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Abstract. Dealing with high conflict evidence, traditional evidence theory sometimes has certain limitations, and results in fusion results contrary to common sense. In order to solve the problem of high conflict evidence fusion, this paper analyzes traditional evidence theory and proposes an evidence fusion method that combines cosine distance and information entropy. Cosine distance can measure the directionality between two vectors. The better the directionality, the more similar the two vectors are. Therefore, this article uses cosine distance to determine the similarity between evidences, and then calculates the credibility of each piece of evidence. Information entropy can calculate the amount of information for each evidence. The greater the information entropy to measure the uncertainty of the evidence. Therefore, this article uses information entropy to measure the uncertainty of the evidence. Therefore, this article uses information entropy to measure the uncertainty of the evidence. Therefore, this article uses information entropy to measure the uncertainty of the evidence. Therefore, this article uses information entropy to measure the uncertainty of the evidence. Therefore, this article uses information entropy to measure the uncertainty of the evidence. Then, the credibility and uncertainty of the evidence are fused to calculate the weight of the evidence. Then we use d-s evidence theory for evidence fusion. The numerical example shows that the method is feasible and effective in dealing with conflict evidence.

Keywords: evidence theory, conflicting evidence, cosine distance, information entropy

1 Introduction

In practical applications, D-S evidence theory is widely used in sensor data processing. As a data collector, sensors can provide users with a large amount of data. Due to environmental and other factors, a single sensor often cannot provide accurate data, so in practical applications, multiple sensors are often used to collect data. However, the data collected by multiple sensors are not always consistent, and when faced with conflicting data, it is necessary to use D-S evidence theory for data fusion to obtain more accurate data. D-S evidence theory is widely applied in various fields of information fusion, such as decision making [1-6], pattern recognition [7-9], information fusion [10-11], supplier management [12-14], risk assessment [15-16], fault diagnosis [17-18] and so on [19-20].

The theory of evidence was first proposed by Dempster [21] in 1967 and further developed by his student Shafer [22] in 1976, also known as Dempster/Shafer evidence theory (D-S evidence theory), which belongs to the category of artificial intelligence and was first applied to expert systems with the ability to process uncertain information. Evidence theory has two main characteristics. One is that he can satisfy weaker conditions than Bayesian probability theory. The other is the ability to express "uncertainty" and "unknown" directly. Although the D-S evidence theorem can be used for evidence fusion, anti-intuitive fusion results sometimes occur when used to fuse highly conflicting evidence. To solve this problem, many scholars have proposed improved evidence fusion algorithms. There are two main types of improvement methods. The first type is to adjust the Dempster's combination rule. Changing the way evidence is combined to solve the problem of evidence combination failure, but this can only eliminate some conflicts, and changing the combination rules also destroys the advantages of the original rules. The second type is to preprocess the evidence. Modifying the probability of evidence can preserve the advantages of combination rules, while also eliminating conflicts between evidences to a certain extent. The first type of improved algorithm includes Yager's, combination rule [23]. The second type of improved algorithm includes Murphy's method [24] and Zhang et al.'s method [25].

In this paper, a new evidence fusion algorithm based on cosine distance and information entropy is proposed. This method not only considers the conflict between evidences, but also considers the impact of the evidence itself on the weight. The cosine distance can represent the similarity between evidences. The average similarity of each evidence can be obtained by calculating the average cosine distance. After normalization, the credibility

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of the evidence can be obtained. The credibility of evidence is measured and determined by information entropy, which is related to the evidence itself. The weight of the evidence is obtained by modifying the credibility with the uncertainty of the evidence. On this basis, the average weighted evidence is calculated. The average weighted evidence is fused using the Dempster's combination rule. The rationality and effectiveness of our proposed method are illustrated through numerical examples.

The rest of this article is organized below. Section 2 briefly introduces the basic concepts of evidence theory. In Section 3, the concept of cosine distance is introduced to measure the degree of similarity between the evidence. Section 4 indicates the uncertainty of the evidence by introducing information entropy. Section 5 proposes new approaches to the fusion of evidence. In section 6, two specific examples are used to verify the feasibility of the algorithm proposed in this paper. Section 7 summarizes the full text.

2. D-S Evidence Theory

In this chapter, this article will briefly introduce the basic concepts and formulas of evidence theory.

2.1 D-S Evidence Theory

Definition 1 (Frame of discernment). Suppose $\Omega = \{a_1, a_2, ..., a_n\}$ is a finite and complete set, the elements in the set are mutually exclusive. Ω is referred to as the identification framework. Each element in the recognition framework can be the result of an event. The set of all the possible subsets in Ω composed of the power set denoted by 2^{Ω} , which include 2^{Ω} elements.

$$2^{\Omega} = \{\phi, \{a_1\}, \{a_2\}, \dots, \{a_n\}, \{a_1, a_2\}, \dots, \{a_1, a_2, a_3\}, \dots \{\Omega\}\}.$$
(1)

Where element ϕ represents the empty set.

