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An Evaluation Method of Teaching Quality Using TOPSIS for Intuitionistic Fuzzy Sets

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Abstract. The current teaching quality evaluation mainly adopts single quantitative or qualitative index type, resulting in the evaluation results are not objective and accurate. In order to improve this deficiency, this paper proposes a new TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method based on intuitionistic fuzzy sets (IFSs), which can solve the hybrid multi-attribute decision-making (MADM) problem with certain and fuzzy indexes. Firstly, we use the intuitionistic fuzzy values (IFVs) to represent the results of qualitative index, and respectively calculate the similarity between the quantitative, qualitative indexes and the ideal, inverse ideal solution using different equations. Through weighted integration, the comprehensive similarity between them can be obtained. Then the proximity between each evaluation object and the ideal object is calculated. Finally, the proposed MADM algorithm is applied to an example of teaching quality evaluation. Through the comparison and analysis of the evaluation results, it is proved that the algorithm proposed in this paper is effective in solving the problem of teaching quality evaluation.

Keywords: evaluation of teaching quality, intuitionistic fuzzy set, hybrid indexes, TOPSIS

1 Introduction

The evaluation of teaching quality is an effective way to improve the quality of personnel training and teachers' teaching ability in colleges and universities [1]. In teaching activities, teachers, students and school administrators play their respective roles. Therefore, the teaching quality evaluation needs their joint participation. Teachers are the organizers of teaching activities and the evaluation objects of teaching quality. Students are the main participants in teaching activities, the main stakeholders and the core members of teaching quality evaluation. The management department is responsible for the daily monitoring of teaching quality, and organizes students and peer reviewers to regularly evaluate the teaching quality of teachers. Satisfactory teaching results need the joint efforts of teachers and students [2]. So it is not objective and comprehensive to measure the teaching quality only by students' learning outcome. Teaching quality evaluation should consider not only the indexes related to learning outcomes, but also the indexes related to teaching process. From the perspective of certainty, there should be both quantitative and qualitative indexes. Thus teaching quality evaluation is a typical hybrid fuzzy MADM problem.

Teaching quality evaluation mainly includes three aspects: index system construction, quantitative expression of index and evaluation method design. According to the different degree of certainty, the evaluation indexes can be divided into three types: quantitative (certain) indexes, qualitative (fuzzy) indexes and hybrid indexes composed of the two mentioned above. The advantage of quantitative index is that it can accurately and uniquely describe the object to be evaluated and is not affected by the subjective factors of the evaluator. It is suitable to describe the indexes related to teaching effect and easy to quantify, such as the lateness rate of teachers, the head-up rate of students. However, some indexes

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which are closely related to teaching effect are difficult to quantify directly. They are not suitable to be described by quantitative indexes, such as the novelty of teaching content, the suitability of teaching methods. These indexes require evaluators to make judgments based on experience, and the results vary with different evaluators. Therefore, this kind of index is called qualitative index, and it has uncertainty and fuzziness. The construction of the evaluation system of the existing methods mostly adopts one of the above two indexes, such as quantitative indexes adopted in ref. [3] and [4] and qualitative indexes adopted in ref. [5] and [6].

To realize quantitative representation of qualitative indexes, these methods usually be used, such as scoring, fuzzy value, IFV. The scoring method requires the evaluator to give a specific evaluation value according to a qualitative index, so it is difficult to make an accurate judgment in the actual evaluation process [7]. The fuzzy value representation method considers the indeterminacy of qualitative index and can give specific evaluation value according to membership function. On the basis of membership attributes of fuzzy sets, IFSs add non-membership and uncertainty attributes [8-11], which can better describe the uncertainty of objects.

For evaluation method design, the current research results mainly focus on the application of neural network, TOPSIS, analytic hierarchy process, fuzzy decision-making [12-14], etc. The evaluation method using neural network can establish the prediction model according to the existing evaluation data, and satisfactory evaluation results can be obtained [15]. However, it is highly dependent on training data. TOPSIS can also get accurate evaluation results without training data [16-20], but the existing methods are only suitable for a single type of evaluation index. Analytic hierarchy process has been widely used in teaching quality evaluation, and generally can achieve satisfactory results. However, due to the lack of effective means to deal with qualitative evaluation indexes, the accuracy of evaluation is restricted. Fuzzy decision-making method introduces the tool of fuzzy mathematics such as fuzzy set and IFS to deal with the expression of qualitative index and MADM, which can achieve better evaluation results [21].

