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Research on the Optimal Configuration of Distributed Energy Storage in Distribution Networks Based on Improved Benders Decomposition Method

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Abstract. This paper proposes a method for optimizing the configuration of distributed energy storage systems (DESS) in distribution networks based on an improved Benders decomposition method. This framework overcomes the limitations of traditional Benders decomposition in terms of convergence speed and computational complexity by combining a dynamic cutting strategy and an adaptive relaxation technique. These improvements enable the algorithm to optimize the solution space more efficiently and simplify the subproblem calculations without compromising accuracy. Extensive simulation results on IEEE 33-node and 69-node test systems demonstrate that this method significantly reduces operating costs, enhances voltage stability, and improves the overall efficiency of power flow management. In addition, the method exhibits strong scalability and robustness in networks with different renewable energy penetration rates. The results confirm that the improved algorithm not only accelerates convergence speed but also achieves the optimal configuration of energy storage, thereby increasing the utilization rate of renewable energy and improving the economic efficiency of modern distribution network operation.

Keywords: distributed energy storage systems, benders decomposition, optimization, renewable energy penetration, operational costs, computational efficiency

1 Introduction

With the ongoing global shift toward renewable energy, integrating distributed renewable sources like wind and solar into power systems has become increasingly important. However, the integration of these renewable energy sources into distribution networks creates several challenges, particularly regarding power fluctuations, voltage stability, and load imbalances [1]. Distributed Energy Storage Systems (DESS) are increasingly used to address these challenges, serving as a flexible means of balancing renewable energy generation, regulating grid loads, enhancing voltage quality, reducing operational costs, and improving grid reliability and efficiency [2]. While DESS offers significant potential, optimizing their capacity, layout, and scheduling remains a difficult task in practice. Most research on DESS optimization in distribution networks focuses on using various algorithms such as dynamic programming, genetic algorithms, and particle swarm optimization [3]. However, these traditional approaches often struggle with long computation times, local optima, and handling uncertainties in large-scale networks. The Benders decomposition method, which has been applied to optimize power systems, works by breaking large, complex problems into main and sub-problems [4]. However, when applied to DESS optimization in distribution networks, traditional Benders decomposition often suffers from slow convergence and high computational costs [5]. To address these issues, this paper proposes an enhanced version of Benders decomposition aimed at improving solution efficiency and optimizing DESS configurations. Key innovations include: firstly, refining the Benders decomposition method by introducing new cutting strategies and relaxation techniques, which improve convergence speed and reduce resource usage. Secondly, adding an adaptive adjustment mechanism that tailors the solution process based on distribution network configurations, energy storage capacities, and renewable energy penetration. Thirdly, considering multiple time scales and

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uncertainties, the paper introduces a robust optimization approach for DESS that can handle load fluctuations and variable renewable energy generation. Finally, the improved algorithm is tested through numerical experiments, confirming its superior performance in terms of grid operation costs, voltage stability, and energy storage configuration, demonstrating its efficiency in large-scale distribution networks. Santos [6] highlight the importance of dynamic distribution system reconfiguration (DDSR) to facilitate greater integration of DRES and Energy Storage Systems (ESS). Their model, which takes into account the cost of emissions and switchgear degradation, introduces a coordination strategy that significantly reduces energy demand by integrating DNSS, DRES, and ESS technologies. This approach provides a foundation for optimizing distribution networks to better accommodate renewable energy sources. Further elaborating on the integration challenges of DRES, particularly photovoltaics (PV), Sharma et al. [7] propose an economic planning method aimed at improving voltage quality and grid efficiency. Their dual-layer optimization model, using enhanced genetic algorithms, addresses the intermittent nature of PV generation and emphasizes the critical role of DESS in stabilizing grid operations. Ahmadi Ahangar et al. [8] offer a location-based methodology for sizing BESS for voltage regulation in MV/LV distribution networks. By analyzing time-series load data and renewable energy generation profiles, their approach identifies optimal locations for BESS deployment to mitigate over voltage issues. This research contributes to understanding how strategically placed DESS can improve voltage profiles in distribution networks affected by renewable energy fluctuations. While DESS holds great promise in addressing the challenges of renewable energy integration, optimizing their capacity, placement, and scheduling remains a complex issue, particularly in terms of grid reliability and efficiency [9]. Babiker et al. [10] work on a distributed robust optimization method for AC/MTDC hybrid power systems, which includes AC/DC optimal power flow models and GBD, is relevant for advancing DESS optimization strategies in the context of renewable energy integration. These studies collectively contribute to the evolving field of distribution network optimization, emphasizing the vital role of DESS in ensuring grid stability, reliability, and efficiency as renewable energy becomes more prevalent. The schematic diagram of the network structure of distributed energy storage is shown in Fig. 1.