Definition 2 (Mass function). Assuming that Ω is the identification frame of the research problem. A mass function is a mapping m from 2^{Ω} to [0, 1], defined by:

$$\begin{cases} \sum_{A \in \Omega} m(A) = 1\\ m(\phi) = 0 \end{cases}.$$
 (2)

m is referred to as basic probability distribution. In the Dempster–Shafer evidence theory, a mass function can be called as a basic belief assignment (BBA).

Definition 3 (Belief function). Assuming that Ω is the identification frame of the research problem. The belief function Bel: $2^{\Omega} \rightarrow [0, 1]$ is defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B), \forall A \subseteq \Omega.$$
 (3)

Definition 4 (Dempster's rule of combination). Assume that there are two independent evidence bodies $m_1 = \{A_1, A_2, ..., A_N\}$ and $m_2 = \{A_1, A_2, ..., A_N\}$ in the identification framework. Dempster's rule of combination can be defined as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j), A \neq \phi \\ 0, A = \phi \end{cases}$$
(4)

$$K = \sum_{A_i \cap B_j = \phi} m_1(A_i) m_2(B_j).$$
 (5)

Where K is the conflict coefficient used to indicate the degree of conflict between evidences. The smaller the conflict between the evidences, the smaller the K. On the contrary, the greater the conflict between evidences, the greater the K. The D-S evidence theory synthesis rule can be seen as an orthogonal sum operation of two evidences, and satisfies the commutative and associative laws. It can expand the synthesis of N evidences.

2.2 Problems with D-S Evidence Theory

When there is a significant conflict between evidences, the results obtained using Dempster's rule of combination often violate common sense. This phenomenon is called "Zadeh Paradox". Suppose there are two pieces of evidence X1 and X2 under the identification framework $\Omega = \{A, B, C\}$.

 $X1: m(A_1) = 0.99, \ m(B_1) = 0.01, \ m(C_1) = 0.$

 $X2: m(A_2) = 0, m(B_2) = 0.01, m(C_2) = 0.99.$

We can see that evidence X1 almost fully supports A, while evidence X2 almost fully supports C. Both of them have a low level of support for B. By using Eq. (4), we can get that m(A) = m(C) = 0 m(B) = 1.00. It can be seen that the evidence after fusion fully supports B, which is obviously inconsistent with the facts. If we add another evidence $m(A_3) = 0.99$, $m(B_3) = 0.01$, $m(C_3) = 0$ here, the synthesized result is still A, and the error in the synthesized result has not been corrected. It can be seen that when Dempster's rule of combination is used to process highly conflicting evidence, results that violate common sense may be obtained. High conflict evidence sources or deficiencies in the Dempster's rule of combination can lead to high conflict in the fusion results.

3. Evidence Distance Measure

The cosine distance can also be called cosine similarity. The angle cosine in geometry can be used to measure the difference in the direction of two vectors, and this concept is borrowed in machine learning to measure the difference between sample vectors. Compared to distance measurement, cosine similarity focuses more on the difference in direction between two vectors than on distance or length.

Definition 5 (Cosine distance). Suppose that $P = \{p_1, p_2, ..., p_n\}$ and $Q = \{q_1, q_2, ..., q_n\}$ are two vectors. Then the cosine distance of P to Q is defined as:

$$R_{PQ} = \frac{\sum_{k=1}^{n} p_k \times q_k}{\sqrt{\sum_{s=1}^{n} p_s \times p_s} \times \sqrt{\sum_{s=1}^{n} q_s \times q_s}}.$$
(6)

Definition 6 (Cosine distance between evidences). Define the system identification framework as $\Theta = \{A_1, A_2, A_3, \dots, A_M\}$, The system has a total of N evidence bodies $E_1, E_2, E_3, \dots, E_N$, and their corresponding mass functions are $m_1, m_2, m_3, \dots, m_N$, respectively, where $m_i = \{m_i(A_1), m_i(A_2), m_i(A_3), \dots, m_i(A_M)\}$. Assuming evidence bodies E_i and E_j , the cosine distance between evidence bodies E_i and E_j is:

$$R_{ij} = \frac{\sum_{k=1}^{M} m_i(A_k) \times m_j(A_k)}{\sqrt{\sum_{k=1}^{M} m_i^2(A_k)} \times \sqrt{\sum_{k=1}^{M} m_j^2(A_k)}}.$$
(7)

The cosine value range is [-1, 1]. The angle between the two vectors is obtained, and the cosine value corresponding to the angle is obtained. This cosine value can be used to represent the similarity of the two vectors. The smaller the included angle, the closer it approaches 0 degrees, the closer the cosine value is to 1, and the more consistent their directions are, the more similar they are; When the directions of two vectors are completely opposite, the cosine of the included angle is taken as the minimum value of - 1; When the cosine value is 0, the two vectors are orthogonal and the included angle is 90 degrees.