This paper proposes a new TOPSIS method to improve the accuracy and objectivity of teaching quality evaluation. The main contributions are as follows:

(1) We construct a set of new evaluation index system of classroom teaching quality. In order to make full use of the advantages of the two indexes and avoid the disadvantages of a single index type, a hybrid index system composed of quantitative and qualitative indexes is proposed. The data of quantitative indexes can generally be obtained through monitoring equipment. The data of qualitative indexes need to be described by evaluators with linguistic variables.

(2) IFVs are used to represent the evaluation results of qualitative index, and cosine function is introduced to measure the similarity of IFS. The results of an evaluation example show that this method can improve the objectivity and accuracy of qualitative evaluation.

(3) A TOPSIS method for hybrid multi-attribute decision making is proposed. The method uses two different similarity measuring methods to obtain the similarity of the quantitative, qualitative indexes and the ideal, inverse ideal solution respectively. Finally, by weighted integration, the comprehensive similarity is obtained, based on which the proximity between each evaluation object and the optimal solution is obtained.

The content of this paper includes five sections. The first section briefly introduces the purpose, significance and research status of teaching quality evaluation. The second section introduces the basic concepts, operations and similarity measure of intuitionistic fuzzy set. In the third section, the TOPSIS method with hybrid evaluation indexes is presented. The fourth section describes the teaching quality evaluation method based on the TOPSIS method proposed in this paper, and verifies the feasibility and rationality of the method through an example. The fifth section summarizes the conclusions of this research work.

2 Preliminaries

In this section, we briefly introduce some basic concepts and operators of IFSs. The equation of similarity measure between two vectors and two IFSs is also presented respectively, which will be applied in the subsequent algorithm.

2.1 Some Concepts of IFSs

Atanassov [22] firstly proposed an IFS concept. It is an expansion of traditional fuzzy sets. The definition of IFS is as follows.

Definition 1. [22]. Suppose that L be a universal set. A IFS Z in L is characterized by two functions $fu_Z(l)$ and $fv_Z(l)$. They respectively represent the membership and non-membership degree of the element l in L to the set Z. Such that Z can be denoted by $Z=\{\langle l, fu_Z(l), fv_Z(l)\rangle | l \in L\}$. And $fu_Z(l)$ and $fv_Z(l)$ should meet the following conditions:

$$fu_Z(l): L \to [0,1],\ fv_Z(l): L \to [0,1], \text{ and } 0 \le fu_Z(l) + fv_Z(l) \le 1.$$

We define $fp_Z(l) = 1 - fu_Z(l) - fv_Z(l)$. $fp_Z(l)$ is called intuitionistic index or a hesitancy degree of the element l to the set Z. Obviously it satisfies $0 \le fp_Z(l) \le 1$ for $l \in L$.

Definition 2. [8]. If $fu_Z(l)$ and $fv_Z(l)$ are the membership degree and the non-membership degree of l in L to the set Z, $\langle fu_Z(l), fv_Z(l)\rangle$ is called IFV. It is denoted by $\alpha_Z = (fu_Z, fv_Z)$ for short. The set of all IFVs in L is called IFS Z, denoted by $Z = IFS(L)$.

Definition 3. [9]. Let $\alpha_1 = (fu_1, fv_1)$ and $\alpha_2 = (fu_2, fv_2)$ be two IFVs in L. For $\forall l \in L$, $\forall \lambda \in R$ and $\lambda > 0$, the following operational relations are defined:

(1) $\alpha_1 \wedge \alpha_2 = (\min\{fu_1, fu_2\}, \max\{fv_1, fv_2\})$

(2) $\alpha_1 \vee \alpha_2 = (\max\{fu_1, fu_2\}, \min\{fv_1, fv_2\})$

(3) $\alpha_1 \oplus \alpha_2 = (fu_1 + fu_2 - fu_1 \cdot fu_2, fv_1 \cdot fv_2)$

(4) $\alpha_1 \otimes \alpha_2 = (fu_1 \cdot fu_2, fv_1 + fv_2 - fv_1 \cdot fv_2)$

(5) $\lambda\alpha_1 = \left(1-(1-fu_1)^{\lambda}, fv_1^{\lambda}\right)$

Definition 4. [10]. Let $Z=\{\langle l, fu_Z(l), fv_Z(l)\rangle | l \in L\}$, $Q=\{\langle l, fu_Q(l), fv_Q(l)\rangle | l \in L\}$ be two IFSs. Then the following relationships are defined:

(1) $Z \subseteq Q$ if and only if $l \in L$, $fu_Z(l) \le fu_Q(l)$ and $fv_Z(l) \ge fv_Q(l)$ for $\forall l \in L$;

(2) $Z = Q$ if and only if $l \in L$, $fu_Z(l) = fu_Q(l)$ and $fv_Z(l) = fv_Q(l)$ for $\forall l \in L$.

