Journal of Computers Vol. 32 No. 6, 2021, pp. 218-226
doi:10.53106/199115992021123206019

Block Compressed Sensing Observation Matrix Optimization Algorithm Based on Block Target

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Received 1 July 2021; Revised 28 August 2021; Accepted 29 August 2021

Abstract. In order to eliminate the blocking effect of block compressed sensing algorithm, an optimization algorithm of block compressed sensing observation matrix based on block target is studied. The theory of block compressed sensing is to segment the original image with fixed size to obtain the sub blocks, arrange the texture of each sub block, use the compressed sensing observation matrix to sample each sub block, and optimize the observation matrix of block compressed sensing to eliminate the blocking effect. The block target method uses sparse orthogonal basis to make the processed block target conform to sparsity and orthogonality. The sparse coefficient vector is obtained by the basis inverse transformation, and the reflection coefficient of block target is obtained by using the sparse coefficient vector to optimize the observation matrix of block compressed sensing. The experimental results show that the peak signal-to-noise ratio (PSNR) is higher than 31dB when the algorithm is applied to image reconstruction, and the relative support set error of different sparsity and observation times is low, which can effectively eliminate the blocking effect of block compressed sensing algorithm.

Keywords: block target, block compressed sensing observation matrix, optimization algorithm, PSNR

1 Introduction

Compressed sensing theory is the theory of image processing by using the sparse characteristic of signal. Compressed sensing theory has been widely used in many fields such as space exploration, compressed imaging and medicine [1]. The compressed sensing theory does not need direct sampling. By selecting the coefficient transformation domain which is not related to the original signal, the collected transformation domain is projected into the waveform, and the waveform signal is observed through the observation matrix [2], so that the high latitude waveform signal is converted to the low dimensional signal, and the structural feature information of the original signal can be obtained by decompressing the compressed projection data obtained [3]. The application of compressed sensing theory in digital acquisition equipment can reduce the sampling rate, computational complexity and energy consumption [4].

In the process of image processing, when the image scale is too large, the observation matrix scale of compressed sensing theory is also improved [5], which improves the computational and storage complexity. Block compressed sensing theory is an important method for processing large-scale images. Block compressed sensing theory can process each image block in blocks [6] and reduce the storage and computational complexity of image processing.

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In recent years, there are many researches on block compressed sensing theory. Zhu Yongjun et al. proposed a new saliency-based adaptive block compressed sensing algorithm. The saliency proposed by the algorithm is based on the gray-scale spatial correlation matrix and Weber's law, and a deterministic orthogonal symmetric Toplitz matrix is used to measure the target image. The adaptive block strategy for minimizing the entropy, the block vector generation method for maximizing the second-order moment of the angle, and the adaptive sampling rate setting based on the synthesis feature are proposed, and different reconstruction algorithms are combined for analysis and verification [7]. Li Yubo constructs a measurement matrix suitable for convolutional compressed sensing based on the cyclotomic class. The measured value is obtained by using a deterministic sequence to cyclically convolve the signal, and then sampling twice at random. The correlation of the measurement matrix constructed in this paper is less than that of the measurement matrix constructed in the existing literature [8].

2 Optimization Algorithm of Block Compressed Sensing Observation Matrix for Block Targets

2.1 Basic Architecture of Compressed Sensing Observation Matrix

The discrete signal is represented by $r \in R^{l\times 1}$ and the length of discrete signal is represented by l. When the number of non-zero values in the discrete signal is K and $K \ll l$, the discrete signal is called K-sparse signal. The formula of K-sparse signal set is as follows:

$$\Lambda_K = \left\{ r : \|r\|_0 \le K \right\} \tag{1}$$

When the discrete signal is a non-sparse discrete signal, a sparse basis Ψ can be used to obtain the signal transformation formula as follows:

$$r = \Psi s \tag{2}$$

When the transformed discrete signal s satisfies $\|s\|_0 \le K \ll l$, the original discrete signal r can be expressed linearly by a small number of elements contained in the sparse matrix. At this time, the discrete signal r can also be called K-sparse signal.

The measurement process formula of compressed sensing theory is as follows:

$$y = \Phi r = \Phi \Psi s \triangleq As \tag{3}$$

In formula (3), r and Ψ represent the original signal and sparse basis respectively, and $\Psi \in R^{n\times l}$. s and Φ represent the sparse signal and observation matrix after transformation, and $\Phi \in R^{m\times n}$. And $A \triangleq \Phi\Psi$ represent the observation matrix.