Example 1: Assume that in the identification framework $\Theta = \{A, B\}$ has three independent evidence bodies m_1, m_2 , and m_3 .

$$m_1 : m_1(A) = 0.7; m_1(B) = 0.3.$$

 $m_2 : m_2(A) = 0.3; m_2(B) = 0.7.$
 $m_3 : m_3(A) = 0.5; m_3(B) = 0.5.$

The cosine distance between the evidences is:

$$R_{12} = \frac{0.7 \times 0.3 + 0.3 \times 0.7}{\sqrt{0.3 \times 0.3 + 0.7 \times 0.7} \times \sqrt{0.7 \times 0.7 + 0.3 \times 0.3}} = 0.724.$$

$$R_{13} = \frac{0.7 \times 0.5 + 0.3 \times 0.5}{\sqrt{0.3 \times 0.3 + 0.7 \times 0.7} \times \sqrt{0.5 \times 0.5 + 0.5 \times 0.5}} = 0.928.$$

$$R_{23} = \frac{0.3 \times 0.5 + 0.7 \times 0.5}{\sqrt{0.3 \times 0.3 + 0.7 \times 0.7} \times \sqrt{0.5 \times 0.5 + 0.5 \times 0.5}} = 0.928.$$

4. Information Entropy

The concept of information entropy was originally proposed by Shannon [26] and used to describe the uncertainty of events. Based on Shannon entropy, Deng proposed a new information entropy called Deng entropy [27] which can be use in evidence theory. Deng entropy can effectively measure uncertain information. In evidence theory, uncertain information is determined by the basic probability distribution. Therefor we can obtain the uncertainty of evidence through the probability distribution of evidence. In this case, using Shannon entropy to calculate the uncertainty is equivalent to using Deng entropy to calculate the uncertainty. The following is an introduction to basic concepts.

Definition 7 (Information entropy). Suppose A_i is a subset of the identification framework, and $|A_i|$ represents the number of elements in subset A_i , then the information entropy Q(m) is:

$$Q_{(m)} = -\sum_{i} m_{(A_i)} \log \frac{m_{(A_i)}}{2^{|A_i|} - 1}.$$
(8)

In this article, the log function takes a bottom of 10. Because when $m_{(A_i)} = 0$, the log function has no meaning. Therefore, when $m_{(A_i)} = 0$ in this article, the probability is modified to $m_{(A_i)} = 1 \times 10^{-12}$. It has been proven that this will not affect the calculation results [28]. When subset contains only one element, the information entropy degenerates to Shannon entropy, and there is:

$$Q(m) = -\sum_{i=1}^{N} m(a) \log_2 m(a).$$
 (9)

Example 2: suppose there is evidence $m_2\{A, B, C\}$ in the recognition framework Θ

$$m_1: m_1(A) = 0.5; \ m_1(B) = 0.2; \ m_1(C) = 0.3$$

$$Q(m_1) = -0.5 \log \frac{0.5}{2^1 - 1} - 0.2 \log \frac{0.2}{2^1 - 1} - 0.3 \log \frac{0.3}{2^1 - 1} = 0.4472.$$

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Example 3: suppose there is evidence $m_1\{A, B, AC\}$ in the recognition framework Θ

$$m_2: m_2(A) = 0.3; m_2(B) = 0.4; m_2(AC) = 0.3.$$

$$Q(m_1) = -0.3\log\frac{0.3}{2^1 - 1} - 0.4\log\frac{0.4}{2^1 - 1} - 0.3\log\frac{0.3}{2^2 - 1} = 0.6160.$$

5. The Proposed Method

This paper proposes an evidence fusion algorithm based on evidence credibility and uncertainty. Credibility is calculated by introducing cosine distance. The greater the cosine distance between an evidence and other evidence, the smaller the conflict between the evidence and other evidence, and greater weight should be given to this evidence. On the contrary, when the cosine distance between one evidence and the rest of the evidence is small, it indicates that there is a significant conflict between this evidence and the rest of the evidence. Therefore, it is necessary to give this evidence a small weight. Secondly, information entropy is used to calculate the uncertainty of evidence. Then, the credibility and uncertainty of the evidence are combined to calculate the weight of each evidence, and the average weighted evidence is calculated. Finally, the average weighted evidence is fused using the Dempster combination rule to obtain the final result. Fig. 1 illustrates the flowchart of the proposed method.

5.1 Calculate the Credibility Degree of the Evidences

Step 1-1: By making use of the Cosine distance measure Eq. (7), the cosine distance between the bodies of evidences m_i (i = 1, 2, 3, ..., N) and m_j (j = 1, 2, 3, ..., N) can be obtained. After that, construct the cosine distance matrix between evidences. A distance measure matrix $T = (R_{ij})_{N \times N}$ can be constructed as follows:

$$T = \begin{bmatrix} 1 & R_{12} & \cdots & R_{1j} & \cdots & R_{1N} \\ R_{21} & 1 & \cdots & R_{2j} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{i1} & R_{i2} & \cdots & 1 & \cdots & R_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{Nj} & \cdots & 1 \end{bmatrix}.$$
(10)

Step 1-2: The average evidence distance S_i of the body of evidence m_i can be calculated by:

$$S_{i} = \frac{\sum_{j=1, j \neq i}^{N} R_{ij}}{N-1}, \ 1 \le i \le N; \ 1 \le j \le N.$$
(11)

Step 1-3: The credibility degree B_i of the body of evidence m_i is defined as follows:

$$B_{i} = \frac{S_{i}}{\sum_{k=1}^{N} s_{k}}, \ 1 \le i \le N.$$
(12)

5.2 Measure the Information Volume of the Evidences

Step 2-1: The belief entropy of the evidence m_i (i = 1, 2, 3, ..., N) is calculated by leveraging Eq. (8).