Definition 5. [23]. If $\beta_i = (fu_i, fv_i)$ (i = 1, 2, …, n) is a group of IFVs in L, we define the intuitionistic fuzzy weighted aggregation operator ($IFWA$) by the following equation:

$$\begin{aligned} IFWA(\beta_1, \beta_2, \cdots, \beta_n) &= w_1\beta_1 \oplus w_2\beta_2 \oplus \cdots \oplus w_n\beta_n \\ &= \left(1-\prod_{i=1}^{n}(1-fu_i)^{w_i}, \prod_{i=1}^{n}(fv_i)^{w_i}\right) \end{aligned} \tag{1}$$

where $w = (w_1, w_2, \ldots, w_n)$ is the weight vector of β_i, $w_i \in [0,1]$, $\sum_{i=1}^{n} w_i = 1$.

2.2 Similarity Measures

The so-called similarity measure refers to the method of evaluating the similarity between two n-dimensional vectors. For certain indexes, we can use the method given in definition 6 to measure similarity. For fuzzy indexes, we can use the method given in definition 7 to measure similarity.

Definition 6. [24]. Assume that there are two n-dimensional vectors $\delta = (\delta_1, \delta_2, \cdots, \delta_n)$, $\varepsilon = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n)$, then the similarity between the two vectors can be calculated by the following equation:

$$S(\delta,\varepsilon)=\frac{\sum_{j=1}^{n} w_j \cdot \delta_j \cdot \varepsilon_j}{\sqrt{\sum_{j=1}^{n}\delta_j^2} \times \sqrt{\sum_{j=1}^{n}\varepsilon_j^2}}, \quad \textbf{(2)}$$

w_j is weight of each index, $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 7. [12]. Suppose that $Z=\{\langle l_j, fu_Z(l_j), fv_Z(l_j)\rangle | l_j \in L\}$, $Q=\{\langle l_j, fu_q(l_j), fv_Q(l_j)\rangle | l_j \in L\}$ are two IFSs in $L = \{l_1, l_2, \ldots, l_n\}$. The measure of similarity $S(Z, Q)$ between Z and Q should satisfy the following conditions:

(1) $0 \le S(Z,Q) \le 1$;

(2) $S(Z,Q)=1$ if and only if $Z = Q$;

(3) $S(Z,Q)=S(Z,Q)$;

(4) if $Z \subseteq Q \subseteq U$, $S(Z,U) \le S(Z,Q)$ and $S(Z,U) \le S(Q,U)$.

Applying the cosine function, we define the similarity $S(Z, Q)$ between Z and Q. The calculation equation is as follows [12]:

$$S(Z,Q)=\frac{1}{n}\sum_{j=1}^{n} w_j \frac{\left(fu_Z(l_j)\cdot fu_B(l_j)+fv_Z(l_j)\cdot fv_Q(l_j)+fp_Z(l_j)\cdot fp_Q(l_j)\right)}{\sqrt{fu_Z(l_j)^2+fv_Z(l_j)^2+fp_Z(l_j)^2}\sqrt{fu_Q(l_j)^2+fv_Q(l_j)^2+fp_Q(l_j)^2}}, \quad \textbf{(3)}$$

where w_j is weight of each index, $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. Obviously, it satisfies the condition (1)-(4).

3 TOPSIS with Hybrid Indexes

TOPSIS is a sort algorithm that approximates the ideal solution. By calculating the distance between the schemes to be evaluated and the best, the worst targets, ones can judge the proximity between them. The closer to the optimal goal and away from the worst target at the same time, the better the scheme is [17].

Assume that there are m evaluation objects, which are represented by A_i (i = 1, 2, …, m). Each evaluation object has n certain indexes represented by Cd_j (j = 1, 2, …, n) and p fuzzy indexes represented by Cf_k (k = 1, 2, …, p).