There is $m \ll n$, so r cannot be solved by y. The signal s has sparse feature [9], and the estimated value $\hat{s}$ of the sparse signal can be obtained by reconstruction algorithm, and finally the estimated value $\hat{r}$ of the original signal can be obtained.

If the random difference sparse signals r_1 and r_2 belong to Λ_K, they must conform to $Ar_1 \ne Ar_2$, otherwise the sparse signals r_1 and r_2 cannot be distinguished by the observed values. The observation matrix should meet the following conditions:

When there is a random K-sparse signal, the signal can be recovered accurately when the sparsity K satisfies the following formula:

$$K < \frac{1}{2}\left[1 + \frac{1}{\eta(A)}\right] \tag{4}$$

In formula (4), $\eta(A)$ refers to the maximum value of the absolute value of the inner product between a_i and a_j in the observation matrix A. The formula is as follows:

$$\eta(A) = \max_{1 \le i \ne j \le l} \frac{\left|\left\langle a_i, a_j \right\rangle\right|}{\left\|a_i\right\|_2 \left\|a_j\right\|_2} \quad \textbf{(5)}$$

Formula (5) can reflect the maximum correlation between the sparse basis and the observation matrix. When only two columns in the observation matrix A have high correlation, the correlation of the other columns is small [10], and there is a case that the $\eta(A)$ value is large but the performance of observation matrix A is high.

The correlation of the observation matrix is adjusted by the t-average correlation η_t of the total correlation, and η_t is defined as the average value of the elements whose inner product absolute value is higher than the threshold value t of all column vectors in the observation matrix A [11]. The formula is as follows:

$$\eta_t(A) = \frac{\sum_{\forall (i,j) \in H_{av}} \left|g_{ij}\right|}{N_{av}} \quad \textbf{(6)}$$

In formula (6), g_{ij} and N_{av} represent the inner product of i column a_i and j column a_j respectively and the number of elements in H_{av}; g_{ij} represents the elements in the gamma matrix $G = A^T A$; N_{av} represents the number of off diagonal elements $\left|g_{ij}\right|$ is higher than the threshold t.

When the absolute value of the inner product of two random columns in the observation matrix A is higher than the average value of threshold t, H_{av} is defined as follows:

$$H_{av} = \left\{ g_{ij}(i,j) > t, i \ne j \right\} \quad \textbf{(7)}$$

By reducing the value of the off diagonal elements of the gamma matrix to reduce the correlation between the two random columns in the matrix A [12], and using the threshold function to shrink the off diagonal elements whose absolute value is higher than the threshold t in the gamma matrix, the formula is as follows:

$$\hat{g}_{ij} = \begin{cases} \xi g_{ij} & t > \left|g_{ij}\right| \\ \xi t \cdot sign\left(g_{ij}\right) & t > \left|g_{ij}\right| > \xi t \\ g_{ij} & \xi t > \left|g_{ij}\right| \end{cases} \quad \textbf{(8)}$$

In formula (8), t and ξ represent the threshold and shrinkage factor respectively.

When the column vectors in the observation matrix A are uncorrelated, the corresponding gamma matrix is converted to the unit matrix I_l by column normalization, and through the gamma matrix approaching the unit matrix [13], the correlation between the sparse basis and the observation matrix is close to 0. The formula is as follows:

$$\min_{\Phi \in R^{m \times n}} \left\| I_l - A^T A \right\|_F^2 \quad \textbf{(9)}$$

When the matrix belongs to the orthogonal matrix, the off diagonal element of the gamma matrix is 0, the compressed sensing inner $m < l$, $A \in R^{m \times l}$ is over complete, and the column normalization processing $A^T A$ cannot be converted into the identity matrix I_l, so the off diagonal element of the gamma matrix cannot be 0.

The minimum value formula of the off-diagonal element in $G = \left\{ g_{ij} \right\}$ is as follows:

$$\eta_E = \pm \sqrt{\frac{l - m}{m(l-1)}} \quad \textbf{(10)}$$

In this case, the equiangular compact frame matrix is as follows:

$$\tilde{g}_{ij} = \begin{cases} \pm\sqrt{\dfrac{l-m}{m(l-1)}} & i \neq j \\ 1 & i = j \end{cases} \tag{11}$$

When $l \leq m(m+1)/2$, the above formula holds.

2.2 Block Compressed Sensing

The theory of block compressed sensing is that the original image is segmented by the sub blocks with the size of $B \times B$, and the texture of each sub block is arranged and expressed by x_i. the sampling formula of each sub block using the compressed sensing observation matrix is as follows:

$$y_i = \Phi_a x_i \tag{12}$$

In formula (12), Φ_a represents the observation matrix, and its size is $m \times B^2$.