Step 2-2: In order to avoid allocating zero weight to the evidences in some cases, the information volume U_{m_i} of the evidence m_i can be obtained as below:

$$U_{m_i} = e^{\mathcal{Q}(m_i)} = e^{-\sum_i m_{(A_i)} \log \frac{m_{(A_i)}}{2^{|A_i|} - 1}}, \ 1 \le i \le N.$$
(13)

Step 2-3: The information volume U_{m_i} of the evidence m_i is normalized as below, which is denoted as V_i :

$$V_{i} = \frac{U_{m_{i}}}{\sum_{s=1}^{N} U_{m_{s}}}, \ 1 \le i \le N.$$
(14)

5.3 Generate and Fuse the Weighted Average Evidence

Step 3-1: Based on the uncertainty V_i , the credibility degree B_i of the evidence m_i will be adjusted, denoted as Ard_i :

$$Ard_i = V_i \times B_i, \ 1 \le i \le N.$$
⁽¹⁵⁾

Step 3-2: The adjusted credibility degree which is denoted as $\overline{Ard_i}$ is normalized that is considered as the final weight in terms of each evidence m_i :

$$\overline{Ard_i} = \frac{Ard_i}{\sum_{s=1}^{N} Ard_i}, \ 1 \le i \le N.$$
(16)

Step 3-3: The weighted average evidence WAE(m) can be obtained as follows:

$$WAM_{(m)} = \sum_{i=1}^{N} \overline{Ard_1} \times m_i, \ 1 \le i \le N.$$
(17)

Step 3-4: The weighted average evidence WAE(m) is fused via the Dempster's combination rule Eq. (4) by k-1 times, if there are k number of evidences. Finally, the result of evidence fusion is obtained.

5.4 Algorithm Framework

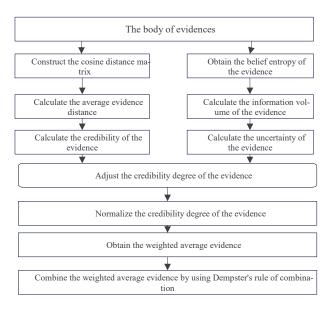


Fig. 1. The flowchart of the proposed method

6. Experiment Analysis

In order to verify the computational effectiveness of the algorithm proposed in this article, this section conducts algorithm analysis through two specific cases. The method used in this article will be compared with algorithms proposed in other literature to demonstrate the superiority of this algorithm.

6.1 Experiment 1

Assume that there is a multi-sensor target recognition problem, each sensor can display the relevant data of the target type being detected. There are three targets in the identification framework Θ , including A, B and C. The system has collected different data from four sensors. The basic probability distribution of each sensor reading is shown in Table 1.

Body	А	В	С
m1	0.41	0.29	0.3
m2	0	0.9	0.1
m3	0.6	0.1	0.3
m4	0.8	0.1	0.1

Table 1. Basic probability distribution of evidences

From the Table 1, we can intuitively see that overall, the evidence has a higher probability of determining A. Next, we will use the improved algorithm proposed in this article to fuse evidence and compare the results with those obtained by other algorithms. The calculation process is as follows:

Step1: Calculate the cosine distance between each piece of evidence and construct a cosine distance matrix $T = (R_{ij})_{4\times 4}$ as follows:

$$T = \begin{bmatrix} 1.0000 & 0.3762 & 0.9806 & 0.8986 \\ 0.3762 & 1.0000 & 0.1954 & 0.1359 \\ 0.9806 & 0.1954 & 1.0000 & 0.9437 \\ 0.8986 & 0.1359 & 0.9437 & 1.0000 \end{bmatrix}$$

Step2: Calculate the average cosine distance S_i between pieces of evidence as follows:

$$S_1 = 0.7518, \ S_2 = 0.2358, \ S_3 = 0.7066, \ S_4 = 0.6594.$$

Step3: Normalize the average cosine distance to obtain the credibility B_i of each evidence as follows:

$$B_1 = 0.3194, B_2 = 0.1002, B_3 = 0.3002, B_4 = 0.2802.$$

Step4: Calculate the information entropy $Q_{(m_i)}$ of each evidence as follows:

$$Q_{(m_1)} = 0.4472, \ Q_{(m_2)} = 0.1412, \ Q_{(m_3)} = 0.3900, \ Q_{(m_4)} = 0.2775.$$