3.1 Calculate Similarity for Certain Indexes

The certain indexes Cd_j are forward processed and normalized to obtain the high excellent indexes set Cd_j^*. Then the set of the certain indexes of the ideal object is $dI^+=(Cd_1^+, Cd_2^+, \cdots, Cd_n^+)$, $Cd_j^+=\max(Cd_j^*)$. The set of the certain indexes of the inverse ideal object is $dI^-=(Cd_1^-, Cd_2^-, \cdots, Cd_n^-)$, $Cd_j^-=\min(Cd_j^*)$.

The equations of the similarity $S_{d,i}^+$, $S_{d,i}^-$ between each evaluation object and the ideal, the inverse ideal object can be derived according to equation (2). They can be denoted by equations (4) and (5):

$$S_{d,i}^+(Cd_i, Cd^+)=\frac{\sum_{j=1}^{n} w_j \cdot Cd_{i,j} \cdot Cd_j^+}{\sqrt{\sum_{j=1}^{n} w_j Cd_{i,j}^2} \times \sqrt{\sum_{j=1}^{n} w_j Cd_j^{+2}}}, \quad \textbf{(4)}$$

$$S_{d,i}^{-}\left(Cd_i, Cd^{-}\right)=\frac{\sum_{j=1}^{n} w_j \cdot Cd_{i,j} \cdot Cd_j^{-}}{\sqrt{\sum_{j=1}^{n} w_j Cd_{i,j}^2} \times \sqrt{\sum_{j=1}^{n} w_j Cd_j^{-2}}}, \tag{5}$$

where $Cd_{i,j}$ represents the *j*-th evaluation index of the *i*-th evaluation object.

3.2 Calculate Similarity for Fuzzy Indexes

The values of fuzzy index are characterized by IFVs, where $Cf_{i,k}$ represents the *k*-th evaluation index of the *i*-th evaluation object. The set of the certain indexes of the ideal object is $fI^{+}=\left\langle fu_k^{+}, fv_k^{+}, fp_k^{+}\right\rangle$ $=\left\langle \max_i fu_{i,k}, \min_i fv_{i,k}, fp_k^{+}\right\rangle$. The set of the certain indexes of the inverse ideal object is $fI^{-}=\left\langle fu_k^{-}, fv_k^{-}, fp_k^{-}\right\rangle$ $=\left\langle \min_i fu_{i,k}, \max_i fv_{i,k}, fp_k^{-}\right\rangle$, where $fp_k^{+}=1-fu_k^{+}-fv_k^{+}$, $fp_k^{-}=1-fu_k^{-}-fv_k^{-}$.

The equations of the similarity $S_{f,i}^{+}$, $S_{f,i}^{-}$ between each evaluation object and the ideal, the inverse ideal object can be derived according to the equation (3). They can be denoted by the equations (6) and (7):

$$S_{f,i}^{+}\left(Cf_i, Cf^{+}\right)=\frac{1}{p}\sum_{k=1}^{p} w_k \frac{\left(fu_{i,k} \cdot fu_k^{+}+fv_{i,k} \cdot fv_k^{+}+fp_{i,k} \cdot fp_k^{+}\right)}{\sqrt{fu_{i,k}{}^2+fv_{i,k}{}^2+fp_{i,k}{}^2}\sqrt{fu_k^{+2}+fv_k^{+2}+fp_k^{+2}}}, \tag{6}$$

$$S_{f,i}^{-}\left(Cf_i, Cf^{-}\right)=\frac{1}{p}\sum_{k=1}^{p} w_k \frac{\left(fu_{i,k} \cdot fu_k^{-}+fv_{i,k} \cdot fv_k^{-}+fp_{i,k} \cdot fp_k^{-}\right)}{\sqrt{fu_{i,k}{}^2+fv_{i,k}{}^2+fp_{i,k}{}^2}\sqrt{fu_k^{-2}+fv_k^{-2}+fp_k^{-2}}}. \tag{7}$$

3.3 Calculate the Proximity

The weights of certain and fuzzy indexes are e_d and e_f respectively. They satisfy the conditions of $0 \le e_d \le 1$, $0 \le e_f \le 1$ and $e_d + e_f = 1$.