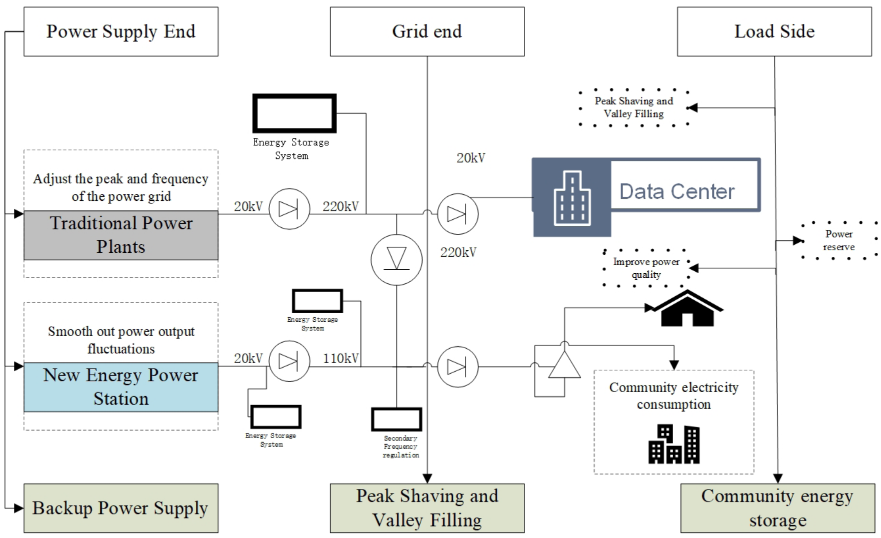

Fig. 1. Schematic diagram of distributed energy storage network structure

This paper first points out in the first part that the integration of renewable energy into the distribution network brings about problems such as power fluctuations, voltage stability, and load imbalance. The DESS is the key solution method, but traditional optimization algorithms face limitations such as high computational complexity and the tendency to get stuck in local optima in large-scale scenarios. Therefore, the motivation for this study to adopt and improve the Benders decomposition method is introduced. In Section 2, a node-branch mathematical model of the distribution network is established, and a DESS optimization model with the goal of "minimum system total cost" is constructed. The constraints include power balance, the evolution of energy storage SOC, and capacity constraints. In the third part, the main problem/sub-problem framework and mathematical form of the traditional Benders decomposition are systematically reviewed, and the core idea of "improved Benders decomposition” is proposed: through dynamic cutting strategies, constraint relaxation, and customized algorithms tailored to the characteristics of the distribution network to accelerate convergence, reduce the complexity of sub-problems, and provide the iterative steps of the improved algorithm. In the fourth part, the example design and simulation platform are introduced. Two standard distribution networks, IEEE 33 nodes and 69 nodes, are selected as the test systems, and time series scenarios containing wind and solar power output and typical loads are constructed. The Benders iteration of the main problem MILP and the sub-problem LP/QP is implemented in MATLAB. In the fifth part, the results are analyzed: the spatial distribution characteristics of the optimal storage location and capacity determination in the 33-node and 69-node systems are presented successively, indicating that the storage configuration is influenced by load, voltage sensitivity, and renewable fluctuations. The scatter relationship between the storage capacity and the load at each node shows that the capacity decision is the result of a multi-factor comprehensive weighing. The running costs of the improved Benders and genetic algorithms during the iterative process are compared, proving that the former converges faster and has lower costs. Further analysis is conducted on the voltage stability and network loss changes under different renewable penetration rates, demonstrating that in modernized and strengthened power grids, by reasonable regulation and storage configuration, good voltage and lower network losses can be maintained at high penetration levels. The convergence stability and robustness of the improved algorithm under different penetration rate conditions are also demonstrated. Finally, in Section 6, the conclusion is drawn: the proposed improved Benders decomposition framework significantly improves computational efficiency and convergence speed while ensuring solution accuracy. It can effectively achieve the optimal configuration of DESS in large-scale distribution networks, reduce operating costs, improve grid stability and the utilization rate of renewable energy, and has good engineering scalability and lays a methodological foundation for the future introduction of uncertainties.

2 Problem Formulation and Assumptions

Mathematical modeling of distribution networks aims to capture their electrical and structural characteristics, including nodes, branches, and key components such as transformers, switches, and capacitors, to ensure voltage stability and operational flexibility. Each node represents a physical location, such as a substation or load center, while branches represent the transmission lines that enable power transfer between nodes. The steady-state operation of the network is governed by Kirchhoff’s laws, which are typically expressed through power flow and voltage balance relationships. These relationships are often linearized using DC power flow approximations to simplify the computation without significantly compromising accuracy. Based on this framework, the optimization of DESS focuses on improving system reliability, reducing operating costs, and supporting the integration of renewable energy. The model’s objective is to minimize total system costs subject to constraints such as power balance, voltage stability, and storage capacity limitations.

2.1 Distribution Network Models

Mathematical models of distribution networks aim to capture the electrical and topological characteristics of the system, including nodes, branches, and various electrical components [11]. In a typical medium-voltage or low-voltage distribution network, each node represents a physical electrical location, such as a substation, distributed generator, or load center; branches correspond to feeders or transmission lines that connect these nodes and allow power to flow between them. The network also includes various auxiliary components, such as transformers, switches, capacitors, and circuit breakers, which are crucial for voltage regulation, protection, and operational flexibility. To describe the power flow in such a system, the distribution network is typically represented by a set of nonlinear algebraic equations based on Kirchhoff’s current and voltage laws. For simplicity, these equations can be linearized using a DC power flow approximation under specific operating conditions, which is computationally more efficient and sufficiently accurate for planning and optimization problems. The power balance equation at node i is as follows:

$$P_i^G(t) - P_i^L(t) - P_i^S(t) = 0, \forall i, t \tag{1}$$

Where $P_i^G(t)$ denotes the generated power at node, $P_i^L(t)$ represents the load demand, and $P_i^S(t)$ denotes the charging or discharging power of the storage system. The voltage constraint ensures stable operation within the acceptable voltage range:

$$V_i^{min} \leq V_i(t) \leq V_i^{max}, \forall i, t \tag{2}$$

Where V_i^{min} and V_i^{max} represent the lower and upper bounds of node voltage, respectively. The branch power flow between node *i* and node *j* can be represented by the linearized DC power flow model:

$$P_{ij}(t) = \frac{V_i(t) - V_j(t)}{X_{ij}} \tag{3}$$

Where X_{ij} is the reactance of the line connecting nodes *i* and *j*. These equations together define the steady-state behavior of the distribution network and form the physical foundation for subsequent optimization of energy storage placement and operation.

2.2 Energy Storage Optimization Model

The optimization of DESS in a distribution network aims to enhance system reliability, reduce operational costs, and facilitate the integration of renewable energy [12]. The objective function typically minimizes the total system cost, including generation cost, storage operation cost, and transmission losses:

$$minC = \sum_t (\sum_i c_i^G P_i^G(t) + \sum_i c_i^S P_i^S(t)) \tag{4}$$

The constraints include: Power balance constraint:

$$P_i^G(t) + P_i^S(t) = P_i^L(t) + \sum_j P_{ij}(t) \tag{5}$$

Storage capacity constraint, describing the dynamic state of charge (SOC) of each energy storage device:

$$E_i(t+1) = E_i(t) + \eta_c P_i^{ch}(t) \frac{1}{\eta_d} P_i^{dis}(t) \tag{6}$$

Where $E_i(t)$ is the stored energy at time *t*, $P_i^{ch}(t)$ and $P_i^{dis}(t)$ denote charging and discharging power, and η_c represent charging and discharging efficiencies, respectively. To solve the resulting large-scale mixed-integer nonlinear optimization problem (MINOP), this study adopts the Benders decomposition method, which separates the original problem into a master problem and several sub-problems. The master problem determines the optimal configuration of storage units, while the sub-problems handle the operation scheduling under given configurations. This decomposition significantly enhances computational efficiency, particularly for multi-period problems with coupling constraints, enabling scalable optimization for complex distribution networks.

3 Improved Benders Decomposition Method Classification and Data-Set

Benders decomposition is an effective method for solving large-scale mixed-integer optimization problems with both discrete and continuous variables, typically structured in blocks or hierarchies in Fig. 2. It decomposes the original problem into two interconnected components: a master problem, which determines key integer decisions, such as the configuration, location, and capacity of DESS; and sub-problems, which evaluate the feasibility and operating costs of these decisions under continuous operational variables. Through an iterative interaction between these two hierarchical layers, the algorithm generates cut-points and progressively refines the master problem until convergence, effectively reducing computational complexity and enhancing scalability for large-scale distribution networks. These improvements enhance the method's adaptability and efficiency, making it particularly suitable for large-scale renewable energy grid-connected systems. The improved algorithm proceeds iteratively: the main problem first determines the layout and size of energy storage units, aiming to minimize total operating costs while maintaining network viability. Sub-problems then verify this configuration through power flow analysis and checking voltage and power balance constraints. Based on the results of the sub-problems, new cut points are added to the main problem to guide subsequent iterations. This process continues until the objective value stabilizes and no further improvement is observed.

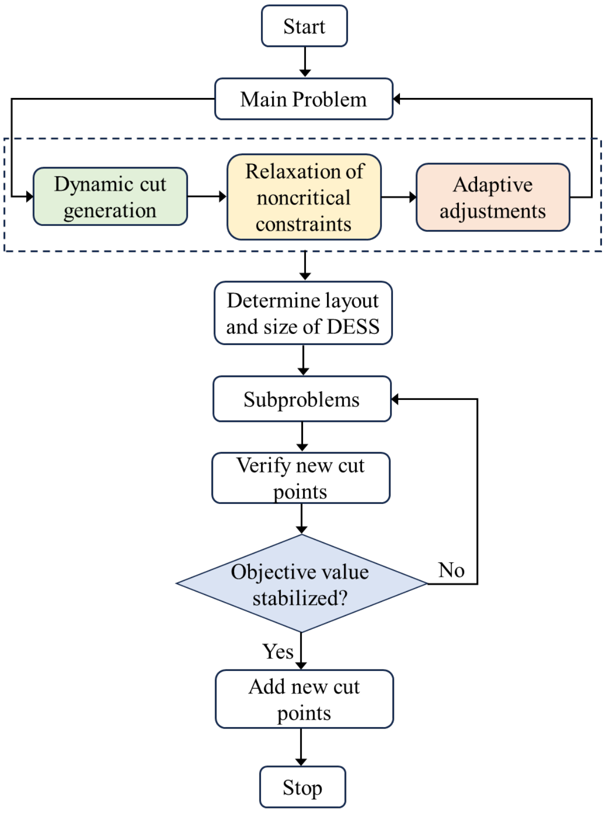

Fig. 2. Flowchart of the improved Benders decomposition method for distributed energy storage optimization

