional network, and SRCNN network model is used for reference. In this paper, an improvement is mainly made in the part of non-linear mapping. The network model of this method has altogether 10 convolution layers, feature extraction is performed in the first convolution layer, the second convolution layer to the ninth convolution layer is used for the non-linear mapping, each two convolution layers constitute a residual block, which is a total of four residual blocks. The jump connection is added between each residual block to form the identity mapping. At the same time, the image information is extracted from each residual block to enter the global feature multiplexing modules, so that the network can learn more image detail characteristics. Finally, the sub-pixel convolution reconstruction is carried out in the tenth convolution layer to obtain clear image. Fig. 3 shows the text method network structure.



Fig. 3. Network structure

3.2 Residual Network and Global Feature Multiplexing Model

When the layers of neural network gradually deepen, the phenomenon of gradient dispersion gradually appears, which seriously affects the training process of neural network. Gradient diffusion is usually caused by sigmod function used in the neural network. This function can map the number from negative infinity to positive infinity to between 0 and 1, and the result of derivation of this function will be expressed as the multiplication of two numbers between 0 and 1. The back propagation of neural network is to multiply the partial derivatives of function layer by layer. When the layer number of neural networks are very deep, the deviation generated by the last layer becomes smaller and smaller, because it is multiplied by many numbers less than 1, and eventually becomes 0. Therefore, the weight of shallow layer is not updated, this is gradient diffusion. When training the neural network model, adding the batch normalization (BN) layer [18] can limit the distribution interval of input values and make them within the region with large activation function derivatives, so as to effectively alleviate the occurrence of gradient dispersion phenomenon and accelerate the speed of network convergence. In addition, the BN layer can also suppress the problem that minor parameter changes are magnified with the deepening of the network, and it has stronger adaptability to parameter changes, easier parameter adjustment, and more stable network. However, for the super-resolution problem, adding BN layer is counterproductive, mainly because the super-resolution reconstruction needs more detailed information, as well as the differences of data. The use of BN layer will not only destroy the information, also ignore the absolute differences of the data, in addition, adding the BN layer will consume more computing resources and prolong the training time of network. Therefore, in order to obtain feature images with more detailed information and increase the training speed of the model, the BN layer is removed from the residual structure of the network in this paper. Fig. 4 shows the structure diagram.



Fig. 4. Residual structure of deleting BN layer

In order to fully utilize the image feature details of different depth and obtain the clear reconstructed image with clearer and continuous texture and better display effect, this paper proposes a global feature multiplexing module, which extracts the image information obtained by feature extraction and the image information output by the first three residual modules, synthesizes a large number of feature image information, and using the global feature multiplexing module 1×1 Convolution checks the information for filtering, and then the filtering result and the image information output by the last residual block are sent to the sub-pixel convolution network for image reconstruction. The structure diagram of residual module and global feature multiplexing model is shown in Fig. 5.



Fig. 5. Residual module and global feature multiplexing model

3.3 Sub-pixel Convolution Reconstruction

Sub-pixel convolution reconstruction is an up-sampling method, whose main function is to obtain clear feature images from vague feature images through convolution and multi-channel reorganization, so as to effectively enlarge the reduced feature images. If the network input a sheet of $H \times W$ low-resolution image, after convolution, it gets the feature image of $r^2 \times H \times W$, and then rearrange the r^2 channels of each pixel on the feature image into an $r \times r$ area. It corresponds to a sub-block of $r \times r$ size in the clear image, so that the feature image of size $r^2 \times H \times W$ is update to a clear image of $1 \times rH \times rW$, r here is the multiple of up-sampling. This transformation, though called a sub-pixel convolution, does not actually have a convolution operation. Compared with deconvolution, the sub-pixel convolution layer. Therefore, it only carries out the convolution operation with a large amount of computation in the low-resolution image, as well as rearrange the pixels in the last layer. Therefore, sub-pixel convolution reconstruction can simplify the complexity of the model, reduce the training time of model, and has high efficiency. Fig. 6 shows the sub-pixel convolution reconstruction structure.



Fig. 6. Sub-pixel convolution reconstruction

4 Experimental Results and Analysis

The hardware environment of this experiment is: PC is configured with NVIDIA GeForce MX450, RAM 16G, Windows10 64-bit operating system, Matlab R2018a, and uses Caffe to build the convolutional neural network framework.