The observation matrix A is a block diagonal matrix relative to the whole image, the formula is as follows:

$$A = \begin{bmatrix} \Phi_a & 0 & \cdots & 0 \\ 0 & \Phi_a & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_a \end{bmatrix} \tag{13}$$

The block compressed sensing algorithm samples and reconstructs each sub block of the image separately, which reduces the complexity of image reconstruction. Moreover, using the same measurement matrix Φ_a to process each sub block, it does not need to store all the observation matrix A, only needs to store Φ_a in the receiver, which can effectively save the storage resources in the image reconstruction process. Using the observation matrix of the same dimension to sample each sub block, the texture difference between the image sub blocks is very great, so the sparsity difference is great, and the reconstruction results of different sub blocks are quite different. When the sub blocks with different reconstruction effects are merged into one image, the false boundary will be formed [14]. The shadow exists in the boundary of each sub block, which leads to the great difference in the shading degree of different sub blocks. This phenomenon is called block effect. In order to eliminate the blocking effect of block compressed sensing algorithm, it is necessary to optimize the observation matrix obtained by the block compressed sensing algorithm.

2.3 Optimization of Block Compressed Sensing Observation Matrix for Block Targets

The block target method is used to optimize the block compressed sensing observation matrix. The reflection coefficient vector is obtained by real-time stretching the block target in the image grid area according to the column real-time stretching. The obtained reflection coefficient is processed by orthogonal projection transformation using orthogonal coefficient basis [15]. The sparse orthogonal basis and the coefficient after projection are expressed by ρ and δ respectively, the sparse transformation formula can be obtained as follows:

$$\theta = \rho\delta \tag{14}$$

In formula (14), θ represents the reflection coefficient of the block target [16].

According to the above formula, it can be seen that the sparse coefficient has strong sparsity characteristic [17], and the compressed sensing imaging model of block target is as follows:

$$Y = \bar{\Theta}\delta \tag{15}$$

In formula (15), $\bar{\Theta}$ represents the corresponding compressed sensing observation matrix of block target, and $\bar{\Theta} = \phi\Psi\delta$. The signal reconstruction of block target belongs to l_2 / l_1 optimization problem [18], and the problem to be solved in signal reconstruction of block target is as follows:

$$\min\|\delta\|, s.t. \|Y - \bar{\Theta}\delta\|_2 \leq \varepsilon \tag{16}$$

In formula (16), ε is the threshold value. Formula (16) can be transformed into quadratic programming problem by convex optimization toolbox to obtain the sparse coefficient vector δ [19]. The reflection coefficient vector θ of block target can be obtained by the sparse coefficient vector obtained by inverse orthogonal basis transformation [20]. The block target 2D image can be obtained by arranging the reflection coefficient vector of block target column by column according to the size of grid area, and the optimization of block compressed sensing observation matrix based on block target is realized.

3 Algorithm Testing

In order to test the effectiveness of the block compressed sensing observation matrix optimization algorithm based on the block target studied in this paper, 8 natural gray images of lenna 、horse 、parrots 、house 、character 、boat 、ship 、cameraman are randomly selected from the network as the test object, and their size is 256 × 256. The data block size is set to 32 × 32.

Normalized mean square error, relative support set error, non- diagonal element energy and observation signal-to-noise ratio are selected as the performance evaluation criteria of the proposed algorithm in image reconstruction.

The normalized mean square error formula is as follows:

$$NMSE = 10\lg\left(\frac{\|x - \tilde{x}\|_2^2}{\|x\|_2^2}\right) \tag{17}$$

The relative support set error formula is as follows:

$$RSSE = 1 - \frac{|\Lambda - \tilde{\Lambda}|}{K} \tag{18}$$

In the above formula, $\tilde{\Lambda}$ represents the estimated support set, Λ represents the real support set, $\tilde{x}$ represents the estimated signal, and x represents the test signal.

The energy of off diagonal elements is used to further describe the performance of the proposed algorithm in image reconstruction. The formula is as follows:

$$E_{OE} = \left\|A^T A - diag\left(A^T A\right)\right\|_F^2 \tag{19}$$

In formula (19), $diag(\cdot)$ is the diagonal matrix established by the diagonal elements of the observation matrix.