3.1 Overview of Benders Decomposition

Benders decomposition is an effective method for solving large-scale optimization problems that involve both continuous and integer variables, often in block-structured formats [13]. The general structure of such a problem can be expressed as:

$$\min_{x,y} f(x,y) = c^T x + d^T y \tag{7}$$

$$s.t. Ax + By \geq b, x \in X \subseteq Z^n, y \in Y \subseteq R^m \tag{8}$$

Where X represents the integer decision variables, and Y denotes continuous operational variables. In Benders decomposition, the original problem is divided into two interconnected parts: Master problem: determines the configuration of decision variables X. Sub-problem: evaluates system feasibility and operational costs given X, using continuous variables y. Mathematically, the master problem can be written as:

$$\min_{x,\theta} Z_{master} = C^T x + \theta \tag{9}$$

$$s.t.x \in X, \theta \in R, \theta \geq \alpha_k + \beta_k^T (x - x^{(k)}) \tag{10}$$

Where θ is an auxiliary variable representing the sub-problem's optimal value, and each term corresponds to a Benders cut generated from a previous iteration k. The sub problem is defined as:

$$Z_{sub}(x^{(k)}) = min d^T y \tag{11}$$

$$s.t.By \geq b - Ax^{(k)}, y \in Y \tag{12}$$

If the sub-problem is feasible, its dual solution generates an optimal cut-point; otherwise, a feasibility cut-point is created to enforce the network constraints [14]. The iterative exchange of cut-points between the MP and SP continues until convergence, at which point no new cut-points improve the primary objective. This iterative framework significantly reduces the computational burden by concentrating integer decisions on the primary problem and continuous decisions on the sub-problems, making it particularly efficient for large-scale distribution networks.

3.2 Proposed Improvements

To enhance computational performance, several improvements are introduced to the traditional Benders decomposition algorithm [15] in Table 1.

Table 1. Proposed improvements

Improvement	Description
Dynamic Cutting Strategy	Replaces static cutting in traditional methods
Relaxation Techniques	Relaxing non-critical constraints reduces the complexity of sub-problems
Tailored Algorithm	The algorithm is adjusted according to the specific characteristics of the distribution network.
Increased Efficiency for Large Networks	The proposed improvements improve the efficiency of the algorithm

First, a dynamic cutting strategy is adopted, replacing the traditional static cut addition rule. Instead of adding all cuts in each iteration, cuts are dynamically selected based on their marginal contribution to the master problem's convergence. The cut selection criterion can be expressed as:

$$\Delta\theta_k = \theta_k - \theta_{k-1} < \varepsilon_{cut} \tag{13}$$

Where ε_{cut} is a convergence threshold that determines whether the new cut significantly affects the master problem. Second, relaxation techniques are introduced to simplify sub-problem computations. For instance, non-critical operational constraints are temporarily relaxed as:

$$V_i^{min} - \delta_v \leq V_i(t) \leq V_i^{max} + \delta_v \tag{14}$$

Where δ_v represents a small relaxation margin. This reduces the feasible region complexity, thereby accelerating subproblem convergence without sacrificing optimality. Third, the tailored algorithm integrates system-specific parameters, storage-to-load ratio, and node degree to adjust the decomposition depth and convergence conditions:

$$\varepsilon_{tol} = f(\rho_R, \rho_S, d_i) \tag{15}$$

These refinements jointly improve computational stability, making the method scalable for large, renewable-integrated distribution networks.

3.3 Algorithm Steps

The modified Benders decomposition method follows a structured approach to solve the optimization problem. Initially, the main problem is set up, focusing on determining the layout and size of the distributed energy storage system. The objective typically aims to minimize operating costs while ensuring voltage stability and meeting network constraints. Decision variables in the main problem include the location and capacity of each storage unit. Next, the sub-problem is solved based on the configuration provided by the main problem, involving power flow equations to check the feasibility of energy distribution. The solution of the sub-problem offers crucial information on power flow and voltage levels across the network. Afterward, cuts are generated from the submersible's solution and added to the main problem. These cuts represent feasibility constraints that help improve the main problem's solution in future iterations. The process repeats, with the main and sub-problems solved alternately. In each iteration, the main problem updates the storage configuration based on the sub-problem's feedback, while the sub-problem refines power flow calculations according to the new configuration. The algorithm terminates when the solution converges, meaning no further improvements in the objective function or cuts. This iterative process ensures that the algorithm minimizes operating costs while maintaining grid reliability. The master problem might be formulated as:

$$\text{Minimize}\, Z_{master} = \sum_k c_k x_k + \sum_t \sum_i P_{storage,t} \cdot \Delta P_{\cos t,i} \tag{16}$$

Where c_k represents the cost of storage devices, x_k the decision variables for their placement and size, $P_{storage,t}$ the power flow through storage devices, and $\Delta P_{cost,\, i}$ the cost associated with storage usage. The sub-problem, on the other hand, solves the power flow equations while maintaining system balance:

$$\text{Minimize}\ Z_{subproblem} = \sum_i P_{load,i} - P_{renewable,i} - P_{storage,i} \tag{17}$$

Subject to the power balance and voltage constraints:

$$\sum_i P_{flow,i} = 0, V_{\min} \le V_i \le V_{\max} \tag{18}$$

Where $P_{load,i}$ and $P_{renewable,i}$ are the loads and renewable generation at node i, and $P_{storage,i}$ represents the discharge from storage. The iterative process between the master and sub-problem, combined with dynamic cutting and relaxation techniques, leads to the optimal configuration of distributed energy storage that minimizes operational costs and ensures grid stability.

4 Case Studies

To evaluate the effectiveness of the proposed optimization algorithm, two standard distribution network benchmarks were employed: the IEEE 33-node system and the IEEE 69-node system. The IEEE 33-node system, consisting of 33 nodes and 32 branches, represents a moderately complex network with typical loads, distributed energy resources, and voltage control devices. The IEEE 69-node system, consisting of 69 nodes and 68 branches, provides a larger and more complex test case suitable for large-scale optimization analysis. Both systems employ standard parameters based on established test case data, including line impedance, load profile, and generation capacity. The IEEE 33-node system is used to evaluate the benchmark performance at a moderate renewable energy penetration, while the IEEE 69-node system incorporates a higher proportion of renewable energy generation to test the robustness of the algorithm under different integration scenarios. The experimental setup includes the modeling of a BESS with a charge and discharge efficiency of 90% and a capacity determined by the optimization results; and renewable energy sources modeled using time-series wind and solar power data. Solar power generation follows a diurnal pattern, peaking at midday, while wind power generation exhibits random hourly fluctuations based on real or statistically simulated weather data. The load profiles reflect typical hourly variations in residential and industrial electricity demand. All simulations were performed in MATLAB, using the Power System Toolbox (PST) for load flow calculations and the Optimization Toolbox for applying the Benders decomposition framework. The optimization process used the MATLAB built-in mixed-integer linear programming (MILP) solver, combined with a modified Benders decomposition algorithm, to ensure efficient and scalable problem solving across the two test networks.