Step5: Calculate the information volume U_{m_i} of the evidence m_i as follows:

$$U_{m_1} = 1.5639, \ U_{m_2} = 1.1517, \ U_{m_3} = 1.4770, \ U_{m_4} = 1.3198.$$

Step6: Calculate the uncertainty V_i of each evidence as follows:

$$V_1 = 0.2837, V_2 = 0.2089, V_3 = 0.2679, V_4 = 0.2394.$$

Step7: Integrate the credibility and uncertainty of each evidence as follows:

$$Ard_1 = 0.0906, Ard_2 = 0.0209, Ard_3 = 0.0804, Ard_4 = 0.0671.$$

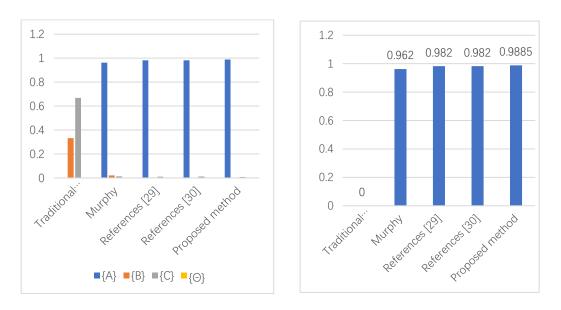
Step8: Calculate the adjusted credibility degree $\overline{A}rd_i$ of each evidence as follows:

$$\overline{Ard_1} = 0.3498, \ \overline{Ard_2} = 0.0807, \ \overline{Ard_3} = 0.3104, \ \overline{Ard_4} = 0.2591$$

Step9: Compute the weighted average evidence as follows:

$$m(\{A\}) = 0.5684, \ m(\{B\}) = 0.1995, \ m(\{C\}) = 0.2320.$$

The weighted averaged mass function is combined with Dempster's combination rule for 3 iterations, and the combined results are shown in Table 2, Fig. 2.



(a) BBAs for different objectives



Fig. 2. The comparison of the fusion results in different methods

Method	m1, m2	m ₁ , m ₂ , m ₃	m ₁ , m ₂ , m ₃ , m ₄
Traditional	m(A)=0	m(A)=0	m(A)=0
D-S methods	m(B)=0.8571	m(B)=0.6667	m(B)=0.3321
	m(C)=0.1429	m(C)=0.3333	m(C)=0.6679
Murphy	m(A)=0	m(A)=0	m(A)=0.2062
	m(B)=0.1543	m(B)=0.3912	m(B)=0.7864

Table 2. Fusion results of different algorithms

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	m(C)=0.7469	m(C)=0.5079	m(C)=0.1752
	m(Θ)=0.0988	m(Θ)=0.1008	m(Θ)=0.0251
Reference [29]	m(A)=0.1176	m(A)=0.5086	m(A)=0.8899
	m(B)=0.6013	m(B)=0.2370	m(B)=0.0540
	m(C)=0.1398	m(C)=0.2543	m(C)=0.0561
	m(Θ)=0.1414	$m(\Theta)=0$	$m(\Theta)=0$
Reference [30]	m(A)=0.2551	m(A)=0.6758	m(A)=0.9431
	m(B)=0.5000	m(B)=0.0960	m(B)=0.0169
	m(C)=0.2449	m(C)=0.2282	m(C)=0.0399
Proposed method	m(A)=0.7749	m(A)=0.8988	m(A)=0.9565
	m(B)=0.0955	m(B)=0.0389	m(B)=0.0145
	m(C)=0.9565	m(C)=0.0145	m(C)=0.0265

6.2 Discussion

As shown in Table 2, of the five algorithms, the Dempster's combination rule takes C as the target. Murphy's method [24] takes B as the target. Whereas, Reference [29], Reference [30] and the proposed method present reasonable results and recognize the target A.

Among all the algorithms, the algorithm proposed in this article has the highest decision probability (95.65%), indicating that the method has the best effect. Among the remaining algorithms, Reference [30] have the medium decision probability (94.31%), References [29] has the lowest decision probability of 88.99%.

From this example, it can be seen that the Dempster's combination rule cannot handle conflict evidence, and may result in results that do not match the actual situation. Although the Murphy's method has improved compared to the Dempster's combination rule, it still cannot yield correct results. References [29] and Reference [30] can select the correct target with a greater probability, but they are not as superior as the methods proposed in this article.

6.3 Experiment 2

In order to further verify the superiority of the algorithm proposed in this paper, we conducted a more complex experiment with reference to Experiment 1. Assuming there are five sensors in a multi-sensor system and the recognition framework Θ contains four targets which include A, B, C, AC. The basic probability distribution of each sensor reading is shown in Table 3.