Using $M_{\max}$ to represent the total observation energy of the observation matrix, the observed signal-to-noise ratio formula can be obtained as follows:

$$SNR = 10\lg\left(\|y\|_2^2 / \left(M_{\max}\sigma_w^2\right)\right) \tag{20}$$

In formula (20), y and σ_w^2 represent the observation vector and Gaussian observation noise variance respectively.

The peak signal-to-noise ratio (PSNR) of eight images reconstructed by the proposed algorithm is statistically analyzed, and compared with the wavelike matrix algorithm (Ref. [7]) and saliency algorithm (Ref. [8]), the statistical results are shown in Table 1.

Table 1. Comparison of PSNR

Image name	The proposed algorithm / dB	Wave matrix algorithm / dB	Saliency algorithm / dB
Lenna	31.8	24.5	28.4
Horse	37.5	26.4	32.4
Parrots	36.5	25.4	31.8
House	32.5	26.4	29.5
Character	32.4	27.8	28.2
Boat	36.5	28.6	30.5
Ship	37.5	30.4	29.5
Cameraman	36.5	31.8	28.4

Experimental results in Table 1 show that the proposed algorithm can effectively improve the peak signal-to-noise ratio of two-dimensional gray-scale natural images, and the improvement of peak signal-to-noise ratio shows that the visual effect of the obtained image is improved. Compared with the other two algorithms, the peak signal-to-noise ratio (PSNR) of the reconstructed image is improved by more than 3dB. For Lenna image with flat texture, the algorithm in this paper still has a high promotion effect. The main reason is that the algorithm uses the block target method to optimize the block compressed sensing and optimize the observation matrix, which effectively improves the image processing effect and eliminates the blocking effect. This method can directly show the line spacing and direction, so that the small image stripes can be effectively displayed, and has a strong subjective visual quality improvement effect. It has higher effect on the reconstruction of straight line and oblique line.

The running time and normalized mean square error of the reconstructed images of the three algorithms under different signal lengths are counted. The statistical results are shown in Table 2.

Table 2. Performance comparison of different algorithms

Signal length	The algorithm in this paper		Wave matrix algorithm		Saliency algorithm	
	Normalized mean square error	Running time / MS	Normalized mean square error	Running time / MS	Normalized mean square error	Running time / MS
50	0.0254	23	0.0584	135	0.0785	195
100	0.0126	34	0.0695	152	0.0854	205
150	0.0234	39	0.0581	167	0.0687	235
200	0.0142	42	0.0687	185	0.0585	259
250	0.0254	45	0.0596	195	0.0695	295
300	0.0135	49	0.0758	205	0.0745	345
350	0.0142	53	0.0695	216	0.0852	395
400	0.0185	62	0.0674	223	0.0741	405
450	0.0136	75	0.0625	265	0.0685	485
500	0.0185	94	0.0852	352	0.0745	596

As can be seen from the experimental results in Table 2, with the increase of signal length, the running time of different algorithms increases. Compared with the other two algorithms, the growth rate of the proposed algorithm is relatively slow; the normalized mean square error of the algorithm in this paper is the lowest when the signal length is different. The proposed algorithm has low normalized mean square error (NMSE) and the shortest running time, which verifies the effectiveness of the proposed algorithm for optimization of block compressed sensing observation matrix. This algorithm can effectively eliminate the blocking effect of the block compressed sensing algorithm, retain a high level of image processing, and improve the performance of image reconstruction.

The reconstruction performance of the three algorithms under different sparsity is counted, and the observation times are set to 24. The statistical results are shown in Fig. 1.

The experimental results in Fig. 1 show that the algorithm in this paper has low relative support set error under different sparsity, and the relative support set error of reconstructed image in this algorithm is lower than that of the other two algorithms under different sparsity degrees, which verifies the effectiveness of the proposed algorithm in optimizing the observation matrix by using block target method, which can effectively improve the image reconstruction accuracy.

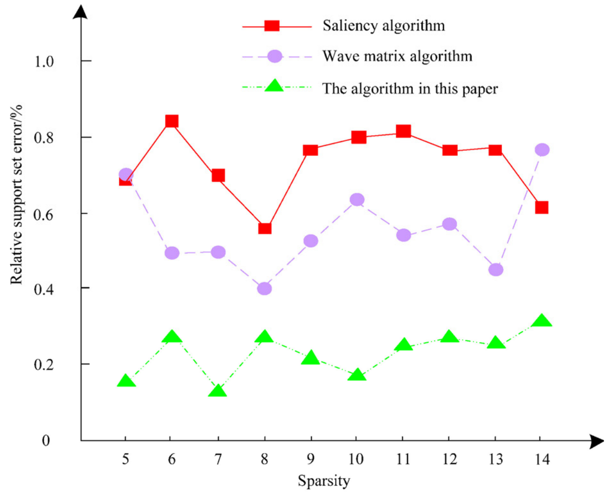

Fig. 1. Error comparison of different sparsity

The number of observations has a great influence on the image reconstruction effect of compressed sensing algorithm. When the sparsity is set to 10, the relative support set error of the three algorithms under different observation times is counted. The statistical results are shown in Fig. 2.