4.1 Test Distribution Networks

To validate the effectiveness and robustness of the proposed optimization framework, this study employed two well-known benchmark distribution systems: the IEEE 33-node and IEEE 69-node test networks in Table 2.

Table 2. Summary of test distribution networks

Network	Nodes / Branches	Base voltage	Total load	Renewable penetration	Key features	Purpose in study
IEEE 33-node system	33 / 32	12.66	3.72 / 2.30	20–30% (PV + wind)	Medium-size radial system; includes OLTCs and capacitor banks; time-series PV/wind data	Validation of optimization performance under moderate complexity and DER integration
IEEE 69-node system	69 / 68	12.66	3.80 / 2.70	50–70% (PV + wind)	Large-scale, deep radial topology; higher voltage drop and power loss;	Scalability and robustness testing under high renewable penetration

These systems are widely used in power system research, providing standardized environments for evaluating optimization and control algorithms under various operating conditions. The IEEE 33-node distribution system represents a medium-sized radial network, commonly used for algorithm validation, voltage stability analysis, and distributed energy resource (DER) integration studies. It consists of 33 nodes and 32 distribution branches, with a baseline voltage of 12.66 kV and total active and reactive loads of approximately 3.72 MW and 2.3 Mvar, respectively. The system includes one substation as a slack node, while the remaining nodes are load points or distributed generation (DG) connection nodes. Each branch is characterized using standardized electrical parameters extracted from a commonly used benchmark data-set in the IEEE distribution system literature. For radial distribution systems, the branch power flow follows the classical DistFlow formulation:

$$P_{ij} = \sum_{k \in \Omega(j)} P_{jk} + P_j + r_{ij} \frac{P_{ij}^{\,2} + Q_{ij}^{\,2}}{V_i^2} \tag{19}$$

$$V_j^{\,2} = V_i^2 - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^{\,2} + x_{ij}^{\,2}) \frac{P_{ij}^{\,2} + Q_{ij}^{\,2}}{V_i^2} \tag{20}$$

This study considers an IEEE 33-node network with moderate renewable energy penetration, assuming photovoltaic (PV) and small-scale wind generation distributed at selected nodes. The renewable energy model is based on time-series generation data reflecting daily irradiance and wind speed variations. The network also incorporates voltage regulation devices such as on-load tap changers (OLTCs) and capacitor banks, enabling the evaluation of coordinated control strategies to maintain voltage stability while optimizing the configuration and operation of DESS. In contrast, an IEEE 69-node system was selected to test the scalability of the proposed algorithm in larger and more complex distribution networks. This system consists of 69 nodes and 68 branches, with a base voltage of 12.66 kV, a total load of approximately 3.8 MW, and a total reactive power of 2.7 Mvar. Compared to the 33-node system, this network has a longer radial structure and higher topological depth, resulting in greater voltage drop and power loss between distant nodes. These characteristics make it an ideal platform for evaluating the proposed method's ability to handle increasingly complex systems, multi-node coordination, and computational challenges. To quantify the voltage deviation highlighted in your description, the per-bus voltage deviation index can be written as:

$$\Delta V = \sum_{i=1}^{N} V_i - V_{ref} \tag{21}$$

In this configuration, higher renewable energy penetration levels, up to 50-70% of total system demand, are introduced and distributed across multiple buses equipped with photovoltaic and wind turbines. This scenario tests the algorithm's ability to manage the uncertainty, variability, and intermittency associated with renewable energy. DESS are optimally deployed to smooth renewable energy output fluctuations, mitigate voltage instability, and reduce network losses. The electrical parameters used in the two test systems, including branch impedances, load profiles, transformer tap settings, and distributed generation capacity, are derived from existing IEEE benchmark datasets and validated using the MATLAB/PST environment. The network is modeled as a balanced radial distribution system, and the nominal voltages of all buses are assumed to remain within acceptable operating limits. This study thoroughly evaluates the proposed improved Benders decomposition algorithm using two representative test networks: an IEEE 33-node network for medium-complexity verification and an IEEE 69-node network for scalability and robustness testing. This dual-system setup enables a detailed comparison of the algorithm's performance, convergence behavior, and computational efficiency under different renewable energy integration levels, demonstrating the method's adaptability to various real-world distribution network conditions.

4.2 Experimental Setup

This study is based on a repeatable simulation implementation, which clarifies the aspects such as energy storage configuration, renewable energy sources, load behavior, and optimization stack. All model inputs have been organized into reusable datasets and scripts, enabling each experiment on the IEEE 33-node and 69-node systems to be re-run under the same conditions, thus ensuring repeatability and facilitating sensitivity analysis.

Energy storage is modeled as a grid-connected battery energy storage system with rated power and energy; the cycle efficiency is fixed at 90% and is applied separately when updating the charging and discharging levels. The charging level is subject to strict constraints such as the boost rate limit and minimum soaking time to prevent rapid cycling. These operational restrictions simulate the actual BESS behavior and avoid aggressive charging-discharging patterns that may overestimate flexibility or ignore aging effects. The initial charging level is set at 50%, unless otherwise specified, providing a neutral starting point for upward and downward adjustments within the optimization period. Renewable energy data includes hourly wind and solar data for several days. The energy variation of solar power follows a clear daily variation curve formed by irradiance and temperature; the generation of wind energy is based on measured values or statistically synthesized through turbine power curves, and incorporates short-term fluctuations. In this way, the simulation can capture predictable daily patterns as well as random changes, which is crucial for evaluating how distributed energy systems alleviate the intermittency of renewable energy. The load exhibits typical characteristics of mixed residential and industrial loads, with peak electricity consumption in the morning and evening; seasonal and weekend scale adjustment coefficients can also be applied, and all demands are mapped to the bus according to standard IEEE standards. This mapping ensures consistency with widely used benchmark distribution models and allows for direct comparison with other studies using the same IEEE 33-node and 69-node systems. Power flow and grid constraints are applied in each step to ensure that each candidate solution can meet the node power balance, voltage limits, and line capacity constraints over the entire time range. All experiments are conducted in the MATLAB environment: the Power System Toolbox is used to perform AC and DC power flow calculations; the Optimization Toolbox is responsible for handling the main problem and sub-problems, which are solved in the Bayesian cycle. The main problem uses MATLAB's mixed integer programming solver to make layout and scale decisions; the sub-problems are solved using LP/QP as needed. This separation leverages the advantages of integer programming in discrete location/scale decision-making, while using faster continuous solvers to handle operation scheduling, thereby improving computational efficiency. Convergence is evaluated by the relative optimality gap and the feasibility tolerance corresponding to the solver's default settings. The relative gap tracks the distance between the current upper and lower bounds of the target value, while the feasibility tolerance ensures that violations of network and storage constraints remain within an acceptable numerical range. The results of the operation are analyzed to compare the original Bendis settings with the improved Bendis settings, focusing on indicators such as convergence speed, total operating cost, voltage curve quality, and storage usage patterns, thereby quantifying the benefits of the proposed improvement measures.

5 Result Analysis

Results demonstrate that the improved Benders decomposition method effectively optimizes the placement and capacity of distributed energy storage devices (ESDs) in IEEE 33- and 69-bus networks. In the 33-bus system, strategic placement of energy storage improves voltage stability and reduces operating costs. In the 69-bus network, the method effectively manages more complex topologies and higher renewable energy penetration. The relationship between energy storage capacity and node load demonstrates that storage configuration depends not only on load demand but also on voltage regulation and renewable energy integration requirements. Compared to genetic algorithms, the Benders method converges faster and has lower operating costs, demonstrating its computational efficiency and robustness. Results also demonstrate that voltage stability and power losses vary with renewable energy penetration: modern grids maintain higher stability and lower losses, while older grids face greater performance challenges.