4.1 Dataset

The training dataset of this experiment consists of BSD200 dataset and DIV2K dataset. A total of 200 images are randomly selected to form a training data set. So as to make the most of this detailed information, data enhancement is performed on them (rotating 90°, 180° and 270° respectively, and scaling according to the coefficient of 0.9 and 0.8 on the basis of the original size). Finally, a dataset composed of 2400 images is obtained. The test set in this paper is composed of SET5, SET14 and BSD100 data set to verify the function of network reconstruction in the training process. Because the image after bicubic interpolation adds unnecessary data and reduces the efficiency of network training, the input image of the method in this paper does not need to be input after bicubic interpolation amplification.

4.2 Network Parameters

The input color image is reduced by bicubic interpolation, and the down sampling factor is 4 to generate a low-resolution image. Then, the reduced image is randomly intercepted 48×48 image blocks are used as the input of the model for image super-resolution reconstruction, and 16 such image blocks are processed in each batch. The network has 1000 iterative training cycles, and the learning rate at the beginning is 10^{-3} . The learning rate will be reduced by half every 200 iterative cycles of training. Optimization method adopts ADAM optimizer [19], and the parameters are set as $\beta_1 = 0.9$, $\beta_1 = 0.999$,

 $\varepsilon = 10^{-8}$. The up-sampling factors are 2, 3, and 4. The activation function adopts the PReLU activation function. MSE is used as the loss function. Since the size of the image will decrease after each convolution, 0 is used to fill the boundary for the input of all the convolution layers, so that the feature image remains unchanged after convolution.

4.3 Evaluation Criteria

The experiment in this paper uses the peak signal to noise ratio (PSNR) and structural similarity (SSIM) to evaluation the reconstruction results.

4.3.1 Peak Signal to Noise Ratio

The peak signal to noise ratio (PSNR) is a common standard to evaluate the results of reconstructed images, which can be defined by the mean square error (MSE). Formula (6) is the definition of mean square error between monochrome original image and super-resolution reconstructed image.

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [X(i, j) - Y(i, j)]^2.$$
 (6)

Where, m represents the length of the image, n represents the width of the image, X represents the reconstructed image, Y represents the high-resolution image. Formula (7) is the definition of the mean square error between the trichromatic original image and the image reconstructed by super-resolution.

$$MSE = \frac{1}{3mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [X(i, j) - Y(i, j)]^2.$$
(7)

The lower the value of MSE, the smaller the distortion of compressed image and the better the final image effect.

Therefore, formula (8) is the definition of PSNR.

$$PSNR = 10\log_{10}\left(\frac{MAX_{X}^{2}}{MSE}\right) = 20\log_{10}\left(\frac{MAX_{X}}{\sqrt{MSE}}\right).$$
(8)

Where, MAX_X is the maximum value 255 that represents the color of the image X points obtained from super-resolution reconstruction. The higher the value of PSNR, the more superior the image whose result is closer to the original image can be obtained.

4.3.2 Structural Similarity

Structural similarity is also an evaluation criterion in the field of image super-resolution. It can directly reflect the effect of human visual observation. Its definition is shown in formula (9).

$$SSIM(X,Y) = \frac{(2\mu_X\mu_Y + C_1)(2\sigma_{XY} + C_2)}{(\mu_X^2 + \mu_Y^2 + C_1)(\sigma_X^2 + \sigma_Y^2 + C_2)}.$$
(9)

Where, the meaning of μ_X and μ_Y are the mean value of pixels of image X and image Y; σ_X^2 and σ_Y^2 denote the variance of image X and image Y pixels, and σ_{XY} means the covariance of image X and image Y pixels; both C_1 and C_2 are constants in the range of [0, 1]. The larger the SSIM value, the closer the reconstructed output image is to the initial high-definition image.

4.4 Result Analysis

So as to test the reconstruction property of this algorithm, experiments are carried out with this method and several other advanced image reconstruction methods. The methods used for comparison include Bicubic algorithm, SRCNN algorithm and VDSR algorithm. The magnification is two, three, and four. Tests were performed on Set5, Set14, and BSD100 test datasets. Table 1 and Table 2 respectively display the values of the two evaluation indexes of the four algorithms after super-resolution reconstruction on the Set5 dataset.

