

Optical Design of Portable LED Nails Lamps

Shu-Ting Chen¹, Shih-Hsin Ma², Ting-Jou Ding^{3*}

¹Department of Fashion Imaging, MingDao University, Changhua 523, Taiwan, ROC
chenshuting899@gmail.com

²Department of Photonics, Feng Chia University, Taichung 407, Taiwan, ROC
shma@fcu.edu.tw

³Department of Intelligent Energy Engineering, MingDao University, Changhua 523, Taiwan, ROC
tjding@gmail.com

Received 15 July 2022; Revised 3 August 2022; Accepted 3 August 2022

Abstract. For the problem of low light efficiency caused by divergent and uneven energy distribution of commercial port-able LED nail lamps, we propose an optical system composed of a Lambertian LED, a total internal reflection lens and a microlens array. According to the simulation results, the optical system can project a 130mm×70mm rectangular light pattern with a uniformity of 85.17% on a screen 70mm away. In addition, the optical utilized factor of the rectangular light pattern is 87.85%, and the optical efficiency of the entire optical system is 83.93%.

Keywords: nail lamps, optical design, LED, optical system, illuminance

1 Introduction

With the improvement of the quality of life, people pay more and more attention to the issues of art and beauty. The presentation of art and beauty can enrich people's hearts, make people happy. In order to establish good interpersonal relationships and enhance self-confidence, in recent years, human-related beauty issues have also been heatedly discussed, such as makeup, dressing, body sculpting, etc. In addition, a person's appearance can also affect the first impression and perception of the others. Therefore, the beauty industry has become one of people's daily needs, and the output value of this industry has continued to grow steadily, especially nail art. Gel manicure has developed rapidly and has gradually become one of the mainstream technologies for nail beauty. The gel nail technique uses a nail polish made of special pigments and resins to spread evenly over the surface of the nail. The coated nails are then placed in a nail lamp and exposed to specific ultraviolet (UV) light to cure the resin [1-6]. The curing of the resin is a very important process for gel nails. Therefore, nail lamps are indispensable for gel nail technology. The conventional nail lamps are bulky and use UV tubes with lower luminous efficiency as light sources. The conventional nail lamps have more power consumption, so they need to be plugged in. The problem of excessive power consumption of nail lamps has always been an interested issue to engineers.

In recent years, with the rapid development of solid-state lighting technology, light-emitting diodes (LEDs) are rapidly replacing conventional tube lights. LEDs have many advantages such as high luminous efficiency, small size and high reliability [7-14]. Today, LED light sources have been widely used in nail lamps [15]. Compared with the conventional tube lights, the LEDs have more advantages in the required ultraviolet spectrum for nail lamps. In ultraviolet region, conventional tube lights need filters to get the desired light wavelengths. During the filtering process, most of the light energy is lost. In terms of the wavelength of light, the spectral range of LEDs is narrower than that of conventional tube lights, and LEDs do not require filters in practical applications. There are different light-emitting mechanisms between conventional tube lights and LEDs, which make LEDs have higher luminous efficiency and lower energy consumption. Therefore, LEDs are a good solution for the commercial nail lamps. The smaller size of LEDs is also an advantage that has less weight and more compact volume, especially suitable for portable nail lamps. Fig. 1 shows a commercial portable nail lamp consisting of UV LEDs.

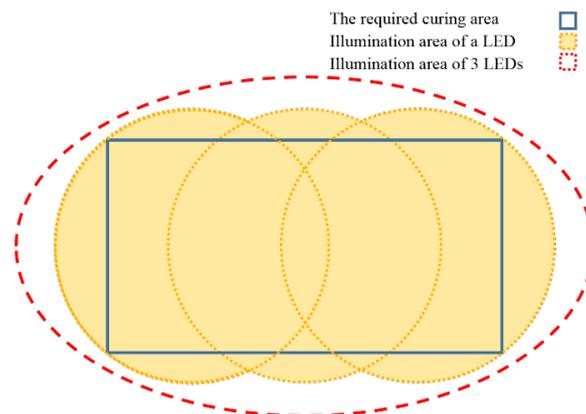
* Corresponding Author



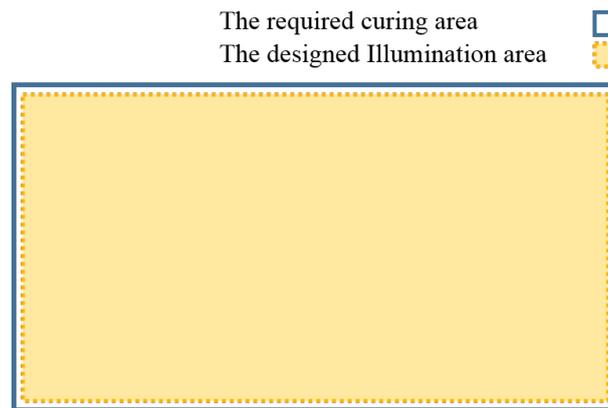
Fig. 1. A commercial portable nail lamp consisting of UV LEDs

In this portable nail lamp, there are UV LEDs array assembled on the underside of top and two folding stands on the left and right side. The two folding stands can support the entire unit and leave it suspended. In the curing process, the coated fingernails can put under the nail lamp with several centimeter distance. The common portable nail lamps on the market usually do not have light irradiation design for LEDs. The LEDs on the nail lamp are simply assembled in a geometric axisymmetric arrangement or a rectangular array arrangement. Typically, these devices utilize the overlapping light of multiple LEDs to increase the irradiance and uniformity of the illuminated area. In addition, in some nail lamps, the two folding stands are also equipped with LEDs to enhance the irradiance on the left and right side faces of the nail. This design allows the light intensity on the side face of the nail to reach the resin curing threshold.

The light illumination area of conventional nail lamps is determined by the original light emission patterns of LEDs, so it lacks the ability to adjust the light distribution. LEDs consist of a light emitting surface and an ax-isymmetric package. In addition, the light emitting surface is composed of countless point light sources, and the light patterns are mostly circular or elliptical. The light superposition of multiple LEDs makes part of the light irradiate outside the required area. Light in the outside area causes additional energy consumption and reduces illuminance [16-17]. During the curing process, the spill light can cause more skin on the back of the hand to darken. This is the result that nail beauticians and customers generally do not want to see [18]. The illumination area of a commercial portable nail lamp is shown in Fig. 2(a).