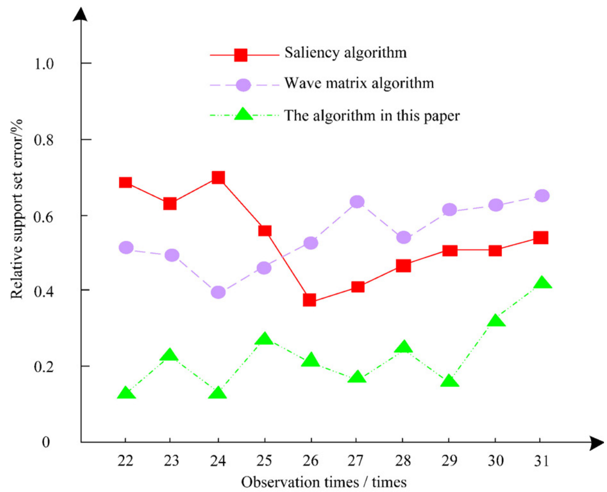

Fig. 2. Comparison of reconstruction errors under different observation times

As can be seen from the experimental results in Fig. 2, the relative support set error of the algorithm in this paper is less than 0.4% under different observation times; when the other two algorithms are applied to image reconstruction, the relative support set error is significantly higher than that of the proposed algorithm. The comparison results in Fig. 2 show that the algorithm can effectively improve the performance of the algorithm in practical application due to the optimization of the observation matrix.

The energy of off diagonal elements is counted for different observation times, and the statistical results are shown in Fig. 3.

As can be seen from the experimental results in Fig. 3, the energy of the off diagonal elements obtained by optimizing the block compressed sensing observation matrix by the proposed algorithm remains unchanged at different observation times; the energy of the off-diagonal elements of the other two algorithms at different observation times increases with the increase of the observation times, which effectively verifies the high operation performance of the algorithm in this paper. In this paper, the optimization algorithm of block compressed sensing observation matrix has high efficiency, the other two algorithms do not optimize the block compressed sensing observation matrix, so the energy of off diagonal elements changes greatly.

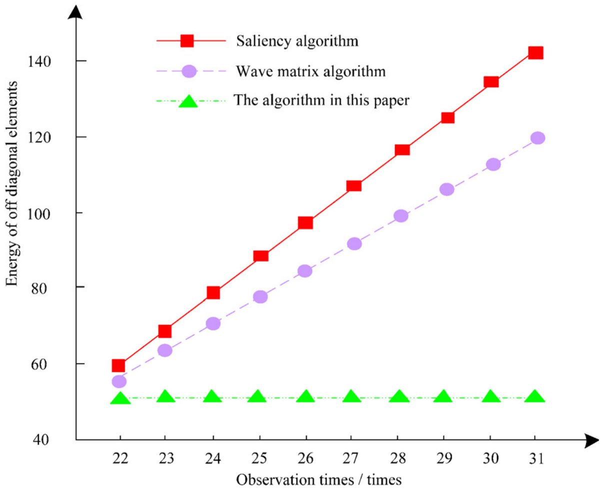

Fig. 3. Energy of off diagonal elements

4 Conclusions

The block target method is used to optimize the observation matrix of block compressed sensing. The algorithm is applied to the practical application of image reconstruction. The experimental results show that the algorithm has high effectiveness in optimizing the observation matrix. The algorithm can effectively remove the block effect of block compressed sensing algorithm, and the reconstructed image line effect is good, which can be applied to the practical application of image processing.

5 Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgements

This work has been supported by the Ningbo National Natural Science Foundation (2019A610124) and General Project of Education Department of Zhejiang Province (Y201737089).

References

[1] F.-H. Tong, L.-X. Li, H.-P. Peng, Y.-X. Yang, Flexible construction of compressed sensing matrices with low storage space and low coherence, Signal Processing 182(2021) 107951.1-107951.13.