5.1 Optimal Configuration of Energy Storage Devices for the IEEE 33-Bus System

Fig. 3 shows the results of optimizing the layout and size of ESDs in an IEEE 33-node system using the proposed Benders decomposition method. Energy storage devices are strategically allocated to specific nodes to maximize cost savings and enhance voltage stability across the network [16]. The installation of each ESD takes into account load levels, voltage sensitivity index, renewable energy distribution, and network topology to ensure that economic and operational goals are achieved. Energy storage capacity is allocated to nodes with high renewable energy penetration, or nodes with large voltage fluctuations, large power flows, and large energy storage capacity. These locations are often critical to maintaining system stability because they are key nodes for the exchange of active and reactive power. Installing higher capacity energy storage devices at these nodes enables the system to absorb excess renewable energy generation during off-peak hours and provide power during high-demand hours, effectively smoothing power fluctuations and minimizing node voltage deviations. This can improve power quality and support the reliable access of distributed generation (DG). Results show that the improved method converges faster than traditional algorithms, achieves more optimized energy storage resource configuration, and ensures that all bus voltage amplitudes remain within the permitted range. This performance demonstrates that the proposed algorithm can effectively handle multi variable coupling relationships and provides a practical and robust approach for integrating distributed energy storage systems into practical distribution systems. From the bar chart in Fig. 3, it can be seen that the distribution of storage capacity for each line is extremely uneven. This indicates that the optimization framework does not simply allocate capacity based on the load ratio, but takes into account factors such as voltage sensitivity, branch impedance, and the renewable energy injection at each node in response. Particularly, some lines with moderate or even relatively low load levels still have large storage units, highlighting their role as voltage vulnerable or topologically critical nodes. At these nodes, small disturbances can have a strong impact on the overall system performance. On the contrary, some lines with higher loads only have medium-sized storage capacity because the nearby resources can already support their local voltage conditions well. This shows that the algorithm has balanced economic investment and marginal grid support benefits. Overall, the spatial distribution pattern of storage scale confirms that the improved Baines decomposition method generates a coordinated configuration pattern: a few high-capacity hub nodes can absorb large fluctuations, while multiple smaller devices provide fine local regulation, jointly enhancing the resilience and operational flexibility of the IEEE 33-node distribution network.

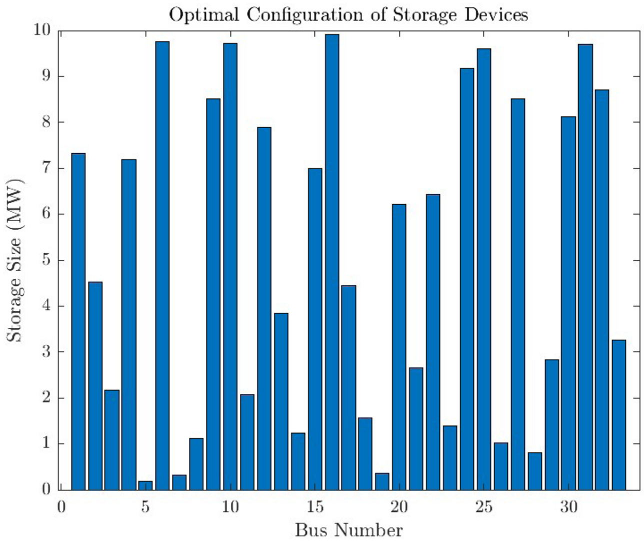

Fig. 3. Optimal configuration of energy storage devices for the IEEE 33-Bus system

5.2 Optimal Configuration of Energy Storage Devices for the IEEE 69-Bus System

Fig. 4 illustrates the optimal placement and sizing of ESDs in a more complex IEEE 69-bus distribution system, obtained using a modified Benders decomposition algorithm. The figure shows the spatial distribution of ESDs in the network, where the size of each storage unit is adjusted based on the local load characteristics, renewable energy generation level, and voltage sensitivity of its corresponding node. The results demonstrate that the optimization framework successfully identifies the most effective ESD placement strategy, ensuring economic efficiency and operational reliability under different renewable energy penetration scenarios. Large-scale energy storage is often deployed at nodes or locations with high renewable energy generation or unstable voltage, where power flow regulation is critical to maintaining network balance. Due to the intermittent output of PV or wind power, these nodes typically experience greater volatility, resulting in fluctuations in both active and reactive power. These smaller units are primarily used to enhance local voltage regulation, mitigate minor load fluctuations, and support reactive power compensation. Large and small ESDs are distributed throughout the network, forming a coordinated hierarchical control structure that allows flexible operation across different voltage levels and load areas. The optimization process performed using the Benders decomposition method ensures that the energy storage system minimizes operating costs while improving grid reliability [17]. By strategically locating storage units, the algorithm effectively addresses the challenges posed by large networks. This approach highlights the ability of the method to optimize energy storage locations in complex systems, demonstrating its practical applicability to real-world grids. A more detailed analysis of Fig. 4 reveals several new insights. Firstly, the storage allocation pattern of the 69 lines is much more different from that of the 33 lines, reflecting a deeper radial topology structure and higher voltage sensitivity of the longer feeder segments. Nodes located at the edge of long feeders tend to receive significantly larger storage units, indicating that this algorithm prioritizes strengthening those areas with weak power supply, as voltage deviations in these areas may spread rapidly. Meanwhile, nodes in the middle sections with moderate renewable energy injection usually receive medium-sized equipment, forming a multi-layer support structure to stabilize local and upstream voltages. Secondly, the existence of multiple high-capacity storage clusters indicates that the improved Baines decomposition algorithm can automatically identify “voltage bottlenecks” and “hotspots of renewable energy fluctuations”, and allocate capacity to break these bottlenecks and make the voltage distribution in the network more gentle. Thirdly, distributing small units on multiple buses indicates that this algorithm has the ability to implement fine regulation, reducing branch flow and alleviating the reverse power problem caused by high photovoltaic penetration. This combination of large hub-level equipment and small distributed equipment proves that this optimization strategy not only reduces costs but also enhances the system’s resilience, ensuring that the network remains stable, well-regulated, and operationally flexible even in a system with 69 nodes and high renewable energy volatility.