(a) The illumination area of the commercial portable nail lamp

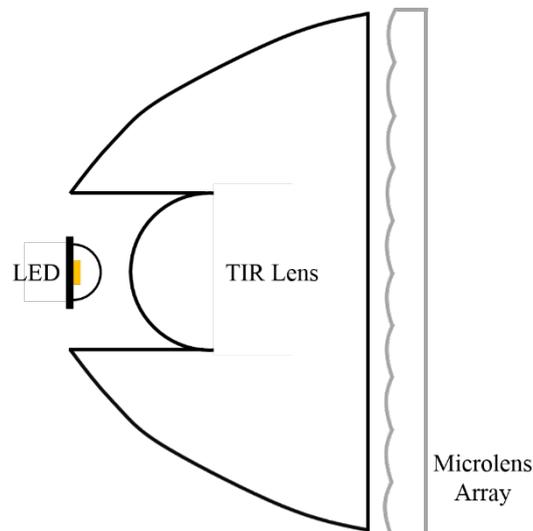


(b) The rectangular illumination area by optical design

Fig. 2. The illumination area of a portable nail lamp

Taking the structure of 3 LEDs as an example, the illumination area is usually oval, which does not match the required curing area. Moreover, the inappropriate spacing of LEDs makes the irradiation areas excessive overlap or no intersection that causes the non-uniformity of illumination. The current solution to uneven illumination is to increase the number of LEDs. However, too many LEDs can make the light too strong and cause the temperature of the resin to rise rapidly.

By optical design, the oval light energy distribution can be modified into a rectangular energy distribution meets the requirements, as shown in Fig. 2(b). The optical utilized factor (OUF) can be improved by collecting light into the required curing area. OUF is defined as the ratio of the light energy on the required area to the light energy emitted by the nail lamp. To solve this problem, we propose an optical system that can turn the divergent light of commercial portable nail lamps into a rectangular illumination area with higher OUF and uniform. This optical system includes a UV LED, a total internal reflection (TIR) Lens and a microlens array. The TIR lens is a circular symmetrical optical component and composed of a cylindrical tube on the left, an axisymmetric quasi-parabolic surface, a small-aperture convex spherical surface on the right of the cylindrical tube, and a planar surface on the right end face. The system structure of UV LED, TIR lens and microlens array is shown in Fig. 3.

**Fig. 3.** Side view of the optical system structure

(This optical system consists of UV LEDs, TIR lenses, and microlens arrays in order from left to right)

A UV LED is put in the cylindrical tube of the TIR Lens, the diverging light from the UV LED could be refracted into the TIR lens and collimated by axisymmetric quasi-parabolic surface. The Collimated light from a TIR lens is projected onto an optical microlens array. The optical microlens array can uniform the light energy

distribution and form a rectangular light energy distribution at the same time. In this optical system, the flat surface on the right side of TIR lens is next to the convex surfaces of the microlens array. A screen is placed behind the microlens array 70mm to observe the irradiance distribution. This study used computer simulation to calculate the light energy distribution, optical efficiency, and the uniformity [19-21].

2 Design Methods

2.1 LED Light Source Model

Before the optical mechanism design, a precision virtual UV LED light source model is needed. A virtual light source model must be built in the computer before optical mechanism design. Generally, a precise and correct light source model can enhance the accuracy of the optical design. [22-24] In this study, the ASAP (Advanced Systems Analysis Program, Breault) optical design development software is used to build the LED light source model and subsequent optical mechanism design. Moreover, it also can help us analyze the optical performance.

In terms of light source selection, the Lambertian LED is used as the light source. We set the parameters of the Lambertian LED light source model according to the specifications of general commercial LEDs. Inside the LED, a 1mm×1mm square light emitting chip is encapsulated at the bottom of a hemispherical silicone lens with a radius of 1.3mm and a refractive index of 1.41. In the ASAP simulation environment, the rays emerged from random locations of the square emitting surface and travel in random directions, it is the so called Monte Carlo ray tracing method. Under this luminous mechanism, from a macro perspective, the rays are arranged in a Lambertian distribution. We set 20 million rays to trace in the LED optical model by ASAP software and 180 detectors to determine the light intensity distribution. The 180 detectors on a semicircular track with a radius of 300mm all face the LED. The simulation setup is shown as Fig. 4.

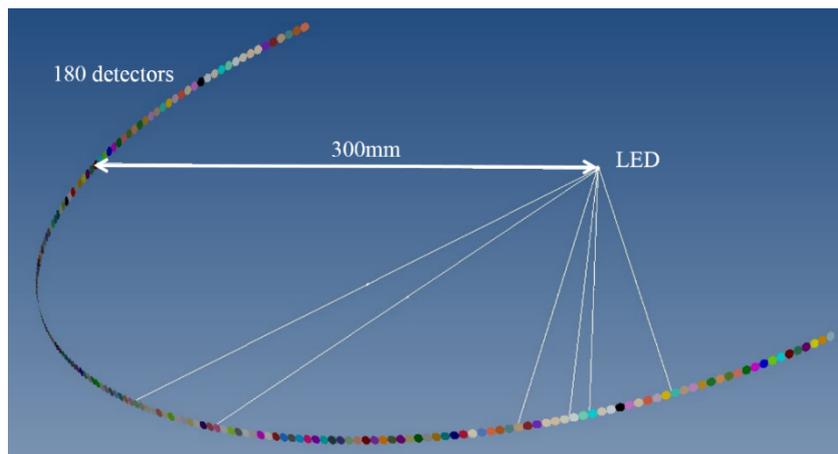


Fig. 4. The simulation setup of the 1D intensity distribution of LED
(The 180 detectors on the semicircular track are all 300mm away from the LED.)

Each detector corresponds to a different divergence angle. The area of each detection is set to the area of the projection plane corresponding to a solid angle of one degree. Hence, the LED intensity distribution can be obtained from the one-dimension intensity measurements. The one-dimension light intensity curve of LED is shown in Fig. 5.

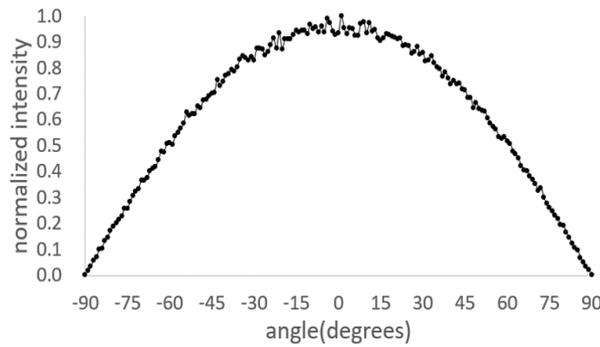


Fig. 5. The simulation result of the 1D intensity distribution of LED

2.2 TIR Lens Design

As the structure of Lambertian LED is axisymmetric, the related spatial energy distribution is also axisymmetric. Under this condition, a large number of rays are distributed in a region with a large divergence angle. To adjust these rays to rectangular light distribution, all the rays must be collected as possible and collimated [25-27]. Therefore, the TIR Lens is used as the main body of optical mechanism for design, as shown in Fig. 6.