image		Bicubic	SRCNN	VDSR	In this paper
	Baby	32.69	35.77	36.87	37.62
	Bird	33.58	36.80	37.88	38.26
×2	Butterfly	33.94	36.62	37.52	38.31
	Head	34.12	37.65	38.03	38.29
	Woman	33.12	35.95	37.19	37.64
	Baby	30.42	32.68	33.36	34.42
	Bird	31.47	34.15	34.84	35.24
×3	Butterfly	29.85	30.16	32.69	33.96
	Head	32.10	34.78	35.14	35.73
	Woman	30.94	31.26	32.28	32.85
×4	Baby	28.46	30.70	32.15	33.10
	Bird	27.66	29.65	31.36	32.87
	Butterfly	27.71	29.85	30.74	31.55
	Head	29.29	31.56	32.22	33.61
	Woman	28.85	30.49	31.08	31.98

Table 1. Comparison of PSNR results reconstructed by different algorithms on Set5

Table 2. Comparison of SSIM results reconstructed by different algorithms on Set5

image		Bicubic	SRCNN	VDSR	In this paper
	Baby	0.9422	0.9548	0.9594	0.9629
	Bird	0.9320	0.9497	0.9584	0.9615
×2	Butterfly	0.9382	0.9576	0.9622	0.9637
	Head	0.9350	0.9499	0.9596	0.9634
	Woman	0.9415	0.9584	0.9613	0.9685

image		Bicubic	SRCNN	VDSR	In this paper
×3	Baby	0.8807	0.9175	0.9213	0.9358
	Bird	0.9128	0.9326	0.9401	0.9487
	Butterfly	0.8356	0.8897	0.9124	0.9321
	Head	0.8449	0.8760	0.9026	0.9267
	Woman	0.9285	0.9332	0.9352	0.9418
×4	Baby	0.8229	0.8753	0.8903	0.8925
	Bird	0.8023	0.8650	0.8912	0.9014
	Butterfly	0.8104	0.8528	0.8797	0.8847
	Head	0.8235	0.8697	0.8886	0.8924
	Woman	0.8112	0.8571	0.8674	0.8762

Table 2. Comparison of SSIM results reconstructed by different algorithms on Set5 (contine)

From the objective evaluation criteria, it can be seen that, in contrast to the Bicubic algorithm, SRCNN algorithm and VDSR algorithm, the method of this paper has higher PSNR values and SSIM values on Set5, Set14 and BSD100 datasets, which can obtain clearer images, rich restored image details and better performance.

In terms of visual display effect, Fig. 7, Fig. 8 and Fig. 9 respectively show the image reconstruction effect of the four algorithms on Set5, Set14 and BSD100 datasets when the magnification is 2. It can be seen that bicubic algorithm has insufficient recovery of detail information, and the display effect of reconstructed image is very blurry. Although SRCNN algorithm and VSDR algorithm have recovered some high-frequency information, the edge sharpening is still poor. In contrast to additional methods, the high-resolution image reconstructed by the proposed method is richer in detail texture, clearer in edge, and more consistent with the perception of human eyes. The image reconstruction effect is obviously better than other algorithms.



(1) Original image

(2) Bicubic



(3) SRCNN



(4) VDSR



(5) In this paper

Fig. 7. Effect picture of parrot super-resolution reconstruction on Set5



(1) Original image

(2) Bicubic

(3) SRCNN



(5) In this paper

Fig. 8. Effect picture of baboon super-resolution reconstruction on SET14



Fig. 9. Effect picture of fish super-resolution reconstruction on BSD100

5 Conclusion

In this paper, a residual sub-pixel convolutional network super-resolution image reconstruction method is proposed based on SRCNN algorithm. This method aims at the problems of low-resolution images such as blurring and virtual edge, in order to obtain clear, high-quality and detailed images. First of all, in the process of feature extraction, the method directly input low-resolution images into the model. It avoids the amount of computation caused by bicubic interpolation. Secondly, in the process of non-linear mapping, deleted the BN layer of the residual network was used to retain more image information, also solved the problem of the network gradient dispersion, in the meantime, so as to fully utilize different depth image information, so as to get a smooth edge reconstruction image, using the global feature multiplexing module. Finally, in the process of reconstruction, the subpixel convolutional layer is introduced to carry out the up-sampling filter to realize the image amplification processing and complete the image reconstruction, which not only improves the reconstruction quality, but also shortens the network training time. It can be seen from the experimental results that compared with the Bicubic algorithm, SRCNN algorithm and VDSR algorithm, the proposed method has higher PSNR values and SSIM values, better reconstruction effect and clearer visual display effect. Although the method in this paper can effectively improve the image super-resolution reconstruction effect, the reconstruction quality needs to be further improved for super-resolution reconstruction of images that are too bright or too dark.

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