[2] J. Zhu, Q. Zhang, X.-M. Meng, Zh.-W. Xu, Vector approximate message passing algorithm for compressed sensing with structured matrix perturbation, Signal processing 166(2020) 107248.1-107248.5.

[3] X.-Q. Yin, J.-Sh. Qian, X.-G. Guo, Ch.-F. Fu, G.-H. Lin, V.-E. Balas, J.-L. Hong, J. Gu, T.-C. Lin, Compressed sensing of remote image based on photograph LDPC codes matrix, Journal of Intelligent and Fuzzy Systems 37(2)(2019) 4605-4613.

[4] R. Li, Y.-G. Sun, Q.-J. Zhang, H.-B. Liu, Adaptive granular reconstruction algorithm in image block compressed sensing, Data acquisition and processing 33(1)(2018) 151-160.

[5] Y.-Q. Li, H. Cui, X.-Y. Gan, R. An, F. Hong, L.-F. Xia, WSN information collection and recovery based on compressed sensing, Computer Engineering 44(7)(2018) 91-97.

[6] C.-P. Shi, H.-H. Liu, Compressed sensing method based on block sparsity and adaptive iteration, Telecommunication technology 60(2)(2020) 216-221.

[7] Y.-J. Zhu, W.-B. Liu, S. Qian, W. Yin, B. Han, JPEG Lifting Algorithm Based on Adaptive Block Compressed Sensing, Mathematical Problems in Engineering (2020) 2873830.1-2873830.17.

[8] Y.-B. Li, J.-J. Zhang, Ch.-H. Han, X.-P. Peng, Construction of Convolutional Compressed Sensing Measurement Matrix Based on Cyclones, Journal of Electronics & Information Technology 43(2)(2021) 419-425.

[9] X.-H. Li, J.-G. Lv, Sh. Jiang, Block compressed sensing image fusion based on particle swarm optimization. Remote sensing information 34(1)(2019) 129-134.

[10] I. Taghavi, M.-F. Sabahi, F. Parvaresh, High Resolution Compressed Sensing Radar Using Difference Set Codes, IEEE Transactions on Signal Processing 67(1)(2018) 136-148.

[11] R. Badveli, V. Jagadish, R. Srikanth, F. Petruccione, Compressed Sensing Tomography for qudits in Hilbert spaces of non-power-of-two dimensions, Physical Review A 101(6)(2020) 062328.1-062328.5.

[12] S. Sharma, A. Gupta, V. Bhatia, Compressed Sensing Based UWB Receiver Using Signal-Matched Sparse Measurement Matrix, IEEE Transactions on Vehicular Technology 68(1)(2019) 993-998.

[13] B. Bernstein, S. Liu, C. Papadaniil, C.-F. Granda, Sparse Recovery Beyond Compressed Sensing: Separable Nonlinear Inverse Problems, IEEE Transactions on Information Theory 66(9)(2020) 5904-5926.

[14] A. Bangun, A. Behboodi, R. Mathar, Sensing Matrix Design and Sparse Recovery on the Sphere and the Rotation Group, IEEE Transactions on Signal Processing 68(2020) 1439-1454.

[15] M. Ndaoud, A.-B. Tsybakov, Optimal Variable Selection and Adaptive Noisy Compressed Sensing, IEEE Transactions on Information Theory 66(99)(2020) 2517-2532.

[16] S. Bharati, P. Podder, Adaptive PAPR Reduction Scheme for OFDM Using SLM with the Fusion of Proposed Clipping and Filtering Technique in Order to Diminish PAPR and Signal Distortion, Wireless Personal Communications 113(2020) 2271–2288.

[17] S. Bharati, M.-A. Rahman, P. Podder, Implementation of ASK, FSK and PSK with BER vs. SNR comparison over AWGN channel, Electrical Engineering and Systems Science 10(2)(2020) 03601.1-03601.5.

[18] G.-L. Duan, W.-X. Hu, J. Wang, Research on the natural image super-resolution reconstruction algorithm based on compressive perception theory and deep learning model, Neurocomputing 208(5)(2016) 117-126.

[19] S. Bharati, P. Podder, M.-R.-H. Mondal. Hybrid deep learning for detecting lung diseases from X-ray images, Informatics in Medicine Unlocked 20(2020) 100391.1-100391.14.

[20] S. Bharati, T.-Z. Khan, P. Podder, N.-Q. Hung, A Comparative Analysis of Image Denoising Problem: Noise Models, Denoising Filters and Applications, Cognitive Internet of Medical Things for Smart Healthcare 311(2020) 49-66.