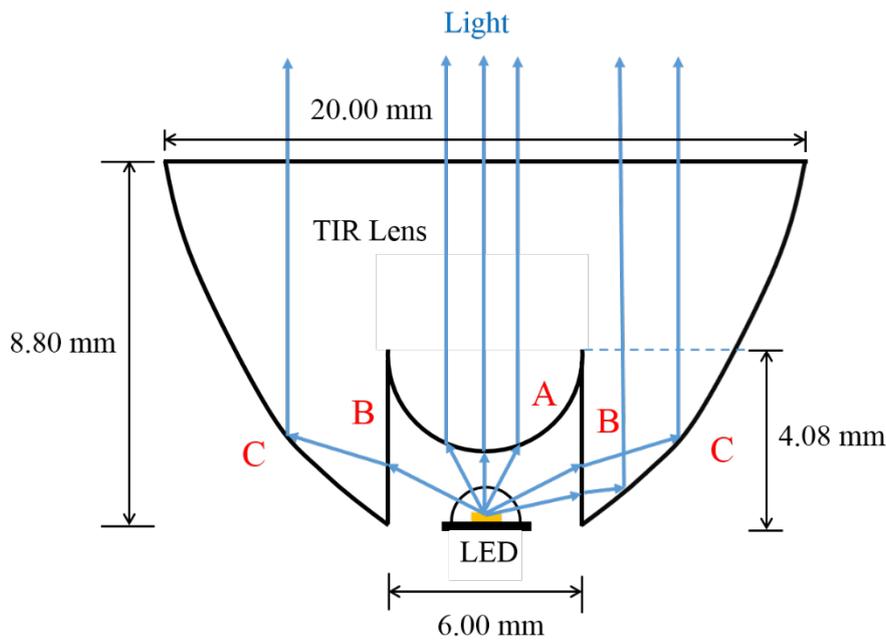


Fig. 6. Side view and the specification of the TIR lens (The rays from LED can be collected and collimated.)

When the large divergence angle rays from LED reach the side B, they are refracted into the TIR lens. The side C of TIR lens is a free-form surface formed of paraboloids in different curvature radii. When these rays irradiate side C, by the total interface reflection of side C, the rays are straight shot out of the lens upwards. On the other hand, the small angle light rays emitted from the LED are directly refracted by the side A and straight shot out of the lens. The side A is a spherical surface, its curvature radius is estimated by Eq.1. Eq. 1 is expressed as follows

$$\frac{1}{f} = \frac{n-1}{R} \tag{1}$$

where f is the distance between the LED chip and side A of TIR lens; n is the refractive index of TIR lens, the value is 1.53; R is the curvature radius of side A. The LED is combined with TIR Lens by ASAP software for simulation, the light distribution curve can be obtained, as shown in Fig. 7.

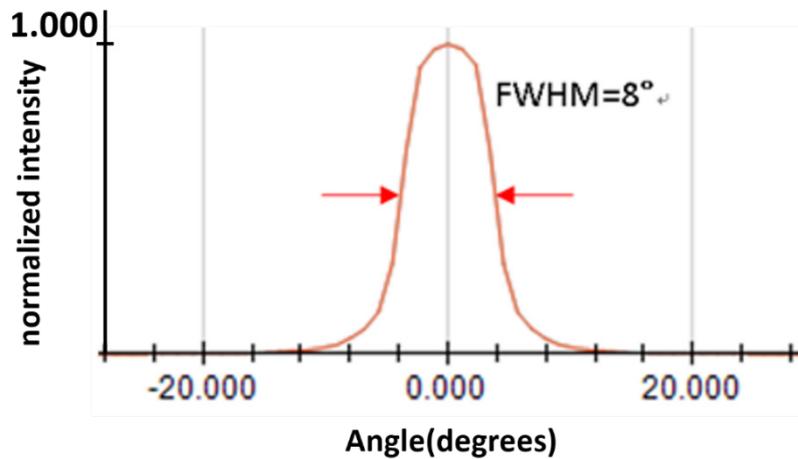


Fig. 7. The light distribution curve of LED combined with TIR lens (The FWHM of this curve is about 8° .)

The FWHM of this light distribution curve is about 8° , and the optical efficiency of system is about 93.7%. In comparison to the light distribution curve result of LED light source in Fig. 5, the total internal reflection lens can effectively reduce the divergence angle of Lambertian LED, and achieve the collimation of light.

2.3 Light Shaping Array Lens Design

The rays from LED collimated by the TIR Lens generate a small divergence angle light beam which can be reshaped by the microarray lens with light reshaping capability [28-29]. In this way, it can achieve the required light shape. The schematic diagram of the light reshaping microarray lens is shown in Fig. 8. This microarray lens is a circle in diameter of 20mm and in thickness of 2.5mm, its area is equal to the exit area of TIR Lens.

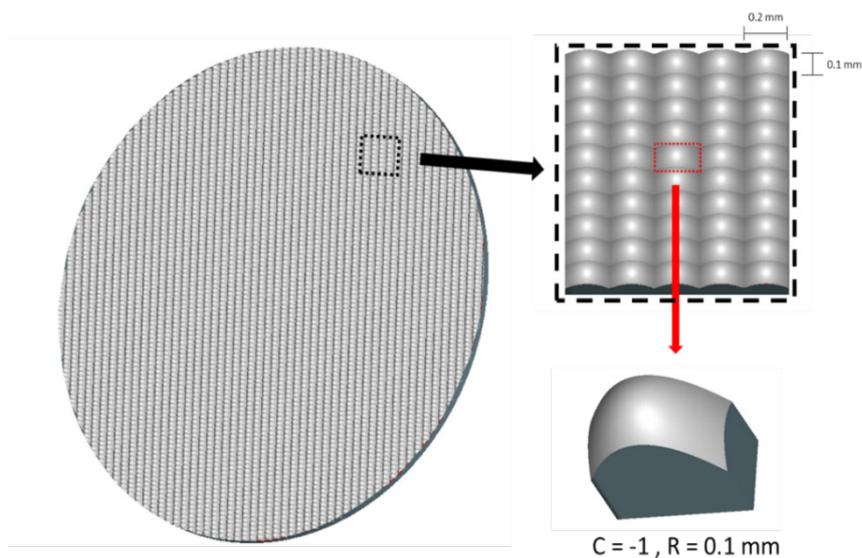


Fig. 8. The schematic diagram of microarray lens, the conic constant of single-element lens is -1, the curved surface curvature radius R is 0.1 mm

The microarray lens component comprises multiple plano-convex lens units (about 19,000 pieces), the aperture of plano-convex lens is a $0.2\text{mm}\times 0.1\text{mm}$ rectangle. The convex surface of plano-convex lens is an aspheric surface, the shape of the curved surface is determined by Eq. 2, expressed as follows.

$$z(r) = \frac{r^2 / R}{1 + \sqrt{1 - (1 + c)r^2 / R^2}}. \quad (2)$$

3 Simulation Results

Fig. 9 shows the three-dimensional graphics of LED light source module and the designed optical mechanism in the ASAP optical simulation software.