The comprehensive similarity WS_i^{+}, WS_i^{-} can be calculated by applying the following weighted aggregation equations:

$$WS_i^{+}=e_d \cdot S_{d,i}^{+}+e_f \cdot S_{f,i}^{+}, \tag{8}$$

$$WS_i^{-}=e_d \cdot S_{d,i}^{-}+e_f \cdot S_{f,i}^{-}. \tag{9}$$

The following equation is used to calculate the proximity R_i between each evaluation object and the ideal one:

$$R_i=\frac{WS_i^{+}}{WS_i^{+}+WS_i^{-}}. \tag{10}$$

4 Evaluation of Teaching Quality

Teaching quality evaluation is not only an important means for teaching department of school to survey teachers' teaching quality, but also an important way to help teachers improve their teaching ability. From the perspective of evaluation methods, teaching quality evaluation is a typical MADM problem. In this paper, the TOPSIS method with the hybrid indexes proposed above will be used to evaluate the teaching quality.

4.1 Evaluation Indexes

In order to evaluate the teaching quality objectively and comprehensively, the evaluation index proposed in this paper takes into account both process factors and outcome ones. From the perspective of certainty, both quantitative and qualitative indexes are considered. The detailed indexes are shown in Table 1.

Table 1. Teaching quality evaluation indexes

Type	Symbol	Name
Quantitative	IU_1	Rate of lateness
	IU_2	Rate of early departure
	IU_3	Rate of head-up
	IU_4	Degree of achievement of teaching objectives
Qualitative	IU_5	Degree of concentration on teaching
	IU_6	Novelty of teaching content
	IU_7	Familiarity with subject knowledge
	IU_8	Suitability of teaching methods
	IU_9	Effectiveness of stimulating learning interest
	IU_{10}	Effectiveness of achieving learning goals
	IU_{11}	Effectiveness of improving learning ability

In Table 1, the quantitative indexes IU_1-IU_4 are calculated according to the following equations:

$IU_1 = N_l / N_t$, N_l is the number of lateness, and N_t is the total number of classes in a teaching cycle.

$IU_2 = N_e / N_t$, N_e is the number of early departure in a teaching cycle.

$IU_3 = \frac{1}{N_t}\sum_{i=1}^{Nt}\left(\frac{1}{m}\sum_{j=1}^{m} Nh_{j,i} \Big/ N_i\right)$, m is the number of head-up rate monitoring during a class time; $Nh_{j,i}$ is the number of head-up at the j-th monitoring in i-th class; N_i is the total number of students attending the i-th class.

$IU_4 = N_f / N_s$, N_f is the number of students who have achieved the teaching objectives; N_s is the total number of students on the course.

In Table 1, IU_5-IU_{11} are the qualitative indexes. In the process of evaluation, the reviewers generally use linguistic variables to express them vaguely. In this paper, the linguistic variables are divided into nine levels, each of which can be characterized by corresponding IFVs. The specific relationship is shown in Table 2.

Table 2. Correspondence between linguistic variable and IFV

No.	Linguistic term	IFV
1	Exceedingly ideal	$<0.99, 0.01>$
2	Very ideal	$<0.90, 0.10>$
3	Ideal	$<0.80, 0.10>$
4	**Relatively ideal**	$<0.65, 0.20>$
5	Ordinary	$<0.50, 0.40>$
6	Relatively poor	$<0.30, 0.60>$
7	Poor	$<0.20, 0.70>$
8	Very poor	$<0.10, 0.90>$
9	Exceedingly poor	$<0.01, 0.99>$

4.2 Evaluation Example

At the end of a semester, it was necessary to evaluate the teaching quality of five teachers, and they were expressed by $T = \{T_1, T_2, T_3, T_4, T_5\}$. The teaching management department selected four student representatives and four teachers to form an evaluation team. Four student reviewers were expressed by $S = \{S_1, S_2, S_3, S_4\}$. Four peer reviewers were expressed by $P = \{P_1, P_2, P_3, P_4\}$. The evaluation data were collected during the semester. The evaluation data of quantitative indexes are shown in Table 3. The evaluation data of qualitative indexes are shown in Tables 4 to Table 11. In order to save space, the numbers in Table 2 are used to represent their corresponding linguistic variables.