Body	Α	В	C	AC
\mathbf{m}_1	0.41	0.29	0.30	0.00
m ₂	0.00	0.90	0.10	0.00
m ₃	0.58	0.07	0.00	0.35
m_4	0.55	0.10	0.00	0.35
m ₅	0.60	0.10	0.00	0.30

Table 3. Basic probability distribution of each sensor reading

Next, we use the method proposed in this article to fuse these five pieces of evidence and compare them with the results obtained by other algorithms. The calculation process is as follows:

Step1: Calculate the cosine distance between each piece of evidence and construct a cosine distance matrix $T = (R_{ij})_{5\times 5}$ as follows:

$$T = \begin{bmatrix} 1.0000 & 0.5493 & 0.6479 & 0.6596 & 0.6931 \\ 0.5493 & 1.0000 & 0.1022 & 0.1507 & 0.1465 \\ 0.6479 & 0.1022 & 1.0000 & 0.9985 & 0.9959 \\ 0.6596 & 0.1507 & 0.9985 & 1.0000 & 0.9948 \\ 0.6931 & 0.1465 & 0.9959 & 0.9948 & 1.0000 \end{bmatrix}.$$

Step2: Calculate the average cosine distance S_i between pieces of evidence as follows:

$$S_1 = 0.6375, S_2 = 0.2372, S_3 = 0.6861, S_4 = 0.7009, S_5 = 0.7076.$$

Step3: Normalize the average cosine distance to obtain the credibility B_i of each evidence as follows:

$$B_1 = 0.2174, B_2 = 0.0799, B_3 = 0.2311, B_4 = 0.2360, B_5 = 0.2383$$

Step4: Calculate the information entropy $Q_{(m_i)}$ of each evidence as follows:

$$Q_{(m_1)} = 0.4715, \ Q_{(m_2)} = 0.1412, \ Q_{(m_3)} = 0.5446, \ Q_{(m_4)} = 0.5694, \ Q_{(m_5)} = 0.5331.$$

Step5: Calculate the information volume U_{m_i} of the evidence m_i as follows:

$$U_{m_1} = 1.6024, \ U_{m_2} = 1.1517, \ U_{m_3} = 1.7239, \ U_{m_4} = 1.7672, \ U_{m_5} = 1.7042.$$

Step6: Calculate the uncertainty V_i of each evidence as follows:

$$V_1 = 0.2016, V_2 = 0.1449, V_3 = 0.2169, V_4 = 0.2223, V_5 = 0.2144.$$

Step7: Integrate the credibility and uncertainty of each evidence as follows:

$$Ard_1 = 0.0433, Ard_2 = 0.0116, Ard_3 = 0.0501, Ard_4 = 0.0525, Ard_5 = 0.0511.$$

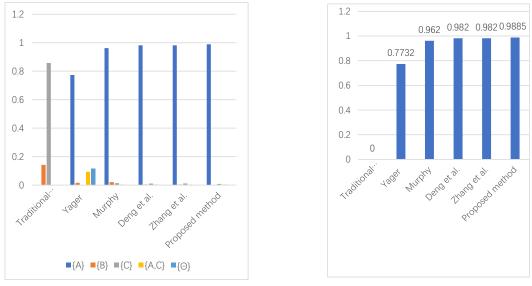
Step8: Calculate the adjusted credibility degree \overline{Ard}_i of each evidence as follows:

$$\overline{Ard_1} = 0.2076, \ \overline{Ard_2} = 0.0556, \ \overline{Ard_3} = 0.2402, \ \overline{Ard_4} = 0.2517, \ \overline{Ard_5} = 0.2450$$

Step9: Compute the weighted average evidence as follows:

$$m(A) = 0.5099, m(B) = 0.1767, m(C) = 0.0678, m(A, C) = 0.2457.$$

Finally, the weighted averaged mass function is combined with Dempster's combination rule for 4 iterations, and the combined results are shown in Fig. 3 and Table 4.