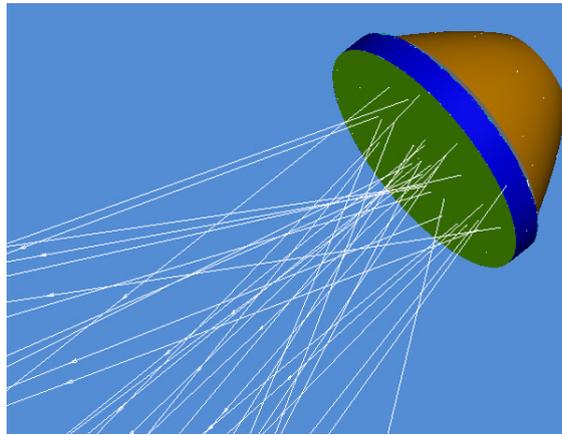


Fig. 9. 3D view of the optical mechanism

In the optical simulation environment of ASAP, the energy distribution of projection plane is calculated by the track down rays, the projection distance is 70mm away from the light emitting surface of TIR Lens, the result is shown in Fig. 10.

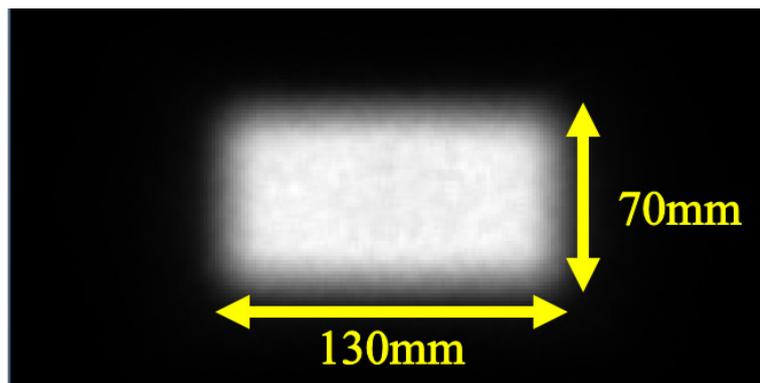


Fig. 10. Light energy distribution of the optical mechanism at projection distance of 70 mm

The result shows that the light energy distribution presents rectangular distribution, its length and width are close to the designed illumination area $130\text{mm}\times 70\text{mm}$. In the analysis of optical efficiency of the optical system, the optical efficiency is defined as the luminous flux of system emitted light divided by the luminous flux of LED emitted light. The optical efficiency is 83.96% according to calculation results. The effective optical utilized factor (OUF) of rectangular light pattern is defined as the luminous flux in the specified region ($130\text{mm}\times 70\text{mm}$) divided by the luminous flux of system emitted light, the value is 87.85%.

Additionally, in the analysis of light energy illuminance uniformity, the normalized irradiated area is divided into 12 homolographic regions, and one point is taken from the center point of each region as the illuminance analysis point, there are 12 points, as shown in Fig. 11.

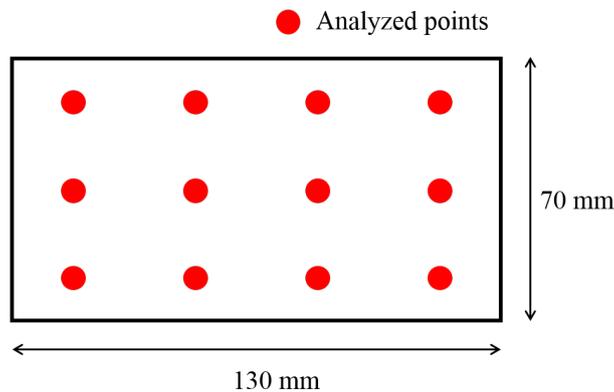


Fig. 11. The irradiated area is divided into 12 homolographic regions to calculate the uniformity

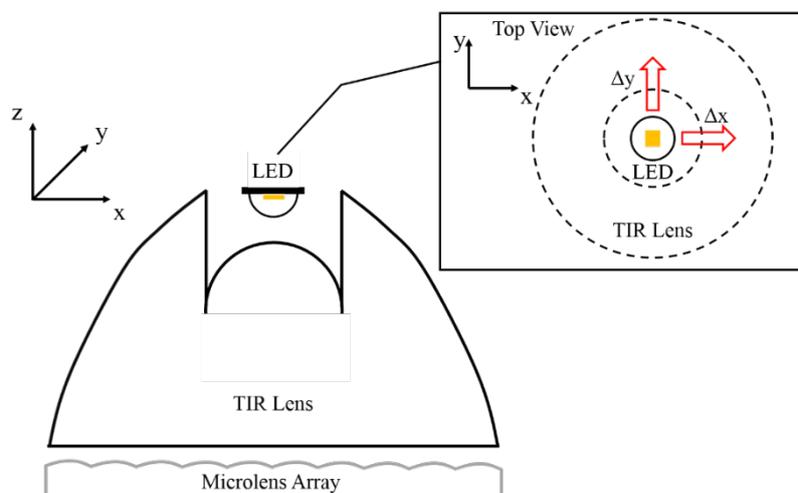
The average value and minimum value of the illuminance values of the 12 points are calculated, and the uniformity is calculated by Eq. 3 [30], expressed as follows:

$$Uniformity = \frac{Minimum\ illuminance\ of\ 12\ measuring\ points}{Average\ illuminance\ of\ 12\ measuring\ points} \quad (3)$$

According to the calculation results, the projected rectangular light uniformity of this optical mechanism system is 85.17%.

