Using Ordinal Representation for Generating Permutations with a Fixed Number of Inversions in Lexicographic Order

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Abstract. An *inversion* occurs between a pair of (π_j, π_k) in a permutation $\pi = (\pi_1\pi_2\cdots\pi_n)$ of {1, 2,…, *n*}, if $j < k$ but $\pi_j > \pi_k$. By using a new representation scheme of permutations, called *ordinal representation*, we propose an algorithm for generating, in *lexicographic* order, the set of all permutations of {1, 2,…, *n*} with a fixed number of *inversions m*, where $0 \le m \le C_2^n$. Then, we derive a theorem that can be used to guarantee that the proposed algorithm is optimal, meaning that it will never visit any of the unqualified permutations. The beauty of the new representation scheme lies not in the result itself, but rather in its arithmetical ability and its wide applicability.

Keywords: inversion, lexicographic order, ordinal representation, permutation.

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