Table 3. Evaluation results of indexes IU_1-IU_4

	IU_1	IU_2	IU_3	IU_4
T_1	0.08	0.10	0.88	0.86
T_2	0	0.06	0.95	0.91
T_3	0	0	0.91	0.94
T_4	0.16	0.15	0.78	0.81
T_5	0.15	0	0.92	0.85

Table 4. Evaluation results of S_1 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	3	2	2	2	3	1	2
T_2	2	2	2	4	3	3	3
T_3	2	1	2	2	1	2	2
T_4	5	6	4	6	7	8	5
T_5	6	5	5	4	4	5	6

Table 5. Evaluation results of S_2 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	2	2	2	3	3	1	3
T_2	3	3	2	2	2	4	3
T_3	3	2	1	2	2	2	2
T_4	5	6	6	7	6	7	6
T_5	6	5	6	6	5	5	7

Table 6. Evaluation results of S_3 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	2	2	2	4	2	2	2
T_2	3	2	2	3	3	2	3
T_3	2	3	2	2	2	3	2
T_4	6	6	5	5	5	6	6
T_5	5	6	6	5	7	6	8

Table 7. Evaluation results of S_4 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	2	3	4	3	5	3	2
T_2	3	3	3	4	4	3	3
T_3	2	1	2	3	4	3	2
T_4	5	6	7	6	8	5	5
T_5	6	5	5	5	4	5	6

Table 8. Evaluation results of P_1 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	1	2	2	4	3	2	2
T_2	2	3	3	3	3	3	3
T_3	2	2	1	1	2	3	2
T_4	4	3	5	6	4	6	4
T_5	5	5	4	6	6	5	5

Table 9. Evaluation results of P_2 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	2	3	3	2	1	2	3
T_2	3	4	3	4	2	4	3
T_3	2	1	2	1	2	2	2
T_4	5	6	5	7	5	6	5
T_5	6	5	6	3	3	6	6

Table 10. Evaluation results of P_3 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	4	2	2	3	4	2	3
T_2	4	4	4	3	3	3	3
T_3	2	2	2	1	3	2	2
T_4	6	7	7	6	6	6	6
T_5	6	5	6	5	4	5	6

Table 11. Evaluation results of P_4 on IU_5-IU_{11}

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	3	3	3	2	2	2	4
T_2	4	4	3	3	2	3	4
T_3	2	2	3	1	1	2	2
T_4	5	5	6	6	5	6	6
T_5	5	6	7	5	5	5	6

The above is the original evaluation data. The specific evaluation steps are given below:

Step 1: Analyze certain indexes

Forward process the data in Table 3. Because the indexes IU_1 and IU_2 are low excellent indexes, transform them into high excellent indexes $\widehat{IU}_{i,j}$ according to the following equation:

$$\widehat{IU}_{i,j} = \left(\max_i\left(IU_{i,j}\right) - IU_{i,j}\right) \Big/ \left(\max_i\left(IU_{i,j}\right) - \min_i\left(IU_{i,j}\right)\right),\ i = 1, 2, 3, 4, 5, j = 1, 2$$

The indexes IU_1 and IU_2 are normalized according to the following equation:

$$\widetilde{IU}_{i,j} = \widehat{IU}_{i,j} \Big/ \sqrt{\sum_{i=1}^{5} \widehat{IU}_{i,j}^{\ 2}}\ ,j = 1, 2$$

The indexes IU_3 and IU_4 are high excellent indexes. They are normalized according to the following equation:

$$\widetilde{IU}_{i,j} = IU_{i,j} \Big/ \sqrt{\sum_{i=1}^{5} IU_{i,j}^{\ 2}}\ ,j = 3, 4$$

The data after forward processing and normalization are shown in Table 12.

Table 12. The processed data

	IU_1	IU_2	IU_3	IU_4
T_1	0.333	0.210	0.442	0.439
T_2	0.666	0.382	0.477	0.465
T_3	0.666	0.636	0.457	0.480
T_4	0.000	0.000	0.392	0.414
T_5	0.040	0.636	0.462	0.434

The set of index value of the ideal teacher is dI^+ = [0.666, 0.636, 0.477, 0.480]. The set of index value of the inverse ideal teacher is dI^- = [0.000, 0.000, 0.392, 0.414]. The weights of IU_1-IU_4 are equal, w_j = 1

(j = 1, 2, 3, 4).

According to equations (4) and (5), the similarity S_d^+, S_d^- between each evaluation object and the ideal, the inverse ideal teacher can be obtained.

$$S_d^+ = [0.9225, 0.9790, 0.9999, 0.592, 0.8371]^T,\ S_d^- = [0.8449, 0.6548, 0.5841, 1.0000, 0.7040]^T.$$

Step 2: Analyze fuzzy indexes

According to the correspondence of Table 2, the numbers in Table 4 to Table 11 are converted into IFVs.