4 Tolerance Analysis of LED Light Source

In the assembly of the light source, the error of LED installation position might cause the potential affections for optical performance, such as the uniformity of the light pattern, the optical efficiency, the center irradiance value, etc. Therefore, position tolerance analysis of the LED can provide an important reference for manufacturers to evaluate the assembly error of light source, so as to improve the yield of products. A slight offset of the LED in the x, y, and z directions can cause deviations in the light travel path in the nail lamp. We traced the light's travel path for tolerance analysis. The schematic diagram of the LED displacement mechanism is shown in Fig. 12.



(a) x- and y-directions

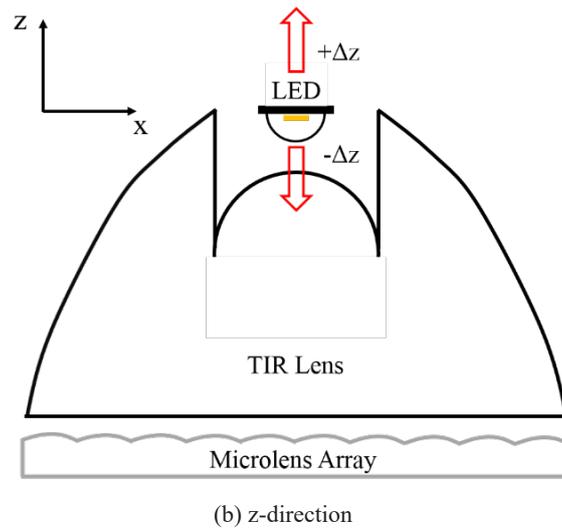
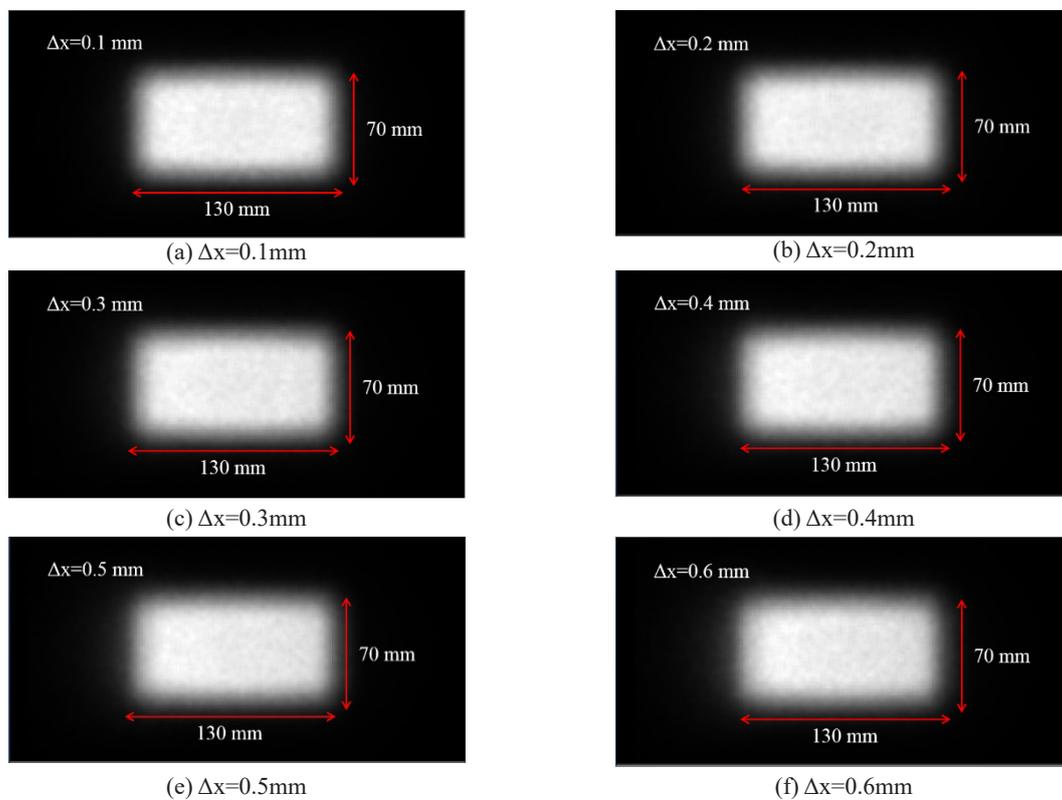


Fig. 12. The schematic diagram of the LED displacement mechanism

Due to the symmetry of TIR lens, we took the positive displacement in both x direction and y direction to simulate, as shown in Fig. 12(a). The displacement Δx and Δy both ranged from 0.1mm to 1mm in 0.1mm increments. In the z direction, as shown in Fig. 12(b), we took two displacements Δz and $-\Delta z$ in opposite direction. The range of displacement of $+z$ direction is from 0mm to 1mm and $-z$ direction is from 0mm to -1mm in both 0.1mm increments.

Fig. 13 shows the simulated light patterns of the LED shifted in different displacements in the x direction. The result shows that with the increase of Δx , the light intensity distribution gradually presents asymmetry.



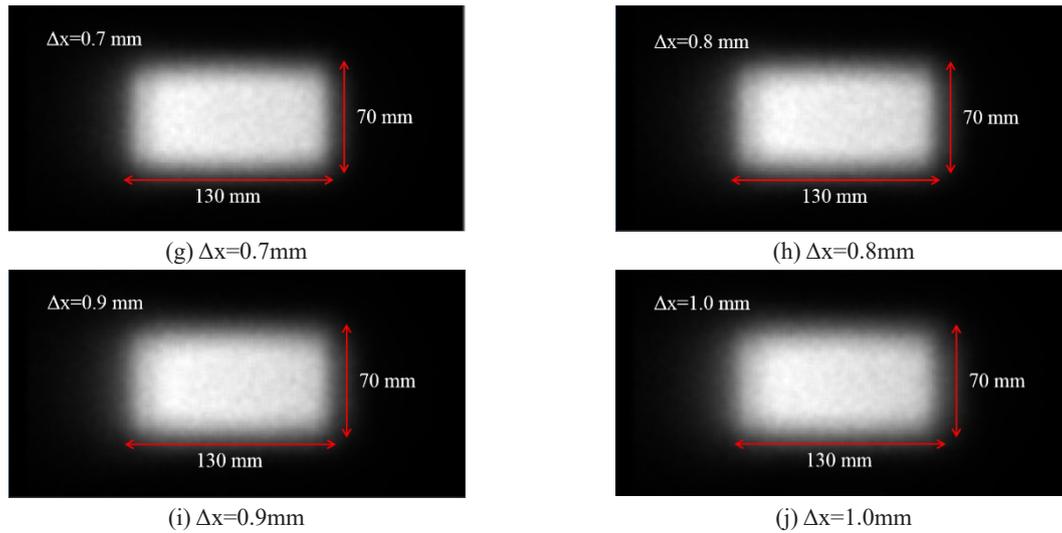


Fig. 13. The light patterns of the LED with different displacements in the x direction

Fig. 14 shows the simulation result of corresponding optical efficiency, uniformity and effective optical utilized factor.