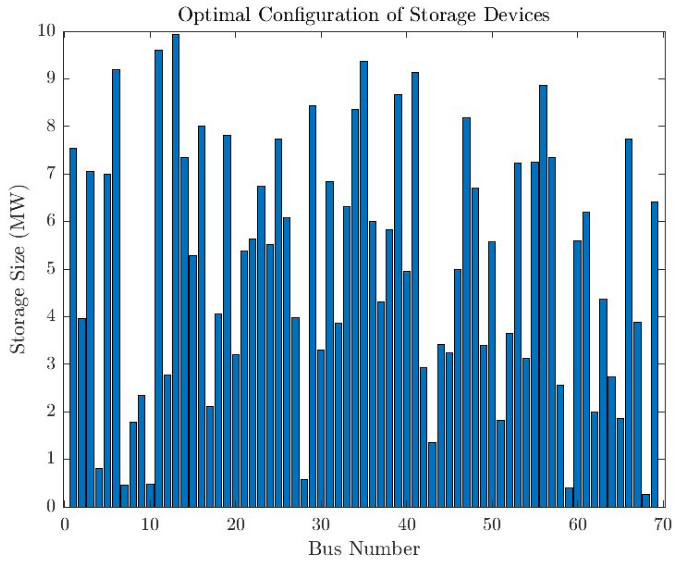

Fig. 4. Optimal configuration of energy storage devices for the IEEE 69-Bus system

5.3 Energy Storage Sizing vs. Load at Each Bus

Fig. 5 shows a scatter plot of the energy storage capacity and load demand at each node in the IEEE 33-node distribution network. The results show that although the general trend is that nodes with higher loads tend to be equipped with larger energy storage capacity, the correlation between the two variables is not strictly linear. This observation suggests that the optimization framework determines the energy storage capacity based on multiple interrelated factors rather than relying solely on the load size. Specifically, the improved Benders decomposition method not only considers the active and reactive power requirements of each node, but also considers key operational indicators such as voltage stability, power loss sensitivity, and renewable energy generation volatility. Therefore, the energy storage capacity configuration of the entire network reflects a holistic optimization strategy aimed at balancing economy and system reliability. Some medium and high load nodes are assigned disproportionately large energy storage capacities, which can be attributed to their role as voltage-sensitive nodes or key points for renewable energy access [18]. These nodes usually act as local hubs, and fluctuations in generation and demand can significantly affect the overall voltage curve. Therefore, assigning larger energy storage capacity to these nodes can help stabilize voltage, reduce line congestion, and improve local power quality. In contrast, the observed that some low-load buses had relatively large energy storage units, indicating that the optimization process prioritized overall network performance over local load size. These units' functions included smoothing large fluctuations in renewable energy output, supporting reactive power compensation, and preventing voltage sags in remote feeder segments. In addition, Fig. 5 also reveals several other characteristics of the system's energy storage allocation behavior. Firstly, the scattered distribution of points, especially those clusters combining medium loads with small-capacity or large-capacity storage units, indicates that the improved Bayes algorithm can dynamically adapt to the sensitivity of the local power grid rather than following simple load-capacity monotonic rules. This suggests that even if the active power demands of these nodes are not extreme, those nodes with high-voltage reactive coupling or branch congestion will be allocated larger storage capacities. Secondly, several high-load nodes have only moderate storage capacities, indicating that their electrical positions in the feeder inherently provide stability, reducing the need for additional storage enhancement. Thirdly, the shape of the scatter cloud implies that this is a multi-objective optimization surface rather than a single dominant factor, further indicating that the storage allocation is generated through a balance among renewable energy smoothing, loss minimization, and grid support goals.

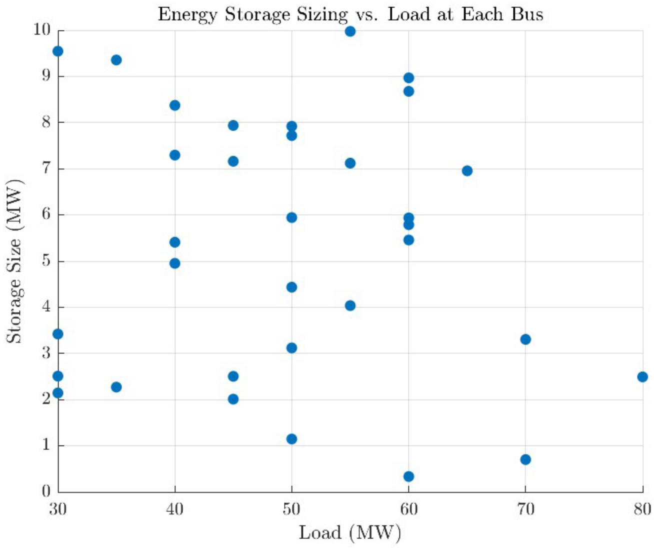

Fig. 5. Energy storage sizing vs. load at each bus

5.4 Operating Cost Comparison Over Iterations

Fig. 6 compares the runtime costs of the Benders decomposition method and a genetic algorithm (GA) over 10 iterations. As shown, the Benders decomposition curve (blue line) exhibits a rapid and stable convergence pattern, with the runtime cost decreasing sharply during the initial iterations, reaching approximately $900 by the 10th iteration. This behavior highlights the computational efficiency and robustness of the Benders-based optimization framework for solving large-scale, mixed-integer, and nonlinear distribution network problems. This method effectively decomposes the main problem into sub-problems, enabling parallel computation and iterative refinement, accelerating convergence and maintaining numerical stability. In contrast, the runtime cost of the genetic algorithm curve (red line) gradually decreases, stabilizing at approximately $1010 after 10 iterations. This slower convergence can be attributed to the stochastic and population-based nature of the GA, where random mutation and crossover operations require a large number of evaluations to effectively explore the solution space. Although genetic algorithms offer global search capabilities and greater flexibility in non-convex optimization scenarios, they typically require longer computation times and more resources to obtain stable solutions compared to deterministic and structure-based Benders methods. The comparison results clearly show that Benders decomposition is superior in terms of convergence speed and computational efficiency, making it particularly suitable for solving optimization problems with strict time constraints or limited computational resources. Its deterministic structure allows for systematic exploration of the feasible region and guarantees convergence to the optimal or near-optimal solution within a limited number of iterations. On the other hand, genetic algorithms remain valuable for highly nonlinear, multi modal, or discontinuous optimization problems, while traditional decomposition methods may face difficulties due to complex constraint coupling or non-differentiable objective functions. Its exploratory search mechanism and ability to escape from local optima make it a strong candidate for exploratory or hybrid optimization strategies [19].