The weights w_i of 8 reviewers are 0.15, 0.15, 0.15,0.15, 0.10, 0.10, 0.10, 0.10 respectively.

Applying equation (1), the evaluation matrix can be obtained by aggregating the data of 8 reviewers. It is shown in Table 13.

Table 13. Evaluation matrix of the fuzzy indexes

	IU_5	IU_6	IU_7	IU_8	IU_9	IU_{10}	IU_{11}
T_1	<0.8929, 0.0851>	<0.8725, 0.1000>	<0.8614, 0.1110>	<0.8195, 0.1189>	<0.8488, 0.1048>	<0.9444, 0.0501>	<0.8555, 0.1072>
T_2	<0.8119 0.1149>	<0.8079, 0.1231>	<0.8452, 0.1072>	<0.7745, 0.1320>	<0.8293, 0.1110>	<0.7927, 0.1189>	<0.7885, 0.1072>
T_3	<0.8890, 0.1000>	<0.9558, 0.0398>	<0.9397, 0.0562>	<0.9558, 0.0398>	<0.9273, 0.0624>	<0.8681, 0.1000>	<0.9000, 0.1000>
T_4	<0.4752, 0.4130>	<0.3948, 0.4891>	<0.4202, 0.4589>	<0.3119, 0.5868>	<0.3849, 0.5073>	<0.2949, 0.6140>	<0.4291, 0.4571>
T_5	<0.3778, 0.5206>	<0.4561, 0.4427>	<0.4017, 0.4834>	<0.5296, 0.3473>	<0.5610, 0.2989>	<0.4561, 0.4427>	<0.2829, 0.6266>

The set of index value of the ideal teacher is fT^+ = [<0.8929, 0.0851>, <0.9558, 0.0398>, <0.9397, 0.0562>, <0.9558, 0.0398>, <0.9273, 0.0624>, <0.9444, 0501>, <0.9000, 0.1000>]. The set of index value of the inverse ideal teacher is fT^- = [<0.3778, 0.5206>, <0.3948, 0.4891>, <0.4017, 0.4834>, <0.3119, 0.5868>, <0.3849, 0.5073>, <0.2949, 0.6140>, <0.2829, 0.6266>]. The weights of IU_5-IU_{11} are equal, $w_k = 1$ (k = 1, 2, …, 7).

According to equations (6) and (7), the similarity S_f^+, S_f^- between each evaluation object and the ideal, the inverse ideal teacher can be obtained.

$$S_f^+ = [0.9976, 0.9924, 0.9996, 0.6480, 0.7221]^T,\ S_f^- = [\ 0.6254, 0.6562, 0.5970, 0.9886, 0.9603]^T.$$

Step 3: Calculate the proximity

According to the ratio of the number of evaluation indexes, the weights of quantitative and qualitative indexes e_d, e_f are determined, e_d = 4/11 = 0.36, e_f = 7/11 = 0.64. According to the equations (8) and (9), the weighted comprehensive distance WS^+, WS^- is obtained.

$$WS^+ = [0.9706, 0.9876, 0.9997, 0.6278, 0.7635]^T,\ WS^- = [0.7044, 0.6557, 0.5924, 0.9927, 0.8680]^T$$

The proximity $R(T_i)$ can be calculated according to equation (10).

$$R(T_i) = [0.5795, 0.6010, 0.6279, 0.3874, 0.4680]^T,\ i = 1, 2, \ldots, 5.$$

Step 4: Sort the results of evaluation

According to the order of proximity $R(T_i)$, the ranking results of teaching quality of 5 teachers can be determined. Because $R(T_3) > R(T_2) > R(T_1) > R(T_5) > R(T_4)$, the ranking of 5 teachers should be T_3, T_2, T_1, T_5, T_4 in terms of teaching quality.

From the values of quantitative and qualitative evaluation indexes in Table 3 to Table 11, it can be seen that the teaching quality of T_3, T_2 and T_1 is relatively good. The same results are obtained by applying the proposed method. In order to further verify the rationality of the proposed method, it is compared with the method in Ref. [25]. An intuitionistic fuzzy TOPSIS method based on Euclidean distance calculation was proposed in the Ref. [25]. Using this method to analyze the data in the above examples, the results are shown in Table 14, and the ranking of 5 teachers T_3, T_1, T_2, T_5, T_4.