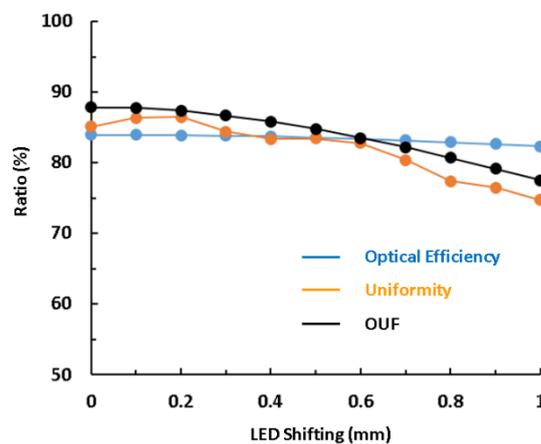
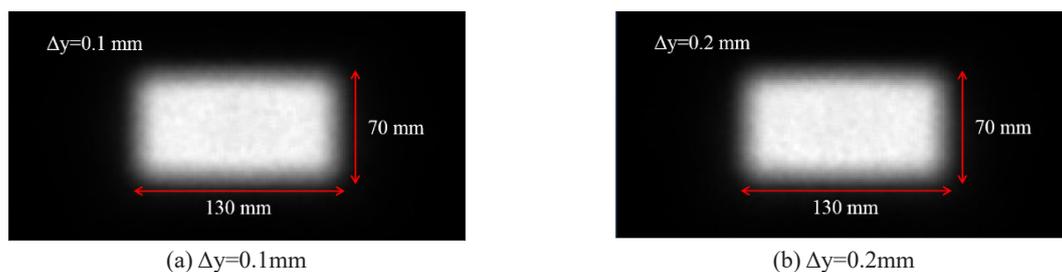


Fig. 14. Comparison of the optical efficiency, the uniformity, and the OUF with LED shifting along x direction

According to the result, there are a slow decline all in the optical efficiency, the uniformity and the effective optical utilized factor. The downward trend of light efficiency is slower and insignificant compared to others. Uniformity will suddenly have a large downward trend at 0.6mm and when it reaches 0.8mm, its value is about 90% of the maximum uniformity of this system. The downward trend of the OUF curve is stable, and the over-all trend is similar to that of uniformity, but the value is slightly higher than that of uniformity. The optical efficiency does not change significantly with the increase of Δx . Therefore, the offset tolerance of the LED mounting position in the x-direction mainly depends on the uniformity and OUF. The results show that the uniformity can be maintained above 90% of the system design value at offsets less than 0.8mm.

In the y direction, the simulation result of light patterns with the increase of Δy is shown in Fig. 15.



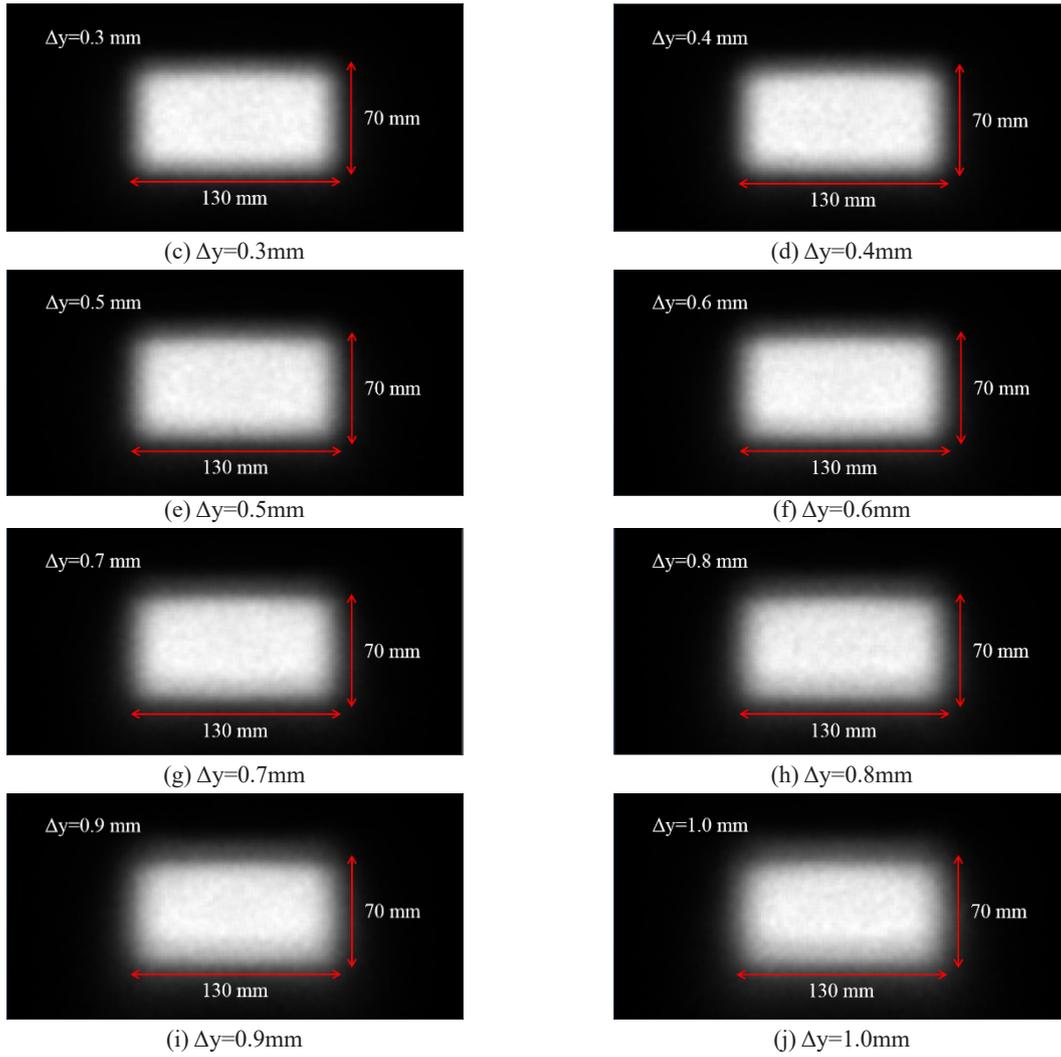


Fig. 15. The light patterns of the LED with different displacements in the y direction

The corresponding optical energy analysis including optical efficiency, uniformity and effective OUF is shown in Fig. 16.