(a) BBAs for different objectives

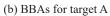


Fig. 3. The	comparison	of the	fusion	results ir	different	methods
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Method	m1, m2	m ₁ , m ₂ , m ₃	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
Traditional	m(A)=0	m(A)=0	m(A)=0	m(A)=0
D-S methods	m(B)=0.8969	m(B)=0.6575	m(B)=0.3321	m(B)=0.1422
	m(C)=0.1031	m(C)=0.3425	m(C)=0.6679	m(C)=0.8578
Yager	m(A)=0	m(A)=0.4112	m(A)=0.6508	m(A)=0.7732
c	m(B)=0.2610	m(B)=0.0679	m(B)=0.0330	m(B)=0.0167
	m(C)=0.0300	m(C)=0.0105	m(C)=0.0037	m(C)=0.0011
	m(AC)=0	m(AC)=0.2481	m(AC)=0.1786	m(AC)=0.0938
	m(Θ)=0.7090	m(Θ)=0.2622	m(Θ)=0.1339	m(Θ)=0.1152
Murphy	m(A)=0.0964	m(A)=0.4619	m(A)=0.8362	m(A)=0.9620
	m(B)=0.8119	m(B)=0.4497	m(B)=0.1147	m(B)=0.0210
	m(C)=0.0917	m(C)=0.0794	m(C)=0.0410	m(C)=0.0138
	m(AC)=0	m(AC)=0.0090	m(AC)=0.0081	m(AC)=0.0032
Deng et al.	m(A)=0.0964	m(A)=0.4674	m(A)=0.9089	m(A)=0.9820
	m(B)=0.8119	m(B)=0.4054	m(B)=0.0444	m(B)=0.0039
	m(C)=0.0917	m(C)=0.0888	m(C)=0.0379	m(C)=0.0107
	m(AC)=0	m(AC)=0.0084	m(AC)=0.0089	m(AC)=0.0034
Zhang et al.	m(A)=0.0964	m(A)=0.5681	m(A)=0.9142	m(A)=0.9820
	m(B)=0.8119	m(B)=0.3319	m(B)=0.0395	m(B)=0.0034
	m(C)=0.0917	m(C)=0.0929	m(C)=0.0399	m(C)=0.0115
	m(AC)=0	m(AC)=0.0084	m(AC)=0.0083	m(AC)=0.0032
Proposed method	m(A)=0.7979	m(A)=0.9203	m(A)=0.9690	m(A)=0.9885
	m(B)=0.0488	m(B)=0.0122	m(B)=0.0029	m(B)=0.0007
	m(C)=0.0593	m(C)=0.0353	m(C)=0.0181	m(C)=0.0086
	m(AC)=0.0943	m(AC)=0.0328	m(AC)=0.0110	m(AC)=0.0036

Table 4. Fusion results of different algorithms

6.4 Discussion

As shown in Table 4, of the six algorithms, only the Dempster's combination rule takes C as the target, which is obviously contrary to people's intuition. Whereas, Murphy's method [24], Deng et al.'s method [25], Zhang et al.'s method [31], Yager's method [23] and the proposed method present reasonable results and recognize the target A.

Among all the algorithms, the algorithm proposed in this article has the highest decision probability (98.85%), indicating that the method has the best effect. Among the remaining algorithms, Zhang et al.'s method [31] and Deng et al.'s method [25] have the same decision probability (98.20%), Murphy's method [24] has a lower decision probability of 96.20%, while Yager's method [23] has the lowest decision probability (77.32%).

The reason is that the algorithm proposed in this paper not only uses cosine distance to calculate the credibility of evidence, but also uses information entropy to calculate the credibility of evidence. The evidence weight obtained by combining these two aspects is more reasonable than other algorithms. Therefore, after evidence fusion, the improved algorithm obtains better results than other algorithms.

7 Conclusion

In this article, we briefly introduce the basic concepts of evidence theory, cosine distance, and information entropy, and propose an improved conflict evidence fusion algorithm based on cosine distance and information entropy. First, by calculating the cosine distance between each evidence, we obtain the cosine distance matrix, and calculate the average cosine distance between the evidence. Normalizing the average cosine distance of evidence can obtain the credibility of the evidence. Secondly, calculate the information entropy of each evidence itself, and use the information entropy of evidence as an indicator to measure the uncertainty of evidence. Normalizing the information entropy of evidence can obtain the credibility of evidence. Then, the evidence credibility and uncertainty are fused to obtain the corrected evidence credibility. Based on the revised credibility of the evidence, we can calculate the average evidence. Using the Dempster's combination rule to perform multiple rounds of fusion of average evidence can obtain the final result. According to the two numerical examples presented in this article, the algorithm proposed in this article can well fuse conflict evidence and has a higher decision probability compared to other algorithms that fuse conflict evidence.

Considering the high superiority of the algorithm proposed in this paper in handling conflict evidence, we will apply this algorithm to more classic conflict cases in future work. The algorithm proposed in this paper is further improved in experiments.

References

- A. Si, S. Das, S. Kar, RETRACTED ARTICLE: Picture fuzzy set-based decision-making approach using Dempster– Shafer theory of evidence and grey relation analysis and its application in COVID-19 medicine selection, Soft Computing 27(6)(2023) 3327-3341.
- [2] Z. Wan, M. Shi, F. Yang, G. Zhu, A novel pythagorean group decision-making method based on evidence theory and interactive power averaging operator, Complexity 2021(2021) 9964422.
- S. Zhang, F. Xiao, A TFN-based uncertainty modeling method in complex evidence theory for decision making, Information Sciences 619(2023) 193-207.
- [4] P. Liu, X. Zhang, A new hesitant fuzzy linguistic approach for multiple attribute decision making based on Dempster– Shafer evidence theory, Applied Soft Computing Journal 86(2020) 105897.
- [5] H. Xu, Y. Deng, Dependent evidence combination based on shearman coefficient and pearson coefficient, IEEE Access 6(1)(2018) 11634-11640.
- [6] J. Zheng, W. Wu, H. Bao, A. Tan, Evidence theory based optimal scale selection for multi-scale ordered decision systems, International Journal of Machine Learning and Cybernetics 13(4)(2022) 1115-1129.
- [7] X. Wang, T. Wang, Research on Face Recognition Algorithm Based on D-S Evidence Theory and Local Domain Pattern, Proceedings - 2021 International Conference on Intelligent Computing, Automation and Applications, ICAA 2021(2021) 261-266.
- [8] W. Wang, M. Zhao, H. Gao, S. Zhu, J. Qu, Human-computer interaction: Intention recognition based on EEG and eye tracking, Hangkong Xuebao/Acta Aeronautica et Astronautica Sinica 42(2)(2021) 286-296.
- [9] W. Wang, F. Chang, Y. Liu, X. Wu, Expression recognition method based on evidence theory and local texture, Multimedia Tools and Applications 76(5)(2017) 7365-7379.