Table 14. Results obtained by the method in the ref [25]

Evaluation objects	Distance to ideal object	Distance to inverse ideal object	Proximity	Ranking
T_1	0.1940	0.3438	0.6393	2
T_2	0.2532	0.3754	0.6272	3
T_3	0.0458	0.3808	0.8926	1
T_4	0.6909	0.2470	0.2634	5
T_5	0.5507	0.2669	0.3264	4

From the calculation results, it can be seen that in the two algorithms, the ranking results of T_3, T_5 and T_4 are the same, while the order of T_1 and T_2 is different. The comparison of the calculation results of the two methods is shown in Fig. 1.

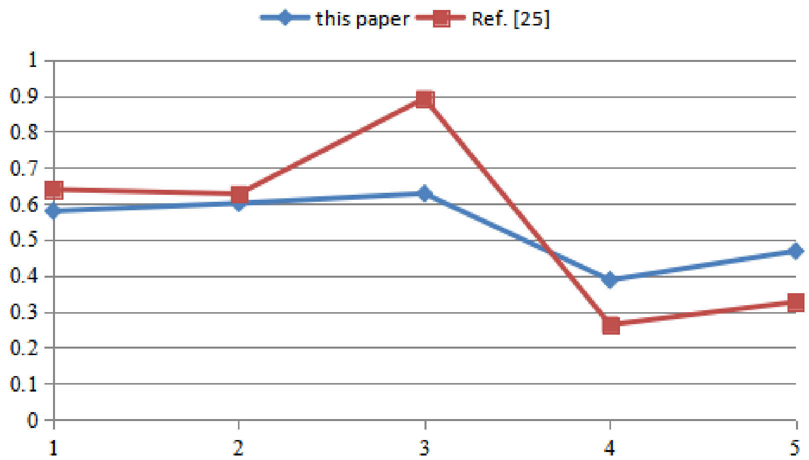

Fig. 1. Comparison of calculation results

The main reason for the difference of $T_1$and T_2 ranking results is that the two algorithms use different similarity measures. The Euclidean distance method is adopted in the Ref. [25], which can reflect the absolute difference of attribute values for each evaluation object. Therefore, it is mostly used in the analysis that needs to reflect the difference according to the absolute size of the index value. Its main disadvantage is that it will result in unreasonable evaluation results when the absolute criteria of the reviewers are not unified. In this paper, the cosine function is used to calculate similarity, which distinguishes the difference between evaluation objects by the angular distance between two vectors, but is not sensitive to the absolute size of their attribute values. Hence, when carrying on the relative evaluation between objects, it is more reasonable to use the cosine function to measure similarity.

In the process of teaching quality evaluation, in order to ensure the objectivity and fairness of the evaluation results, a number of reviewers are usually needed. And different reviewers may have different views on the criteria of "good" and "bad", and there may be great differences in absolute values in the evaluation, but the relative change trend is basically the same. In this case, different evaluation results may occur by using two different similarity measures. Compared with the method based on the Euclidean distance, the result obtained by the cosine function method is more reasonable. The inconsistency of T_1 and T_2 ranking results the two methods is caused by the above reasons, so the calculation results of the method in this paper are more reasonable and reliable.

5 Conclusion

On the basis of summarizing the existing methods, a TOPSIS method was proposed, which is suitable for solving MADM problems with hybrid indexes (quantitative and qualitative). In this paper, the cosine similarity measure was used to replace the Euclidean distance in traditional TOPSIS methods. It pays more attention to the change of relative trend of data rather than the absolute value. The corresponding relationship between IFVs and linguistic variables was established, and the quantitative representation of qualitative indexes was realized. In the process of teaching quality evaluation, the method presented in this paper can effectively eliminate the impact of different evaluation standards of reviewers on the decision results. It can ensure that the evaluation results are more reasonable and credible.

The main defect of this paper is to use single-valued IFS to represent the evaluation index. In fact, interval IFS can better describe the uncertainty and fuzziness of evaluation index values. In the future research work, the expression method of teaching quality evaluation index based on interval IFSs will be further studied. On this basis, a hybrid multi-index TOPSIS decision method based on interval intuitionistic fuzzy sets is designed. It can further extend the method proposed in this paper, and make it have better adaptability.

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