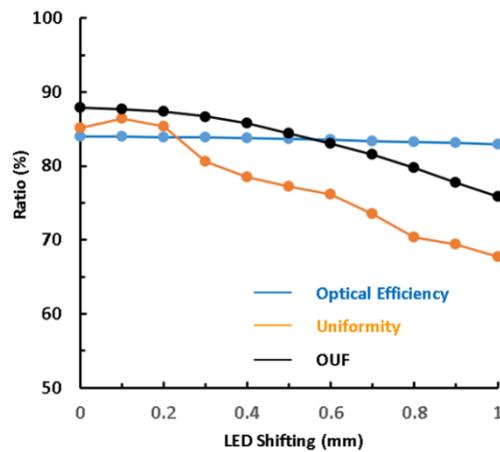


Fig. 16. Comparison of the optical efficiency, the uniformity, and the OUF with LED shifting along y direction

Similar to the simulation result of x-direction, the light patterns gradually exhibit asymmetry as Δy increases. According to the data, the optical efficiency and OUF, similar to the x-direction, both tend to decrease with the increasing displacement. The ratio of optical efficiency and OUF is about the same value near the 0.6mm offset value. In the graph, when the offset of the installation position in the y direction is 0.2mm, the uniformity will suddenly have a large downward trend. And when the offset reaches 0.3mm, the ratio value of uniformity will drop to 80%. When the offset value is 1mm, the value of uniformity drops below 70%. The uniformity in y direction drops faster than the one in x direction. The offset tolerance of the LED along the y direction is mainly determined by the drop in uniformity. When the offset setting is less than 0.5mm, the uniformity can be maintained above 90% of the system design value. Due to the eccentricity of the LED both in the x and y directions, the optical performance of nail lamps is affected and deteriorated.

The assembly error in the z-direction also affects the optical performance of the nail lamps. The simulated light patterns of z-direction are shown in Fig. 17.



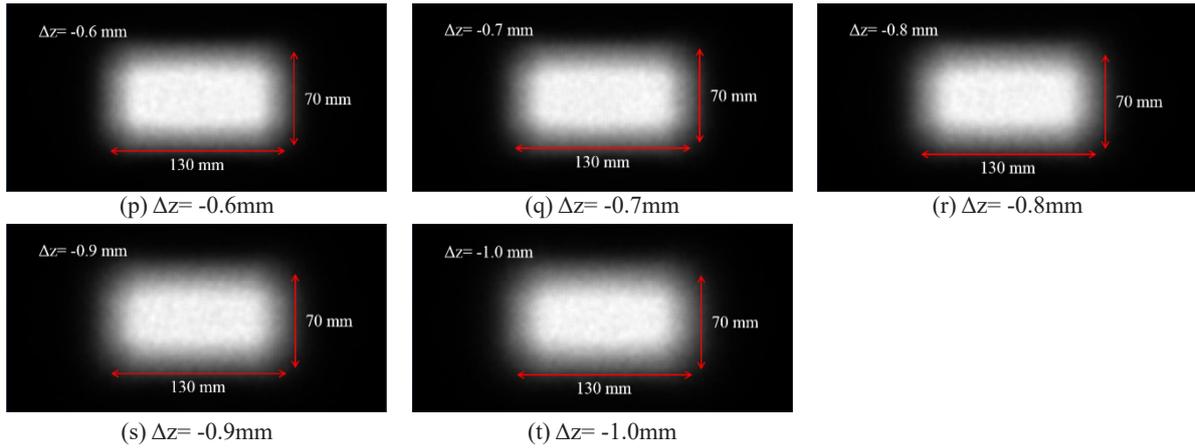


Fig. 17. The light patterns of the LED with different displacements in the z direction

In Fig. 17, the edge sharpness of light patterns is gradually improved as the displacement increased along +z-direction. Conversely, as the displacement increased along $-z$ direction, the edge of light patterns gradually blurs. The comparison graph of the corresponding optical efficiency, uniformity and OUF is shown in Fig. 18.

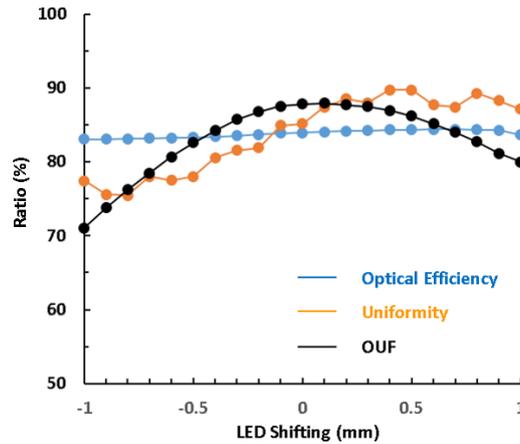


Fig. 18. Comparison of the optical efficiency, the uniformity, and the OUF with LED shifting along z direction

According to the data in Fig. 18, there is only small-scale changes in optical efficiency. The uniformity shows an asymmetric trend, it approximately increases about 5% in the +z-direction and decreases 10% in the $-z$ -direction. With the increase of displacement in +z and -z, OUF will show a downward trend, and the drop rate in the -z direction is larger. In the +z direction, when the displacement reaches the set maximum value of 1mm, the OUF remains above the 90% threshold of the system. In the -z direction, due to the decrease of uniformity, the shifting tolerance depends on both uniformity and OUF. Under the set 90% threshold, the displacement value is recommended to be below 0.5mm.

5 Conclusion

In this study, we propose an optical system for commercial portable nail lamps that includes Lambertian LED, TIR lens, and microlens array. We set a Lambertian LED model composed of a 1mm*1mm emitting chip and a hemispherical packaging lens with 1.41 refractive index at fist. The TIR lens can collimate the divergent light emitted by the Lambertian LED. By TIR lens, the FWHM value of the intensity distribution of the Lambertian LED can be effectively converged to 8° . The collimated light passes through an microlens array composed of a periodic plano-convex lens and a rectangular aperture can be reshaped into a rectangular light pattern. According to the simulation and analysis results of optical software, at the preset projection distance of 70mm, a rectangular light distribution at illuminance of 85.17% uniformity can be obtained. The size thereof is 130mm×70mm, the

OUF in this area is 87.85%. The optical efficiency of the overall system is 83.96%.