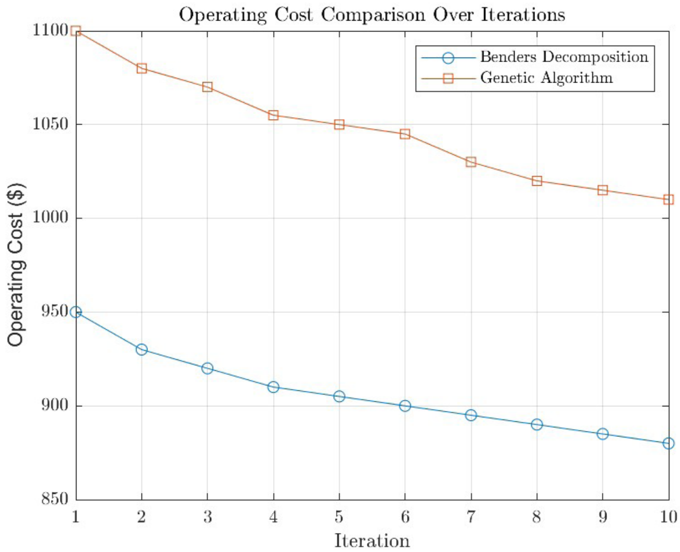

Fig. 6. Operating cost comparison over iterations

5.5 Voltage Stability vs. Renewable Energy Penetration

Fig. 7 shows how grid strength, control quality, and reactive power supply affect voltage stability as renewable energy penetration increases. The blue curve represents a modern grid with high renewable energy penetration, where voltage stability remains nearly flat even at near-100% penetration. This is due to: grid-connected inverters regulating voltage and frequency to the actual source; ample and rapid reactive power support provided by STATCOMs, SVCs, and the inverter's Volt VAR function; higher short-circuit robustness provided by synchronous condensers; and a stronger and more tightly meshed grid topology. Wide-area adaptive control protection and coordinated tap switching prevent tap chasing. This results in improved source voltage margin and reduced reactive voltage sensitivity, enabling rapid correction of voltage dips caused by clouds or gusty winds, requiring power curtailment or energy storage only in extreme cases. The green curve represents a traditional hybrid grid with moderate renewable energy penetration, where the voltage decline becomes more pronounced with increasing penetration. Control is primarily provided by grid-tracking inverters and traditional automatic voltage regulators. Reactive power flexibility is uneven, with feeders farther from the main grid experiencing weaker power. As the non-synchronous share increases, inertia and fault currents slow recovery, pushing the operating point closer to the peak supply voltage. Carrying capacity limitations manifest as sharp inflection points. Local STATCOM synchronous capacitors are often required to optimize voltage and reactive power set points or limited curtailment to maintain stability. The red curve represents an aging grid with fewer renewable energy sources, which experiences the fastest decline. Short-circuit strength is low, dynamic reactive power devices are scarce, and inverter coordination is minimal. Voltage regulation relies on slow mechanical taps and switched capacitors, so rapid rises or faults can lead to deeper voltage sags and tap oscillations. Weak feeders exhibit high reactive voltage sensitivity, and even moderate penetration can push operations to the brink of collapse. Required remedial measures are structural, such as rewiring and looping to strengthen synchronous capacitors or flywheel placement lines, forced grid-forming mode for new inverter banks, and feeder-level voltage and reactive power optimization using PMU or micro-PMU feedback.

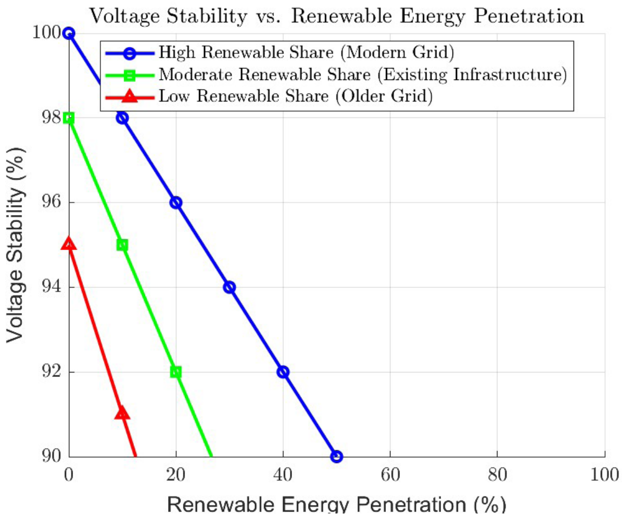

Fig. 7. Voltage stability vs. renewable energy penetration

5.6 Power Losses vs. Renewable Energy Penetration

Fig. 8 depicts the changing trends in network power losses as renewable energy penetration increases, and the reasons for the differences in trends across different grid archetypes. In traditional systems with low renewable energy penetration, losses rise sharply once distributed generation exceeds local demand. Long to medium-sized radial feeders typically experience reverse currents and higher branch currents, which amplify I-squared R losses. Voltage excursions trigger greater reactive power exchange between feeders and substations, leading to tap-changer oscillations to maintain set points and sub-optimal capacitor switching, all of which increase copper losses and control-related losses. Transformers and lines often operate away from their optimal design points, resulting in poor phase balance and elevated neutral currents. Poorly coordinated inverter banks generate additional harmonic content and non-unity power factors, further increasing effective current amplitudes. Due to scarce reactive resources and slow control, variations in cloud cover and wind gusts force the grid to repeatedly take corrective actions, exacerbating power losses throughout the day. Grids with moderate renewable energy penetration and gradual upgrading exhibit a more gradual slope. Some feeders equipped with local reactive power devices, improved capacitor placement, and improved voltage profiles have resulted in slower current growth for the same amount of renewable energy injected. Carrying capacity studies have informed feeder reconfiguration and daytime set-point adjustments, reducing unnecessary reactive power cycling. However, partial gridization and uneven controller deployment have left some weak links, leading to congestion and brownouts during peak hours. As a result, losses continue to grow, albeit less dramatically than in conventional scenarios. Modern grids with high renewable energy penetration and advanced control technologies minimize losses. Smart inverters operate using coordinated voltage-var and voltage-watt functions, maintaining a near-unity power factor or providing local support, keeping reactive power flows localized rather than propagating upstream. Synchronous condensers or virtual synchronization modes enhance strategic bus bars, limiting voltage differences and keeping branch currents low. Feeder topologies are becoming more gridded, phase balance is being actively managed, and transformer loading is being brought closer to the optimal range through dynamic reconfiguration, dynamic line rating, and energy storage technologies, resulting in smoother changes. Wide-area optimal dispatch set points minimize the sum of the squares of current and resistance while meeting voltage and thermal constraints. Dispatch uses predictive forecasts, making corrective actions proactive rather than reactive. Therefore, additional renewable energy capacity does not translate linearly into additional current, and the incremental losses remain small for each 1% increase in renewable energy penetration. The comparative information in Fig. 7 is useful. The main reasons for the increase in losses are volatility and insufficient local reactive power support. Modern technologies reduce both of these effects by localizing support, strengthening weak nodes, and optimizing power flows in time and space. The most effective measures to move the yellow or orange curve to the blue curve include: deploying reactive resources near renewable energy clusters, enforcing the use of smart inverters with coordinated droop, re-configuring feeders to shorten reverse paths, balancing phases, improving transformer efficiency, and limiting energy storage to suppress voltage slope. When these measures are prioritized on the most sensitive feeders identified by load capacity analysis, the grid can accommodate a higher share of renewable energy with only a slight increase in losses, confirming that the impact of efficient integration on losses is more important than the impact of the share of individual renewable energy sources [20].

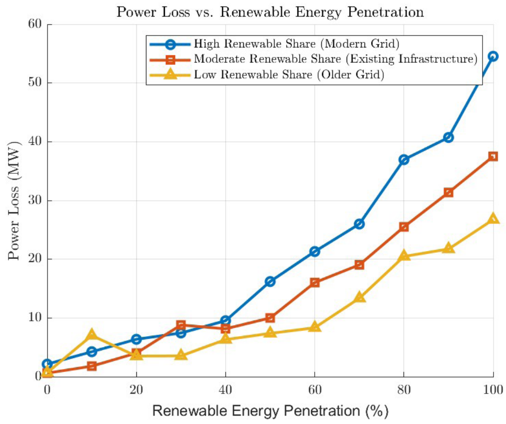

Fig. 8. Power losses vs. renewable energy penetration

5.7 Different Levels of Renewable Energy Penetration

Based on the convergence behavior of the modified Benders decomposition method at different renewable energy penetration levels in Fig. 9, the team can conclude that the algorithm shows effective convergence at all levels of renewable energy integration. As observed, the objective function steadily decreases with each iteration, indicating that the algorithm successfully finds better solutions as it progresses. The convergence seems to stabilize within a relatively small number of iterations, which shows that the modified Benders decomposition method is not only fast but also reliable in providing the best solution for different renewable energy penetration levels. Interestingly, despite the different levels of renewable energy penetration, the algorithm shows consistent performance without large fluctuations in the convergence speed, which means that the algorithm is robust to changes in renewable energy availability. Higher renewable energy penetration can reduce operating costs, which is beneficial for optimizing the power system for sustainability [21]. This means that the method is particularly effective in large-scale optimization problems involving renewable energy integration, and its solution is both fast and accurate.