- [10] L. Pan, Y. Deng, D. Pelusi, A Similarity Measure of Complex Evidence Theory for Multi-Source Information Fusion, SSRN (2022).
- [11] X. Zhang, Y. Ma, Y. Li, C. Zhang, C. Jia, Tension prediction for the scraper chain through multi-sensor information fusion based on improved Dempster-Shafer evidence theory, Alexandria Engineering Journal 64(2023) 41-54.
- [12] C.L. Green, Identifying Supplier Management Best Practices to Sustain Organization Resilience: A Systematic Review, [dissertation] University pf Maryland Global Campus, 2022.
- [13] L. Li, H. Wang, A green supplier assessment method for manufacturing enterprises based on rough ANP and evidence theory, Information (Switzerland) 9(7)(2018) 162.
- [14] L. Li, H. Wang, A parts supplier selection framework of mechanical manufacturing enterprise based on D-S evidence theory, Journal of Algorithms and Computational Technology 12(4)(2018) 333-341.
- [15] T. Liu, Y. Zhou, J. Bao, X. Wang, P. Zhang, A Novel Approach to Ship Operational Risk Analysis Based on D-S Evidence Theory, Communications in Computer and Information Science 1449(2021) 728-741.
- [16] J. Ge, A. Wan, B. Peng, G. Wei, Risk evaluation of energy investment projects along the Belt and Road based on cloud model and evidence theory, Journal of Renewable and Sustainable Energy 14(5)(2022) 055903.
- [17] R. Liu, S. Cui, A. Lin, Y. Liao, Transformer Fault Synthetic Diagnosis Method Based on Fusion of Multi-Neural Networks and Evidence Theory in Cloud Computing, Journal of Physics: Conference Series 2433(1)(2023) 012031.
- [18] Y.-S. Wang, T.-S. Sun, A method of actuator fault diagnosis based on evidence fusion, Kongzhi yu Juece/Control and Decision 37(8)(2022) 2026-2032.
- [19] Y. Su, X. Gao, H. Qian, X. Su, Handling Uncertainty in Human Cognitive Reliability Method for Safety Assessment Based on DSET, CMES - Computer Modeling in Engineering and Sciences 132(1)(2022) 201-214.
- [20] X. Zhou, X. Deng, Y. Deng, S. Mahadevan, Dependence assessment in human reliability analysis based on D numbers and AHP, Nuclear Engineering and Design 313(2017) 243-252.
- [21] A.P. Dempster, Upper and Lower Probabilities Induced by a Multivalued Mapping, Annals of Mathematics 38(2)(1967) 325-339.
- [22] G. Shafer, A mathematical theory of evidence, Technometrics 20(1)(1978) 106.
- [23] R.R. Yager, On the dempster-shafer framework and new combination rules, Information Sciences 41(2)(1987) 93-137.
- [24] C.K. Murphy, Combining belief functions when evidence conflicts, Decision Support Systems 29(1)(2000) 1-9.
- [25] Y. Deng, W. Shi, Z. Zhu, Q. Liu, Combining belief functions based on distance of evidence, Decision Support System 38(3)(2004) 489-493.
- [26] C.E. Shannon, A mathematical theory of communication, ACM SIGMOBILE Mobile Computing and Communications Review 5(1)(2001) 3-55.
- [27] Y. Deng, Deng entropy, Chaos, Solitons&Fractals 91(2016) 549-553.
- [28] X.Z. Guo, X.L. Xin, Partial entropy and relative entropy of fuzzy sets, Fuzzy Systems and Mathematics 19(2)(2005) 97-102.
- [29] D.-Q. Han, Y. Deng, C.-Z. Han, Z.-Q. Hou, Weighted evidence combination based on distance of evidence and uncertainty measure, Hongwai Yu Haomibo Xuebao/Journal of Infrared and Millimeter Waves 30(5)(2011) 396-400+468.
- [30] Y. Meng, L. Xu, M. Ren, Y. Wang, D-S Evidence Fusion Method Based on High Conflict Correction, Computer Engineering 44(1)(2018) 79-83+90.
- [31] Z. Zhang, T. Liu, D. Chen, W. Zhang, Novel algorithm for identifying and fusing conflicting data in wireless sensor networks, Sensors 14(6)(2014) 9562-9581.