Furthermore, considering the actual assembly, we perform a tolerance analysis to understand how the optical performance of this optical system changes due to misalignment of the LED. According to the simulation results, asymmetrical light pattern is occurred in a specific direction when the LED is shifted in the x-y plane, the edge sharpness is influenced by the LED shifting along z direction. We evaluate and set 90% of the system design value as a tolerance threshold based on optical efficiency, uniformity, and OUF. According to the simulation results, the tolerable installation error values of the LED in the x-direction and the y-direction are respectively within 0.8mm and 0.5mm. In the Z direction, due to the asymmetric structure, the tolerable installation error values in the +z direction and -z direction are within 1mm and 0.5mm, respectively. The optical system has a simple structure without highly complex optical components, making it ideal for commercial portable nail lamps.

References

- [1] M.S. Shur, R. Gaska, III-nitride based deep ultraviolet light sources, In: Gallium Nitride Materials and Devices III. SPIE, 2008.
- [2] M.J. Dvorchak, M.L. Clouser, UV curing of nail gels by light emitting diode (LED) and fluorescent (FL) light sources, *Surface Science and Adhesion in Cosmetics* (2021) 73-107.
- [3] F. Zareanshahraki, V. Mannari, "Green" UV-LED gel nail polishes from bio-based materials. *International journal of cosmetic science* 40(6)(2018) 555-564.
- [4] J.Y. Bae, Y. Kim, H. Kim, Y. Kim, J. Jin, B.S. Bae, Ultraviolet light stable and transparent sol-gel methyl siloxane hybrid material for UV light-emitting diode (UV LED) encapsulant, *ACS applied materials & interfaces* 7(2)(2015) 1035-1039.
- [5] R.N. Kumar, L.Y. Keem, N.C. Mang, A. Abubakar, Ultraviolet radiation curable epoxy resin encapsulant for light emitting diodes, *Journal of applied polymer science* 100(2)(2006) 1048-1056.
- [6] K.S. Vandewalle, H.W. Roberts, J.L. Andrus, W.J. Dunn, Effect of light dispersion of LED curing lights on resin composite polymerization, *Journal of Esthetic and Restorative Dentistry* 17(4)(2005) 244-254.
- [7] L.V. Kerai, S. Hilton, S. Murdan, UV-curable gel formulations: Potential drug carriers for the topical treatment of nail diseases, *International Journal of Pharmaceutics* 492(1-2)(2015) 177-190.
- [8] M.R. Krames, O.B. Shchekin, R. Mueller-Mach, G.O. Mueller, L. Zhou, G. Harbers, M.G. Craford, Status and future of high-power light-emitting diodes for solid-state lighting, *Journal of display technology* 3(2)(2007) 160-175.
- [9] S. Pimplutkar, J.S. Speck, S.P. DenBaars, S. Nakamura, Prospects for LED lighting, *Nature photonics* 3(4)(2009) 80-182.
- [10] S. Liu, X. Luo, LED packaging for lighting applications: design, manufacturing, and testing, John Wiley & Sons, 2011.
- [11] C.M. Bourget, An introduction to light-emitting diodes, *Hortscience* 43(7)(2008) 1944-1946.
- [12] G. Held, Introduction to light emitting diode technology and applications, Auerbach publications, New York, 2016.
- [13] J. Cho, J.H. Park, J.K. Kim, E.F. Schubert, White light-emitting diodes: history, progress, and future, *Laser & photonics reviews* 11(2)(2017) 1600147.
- [14] Y. Peng, R. Liang, Y. Mou, J. Dai, M. Chen, X. Luo, Progress and perspective of near-ultraviolet and deep-ultraviolet light-emitting diode packaging technologies, *Journal of Electronic Packaging* 141(4)(2019) 040804.
- [15] Y. Muramoto, M. Kimura, S. Nouda, Development and future of ultraviolet light-emitting diodes: UV-LED will replace the UV lamp, *Semiconductor Science and Technology* 29(8)(2014) 084004.
- [16] Y. Ding, X. Liu, Z.R. Zheng, P.F. Gu, Freeform LED lens for uniform illumination, *Optics express* 16(17)(2008) 12958-12966.
- [17] K. Wang, F. Chen, Z. Liu, X. Luo, S. Liu, Design of compact freeform lens for application specific light-emitting diode packaging, *Optics Express* 18(2)(2010) 413-425.
- [18] D.F. MacFarlane, C.A. Alonso, Occurrence of nonmelanoma skin cancers on the hands after UV nail light exposure, *Archives of dermatology* 145(4)(2009) 447-449.
- [19] Z.M. Zhu, H. Liu, S.M. Chen, The design of diffuse reflective free-form surface for indirect illumination with high efficiency and uniformity, *IEEE Photonics Journal* 7(3)(2015) 1-10.
- [20] Z. Zhu, Z. Wang, F. Zhang, Design method of diffuse transmission free-form surface combined with collimation lens array, *Lighting Research & Technology* 53(4)(2021) 333-343.
- [21] F. Wu, S. Li, X. Zhang, W. Ye, A design method for LEDs arrays structure illumination, *Journal of Display Technology*, 12(10)(2016) 1177-1184.
- [22] W.T. Chien, C.C. Sun, I. Moreno, Precise optical model of multi-chip white LEDs, *Optics express* 15(12)(2007) 7572-7577.
- [23] I. Moreno, C.C. Sun, Modeling the radiation pattern of LEDs, *Optics express* 16(3)(2008) 1808-1819.
- [24] C.C. Sun, C.Y. Chen, H.Y. He, C.C. Chen, W.T. Chien, T.X. Lee, T.H. Yang, Precise optical modeling for silicate-based white LEDs, *Optics express* 16(24)(2008) 20060-20066.
- [25] J. Jiang, S. To, W.B. Lee, B. Cheung, Optical design of a freeform TIR lens for LED streetlight, *Optik* 121(19)(2010) 1761-1765.

- [26] C. Chen, X. Zhang, Design of optical system for collimating the light of an LED uniformly, *JOSA A* 31(5)(2014) 1118-1125.
- [27] P. Liu, R.M. Wu, Z.R. Zheng, H.F. Li, X. Liu, Optimized design of LED freeform lens for uniform circular illumination, *Journal of Zhejiang University Science C* 13(12)(2012) 929-936.
- [28] X. Deng, X. Liang, Z. Chen, W. Yu, R. Ma, Uniform illumination of large targets using a lens array, *Applied optics* 25(3) (1986) 377-381.
- [29] X.H. Lee, I. Moreno, C.C. Sun, High-performance LED street lighting using microlens arrays, *Optics express* 21(9) (2013) 10612-10621.
- [30] Y. Luo, Z. Feng, Y. Han, H. Li, Design of compact and smooth free-form optical system with uniform illuminance for LED source, *Optics express* 18(9)(2010) 9055-9063.