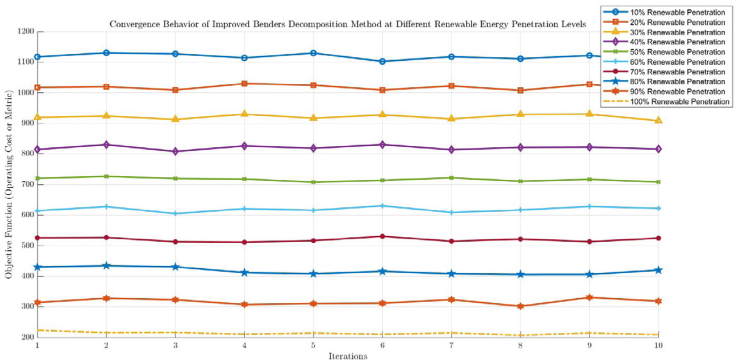

Fig. 9. Different levels of renewable energy penetration

6 Conclusion

In summary, this study proposes an improved Benders decomposition algorithm for the optimal placement of distributed energy storage systems (DESS) in active distribution networks. By integrating an adaptive cutting plane strategy, enhanced relaxation techniques, and complex master-sub problem coordination, this approach successfully addresses challenges often encountered by traditional Benders decomposition algorithms in large-scale nonlinear optimization applications, such as slow convergence, heavy computational burden, and unstable solutions. Comprehensive case studies validate that this framework not only reduces total operating costs, improves grid stability and energy efficiency, but also achieves significant computational speedup, enabling real-time or near-real-time decision-making. Results further demonstrate that this approach maintains high solution accuracy while increasing renewable energy penetration, and achieves even greater cost savings through improved flexibility and optimized storage scheduling. These findings highlight the robust scalability and engineering applicability of this approach, providing a reliable tool for the integrated planning and operation of future low-carbon power systems. Future work will expand the model to incorporate multiple uncertainties, such as renewable generation, load variations, and market dynamics, and develop stochastic and robust algorithm variants to enhance its adaptability to highly variable grid conditions. Therefore, this study not only provides a methodological breakthrough but also lays a practical foundation for accelerating the deployment of intelligent, optimization-driven energy storage planning in renewable energy-rich power systems.

References

[1] X.-D. Liang, Emerging Power Quality Challenges Due to Integration of Renewable Energy Sources, IEEE Transactions on Industry Applications 53(2)(2017) 855-866.

[2] H.-B. Sun, Q.-L. Guo, J.-J. Qi, V. Ajjarapu, R. Bravo, J. Chow, Z.-S. Li, R. Moghe, E. Nasr-Azadani, U. Tamrakar, G.N. Taranto, R. Tonkoski, G. Valverde, Q.-W. Wu, G.-Y. Yang, Review of Challenges and Research Opportunities for Voltage Control in Smart Grids, IEEE Transactions on Power Systems 34(4)(2019) 2790-2801.

[3] M. Mahdavi, H.H. Alhelou, A. Bagheri, S.Z. Djokic, R.A.V. Ramos, A Comprehensive Review of Metaheuristic Methods for the Reconfiguration of Electric Power Distribution Systems and Comparison with A Novel Approach Based on Efficient Genetic Algorithm, IEEE Access 9(2021) 122872-122906.

[4] N. Kagan, R.N. Adams, A Benders' Decomposition Approach to the Multi-objective Distribution Planning Problem, International Journal of Electrical Power & Energy Systems 15(5)(1993) 259-271.

[5] A. Ibrahim, O.A. Dobre, T.M.N. Ngatched, A.G. Armada, Bender's Decomposition for Optimization Design Problems in Communication Networks, IEEE Network 34(3)(2020) 232-239.

[6] S.F. Santos, M. Gough, D.Z. Fitiwi, J. Pogeira, M. Shafie-khah, J.P.S. Catalão, Dynamic Distribution System Reconfiguration Considering Distributed Renewable Energy Sources and Energy Storage Systems, IEEE Systems Journal 16(3)(2022) 3723-3733.

[7] A. Sharma, B.S. Rajpurohit, S.N. Singh, A review on economics of power quality: Impact, assessment and mitigation, Renewable and Sustainable Energy Reviews 88(2018) 363-372.

[8] R. AhmadiAhangar, F. Plaum, T. Haring, I. Drovtar, T. Korotko, A. Rosin, Impacts of grid-scale battery systems on power system operation: case of Baltic region, IET Smart Grid 7(2)(2024) 101-119.

[9] M. Caramanis, E. Ntakou, W.W. Hogan, A. Chakrabortty, J. Schoene, Co-Optimization of Power and Reserves in Dynamic T&D Power Markets With Nondispatchable Renewable Generation and Distributed Energy Resources, Proceedings of the IEEE 104(4)(2016) 807-836.

[10] A. Babiker, S.S. Ahmad, I. Ahmed, M. Khalid, M.A. Abido, F.S. Al-Ismail, Optimal Power Flow: A Review of State-of-the-Art Techniques and Future Perspectives, IEEE Access 13(2025) 60012-60039.

[11] G.J. Peponis, M.P. Papadopulos, N.D. Hatziargyriou, Optimal operation of distribution networks, IEEE Transactions on Power Systems 11(1)(1996) 59-67.

[12] L.-P. Wang, Q.-F. Zhang, A.-M. Zhou, M.-G. Gong, L.-C. Jiao, Constrained Subproblems in a Decomposition-Based Multiobjective Evolutionary Algorithm, IEEE Transactions on Evolutionary Computation 20(3)(2016) 475-480.

[13] R. Lusby, L.F. Muller, B. Petersen, A solution approach based on Benders decomposition for the preventive maintenance scheduling problem of a stochastic large-scale energy system, Journal of Scheduling 16(2013) 605-628.

[14] A. Kargarian, J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, R. Baldick, Toward Distributed/ Decentralized DC Optimal Power Flow Implementation in Future Electric Power Systems, IEEE Transactions on Smart Grid 9(4)(2018) 2574-2594.

[15] S.-Z. Zhao, P.N. Suganthan, S. Das, Self-adaptive differential evolution with multi-trajectory search for large-scale optimization, Soft Computing 15(2011) 2175-2185.

[16] K. Kasturi, C.K. Nayak, S. Patnaik, M.R. Nayak, Strategic integration of photovoltaic, battery energy storage and switchable capacitor for multi-objective optimization of low voltage electricity grid: Assessing grid benefits, Energy Reports Focus 41(2022) 104-117.
[17] D.-P. Chen, Z.-X. Jing, H.-J. Tan, Optimal Siting and Sizing of Used Battery Energy Storage Based on Accelerating Benders Decomposition, IEEE Access 7(2019) 42993-43003.
[18] S.A. Khajehoddin, M. Karimi-Ghartemani, P.K. Jain, A. Bakhshai, DC-Bus Design and Control for a Single-Phase Grid-Connected Renewable Converter with a Small Energy Storage Component, IEEE Transactions on Power Electronics 28(7)(2013) 3245-3254.
[19] M. Alimohammadi, J. Behnamian, Generalized benders decomposition approach for designing a reverse logistics network for unused drugs within a circular economy framework, RAIRO-Operations Research 59(5)(2025) 2633-2656.
[20] A. Ipakchi, F. Albuyeh, Grid of the future, IEEE Power and Energy Magazine 7(2)(2009) 52-62.
[21] A.A. Bazmi, G. Zahedi, Sustainable energy systems: Role of optimization modeling techniques in power generation and supply-A review, Renewable and Sustainable Energy Reviews 15(8)(2011) 3480-3500.